

# Effects of Non-Hermitian Bilayer Medium on the Second-Order Coherence of the Transmitted Coherent and Number States

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Regular paper: Received: Mar. 19, 2022, Revised: May. 4, 2022, Accepted: May. 7, 2022,  
Available Online: May. 9, 2022, DOI: 10.52547/ijop.15.2.219

**ABSTRACT**— We investigate the propagation of the normal two-photon number state and coherent state of light through a dispersive non-Hermitian bilayer structure composed of gain and loss layers, particularly at a discrete set of frequencies for which this structure holds PT-symmetric. We reveal how dispersion and gain/loss-induced noises in such a bilayer structure affect the antibunching property of the incident light. For this purpose, we have calculated the second-order coherence of the output state of the bilayer. Varying the loss layer coefficient, we show that the antibunching property of the incident light only retains to some extent, for small values of loss coefficient for the transmitted number state.

**KEYWORDS:** Antibunching, Coherent state, Non-Hermitian, Number state, Parity-time (PT)-symmetric, Second-order coherence.

## I. INTRODUCTION

Parity-time (PT-) symmetric systems are non-Hermitian but can exhibit entirely real spectra as long as they respect the conditions of PT-symmetry [1], [2]. A PT-symmetric Hamiltonian is invariant under the combination of the parity operator,  $\hat{P}$ , — i.e.,  $\mathbf{p} \rightarrow -\mathbf{p}$  and  $\mathbf{r} \rightarrow -\mathbf{r}$  — and the anti-linear time-reversal operator,  $\hat{T}$ , — i.e.,  $\mathbf{p} \rightarrow -\mathbf{p}$  and  $\mathbf{r} \rightarrow -\mathbf{r}$  and  $i \rightarrow -i$  — implying the satisfaction of the condition  $V(\mathbf{r})=V^*(-\mathbf{r})$  for the complex

quantum potential,  $V(\mathbf{r})$  should satisfy [2]. In other words, the real (imaginary) part of the potential, is an even (odd) function of position  $\mathbf{r}$ . Although complex quantum potentials do not exist in nature [2], their analogs have been realized in optical systems owing to the formal equivalence between the time-dependent Schrödinger equation and the optical paraxial wave equation. In this equivalence, an artificially made refractive index,  $n(\mathbf{r})$ , plays the role of the potential, particularly in multilayered photonic metamaterials with balanced gain and loss, satisfying  $\text{Re}\{n(\mathbf{r})\}=\text{Re}\{n(-\mathbf{r})\}$  and  $\text{Im}\{n(\mathbf{r})\}=-\text{Im}\{n(-\mathbf{r})\}$ . Such metamaterials render non-Hermitian systems with real eigenvalues [3]-[5]. The two latter relations, together, solely represent the necessary condition for the so-called *exact phase regime*. Nonetheless, beyond a critical value of gain/loss strength (i.e., the so-called *exceptional point*), the system eigenvalues become complex, in which case the system is in a broken PT-symmetric phase [1], [2].

The effects linked with PT-symmetric systems have been investigated comprehensively in classical optics during the past decade [6]-[10]. It has been revealed that these media can exhibit exotic features, like optical switching [6], nonreciprocal propagation [7], reflectionless

unidirectional transmission [8], and optical isolation [9].

For the incident light of a nonclassical nature, there are some features such as quadrature squeezing, photon statistics, and second-order coherence that can only be described in the framework of the full quantum theory. Over the past decade, only a few research groups have focused on the nonclassical effects of the propagation of the optical pulses through structures that hold PT-symmetry [11]-[17]. In our recent work [16], we have extensively studied the behavior of obliquely incident *s*- and *p*-polarized quantum states after transmitting through dispersive non-Hermitian multilayered structure, particularly at discrete frequencies that the medium holds PT-symmetry. We have investigated to see to what extent the transmitted light could retain its original nonclassical features, like the squeezing and sub-Poissonian photon statistics. Our findings show one cannot implement PT-symmetry at any arbitrary angle of squeezed states of incidence for either polarization in the quantum optics domain as far as the squeezing feature of outgoing light is concerned. Although this situation is changed if one only probes the sub-Poissonian photon statistics of outgoing light, it seems the structure whose incidence frequency is far from the emission frequency of the gain layer. Here, we focus on the second-order coherence of the normally transmitted coherent and *M*-photon number states through a dispersive non-Hermitian bilayer medium, which holds PT-symmetric at a particular frequency. Besides, we study the effects of the dispersion and the loss(gain)-induced noises on the antibunching property of incident light for various loss coefficients to study the coherence modifications with time delays between two temporally separated intensity signals with the time difference from one input.

## II. METHOD

### A. Bilayer Structure

We consider a bilayer structure that is composed of two gain and loss slabs of identical thickness *l* (and infinite extent along the *x* and *y*

directions) paired along  $-l < z < 0$  and  $0 < z < l$ , respectively. The bilayer is embedded in a vacuum for  $|z| > l$  (Fig. 1). Here, the complex permittivity of the gain/loss (*g/l*) slabs can be written as [18], [19],

$$\varepsilon_{g/l}(\omega) = \varepsilon_{0g/l} - \frac{\alpha_{g/l} \omega_{0g/l} \gamma_{g/l}}{\omega^2 - \omega_{0g/l}^2 + i\omega \gamma_{g/l}}. \quad (1)$$

where  $\varepsilon_0$  represents the medium background permittivity,  $\omega_0$  is the emission frequency,  $\gamma_{g/l}$  indicates the gain/absorption linewidth, and  $\alpha_{g/l}$  is the gain/absorption coefficient. Due to the causality principle, the loss slab parameters satisfy  $\alpha_l > 0$  and  $\gamma_l > 0$ , while those of the gain slab satisfy  $\alpha_g < 0$  and  $\gamma_g > 0$ . To guarantee the structure to be PT-symmetric, we require an exact balance between the gain and loss of two slabs as follows:

$$\text{Re } \varepsilon_g(\omega) = \text{Re } \varepsilon_l(\omega) \text{ and } \text{Im } \varepsilon_g(\omega) = -\text{Im } \varepsilon_l(\omega). \quad (2)$$

which can be satisfied only for a discrete set of real frequencies [20].

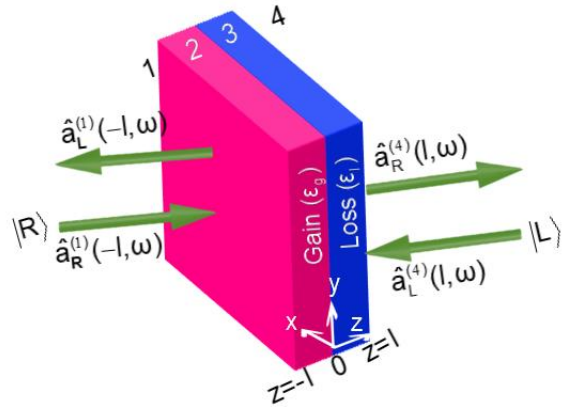


Fig. 1. A 3D representation of a non-Hermitian bilayer structure consists of gain and loss slabs with an identical thickness of *l* along the *z*-direction. The arrows normal to the *x*-*y* plane represent the bosonic operators of the input and output modes.

### B. The Exact Multilayer Theory

Consider an optical beam of light normally incident upon the bilayer. According to the canonical quantization of the electromagnetic field in the presence of a medium, the positive frequency component of the electric field operator is [21]:

$$\hat{E}_+^{(j)}(z, t) = i \int_0^\infty d\omega e^{-i\omega t} \left\{ \sqrt{\hbar\omega/4\pi\epsilon_0 c A} \times \right. \\ \left. \times \left( \hat{a}_R^{(j)}(z, \omega) e^{i\omega l/c} + \hat{a}_L^{(j)}(z, \omega) e^{-i\omega l/c} \right) \right\}, \quad (3)$$

where  $\hbar$ ,  $\epsilon_0$ ,  $A$ , and  $c$  are the reduced Planck's constant, the vacuum permittivity, the area of quantization in the  $x$ - $y$  plane, and free space light velocity, and  $R(L)$  denotes the right (left) going propagation wave. The negative frequency component of the electric field operator is obtained by taking the Hermitian adjoint of (2) — i.e.,  $\hat{E}_\sigma^{- (j)}(z, t) = \hat{E}_\sigma^{+ (j)\dagger}(z, t)$ .

An explicit characterization of the structure can be obtained by assigning the input-output mapping that links the output bosonic annihilation operators  $\hat{a}_L^{(1)}(-l, \omega)$  and  $\hat{a}_R^{(4)}(l, \omega)$  with their input operators  $\hat{a}_R^{(1)}(-l, \omega)$  and  $\hat{a}_L^{(4)}(l, \omega)$ , and also the noise operators,  $\hat{F}_{R(L)}(\omega)$ , as follows [22]-[23],

$$\begin{pmatrix} \hat{a}_L^{(1)}(-l, \omega) \\ \hat{a}_R^{(4)}(+l, \omega) \end{pmatrix} = \mathbb{S} \begin{pmatrix} \hat{a}_R^{(1)}(-l, \omega) \\ \hat{a}_L^{(4)}(+l, \omega) \end{pmatrix} + \begin{pmatrix} \hat{F}_L(\omega) \\ \hat{F}_R(\omega) \end{pmatrix}, \quad (4a)$$

$$\text{where } \mathbb{S} \equiv \begin{pmatrix} r_L & t \\ t & r_R \end{pmatrix}, \quad (4b)$$

Moreover, the multiple transmissions  $t$  and right(left) reflections,  $r_{R(L)}$  of the incident light through/from the bilayer binaries at  $-l$  and  $l$  are described by the same scattering matrix  $\mathbb{S}_\sigma$  analogous to the classical optics. Albeit, the quantum noise  $\hat{F}_{R(L)}$  originating from all layers with either gain or loss has no classical analogous. The optical input operators satisfy the bosonic commutation relations:

$$\left[ \hat{a}_R^{(1)}(\omega), \hat{a}_R^{(1)\dagger}(\omega') \right] = \left[ \hat{a}_L^{(4)}(\omega), \hat{a}_L^{(4)\dagger}(\omega') \right] \\ = \delta(\omega - \omega'). \quad (5)$$

substituting Eq. (4a) into (5) results in a similar bosonic commutation relation for the outgoing operators:

$$\left[ \hat{a}_R^{(4)}(\omega), \hat{a}_R^{(4)\dagger}(\omega') \right] = \left[ \hat{a}_L^{(1)}(\omega), \hat{a}_L^{(1)\dagger}(\omega') \right] \\ = \delta(\omega - \omega'). \quad (6)$$

### III. THE SECOND-ORDER COHERENCE

To illustrate the effects of the bilayer structure in quantum optics, we consider the specific case (same as those used in [13]-[19]) that satisfy the necessary condition for the PT-symmetry system in (1)-(2):  $\omega_{0l} = \omega_{0g} = 1$  PHz,  $\gamma_l = \gamma_g = 0.067$  PHz, and  $\epsilon_{0l} = \epsilon_{0g} = 2$  i.e., ( $\Delta\epsilon = 0$ ), we achieve PT-symmetry at  $\omega_{PT}/\omega_{0g} = 1$  for arbitrary values of  $|\alpha_g| = \alpha_l$ . The thickness of the loss/gain slab along the  $z$ -direction is 10 nm. A practical example of the proposed bilayer structure could be the plasmonic metamaterial suggested by [24]-[26], grown on a lossless glass substrate wherein the quantum noise flux vanishes.

In what follows, we use the above parameters to numerically analyze the structure under study. We know that the thermal noise effect at room temperature and zero Kelvin, for the given incident frequency, are both insignificant [15]. Henceforward, we consider keeping the gain and loss layers at 0 K. In this section, we study how the second-order correlation function [21]

$$g^2(z, t, \tau) = \frac{\langle \hat{E}_-^{(4)}(z, t) \hat{E}_-^{(4)}(z, t + \tau) \hat{E}_+^{(4)}(z, t + \tau) \hat{E}_+^{(4)}(z, t) \rangle}{\langle \hat{E}_-^{(4)}(z, t) \hat{E}_+^{(4)}(z, t) \rangle \langle \hat{E}_-^{(4)}(z, t + \tau) \hat{E}_+^{(4)}(z, t + \tau) \rangle}. \quad (7)$$

of a quantized electromagnetic field is modified by the passage through the structure of Fig. 1. The intensity correlation in (7) is proportional to the joint probability of detecting photons at two times  $t$  and  $t + \tau$ . This function quantifies, how the detection of one photon from a light source influences the probability to detect another one. It usually decays to 1 on timescale  $\tau$  comparable to the coherence time of the light field. The intensity fluctuations for the transmitted light already allow one to distinguish between bunching ( $g^{(2)}(\tau=0) > 1$ ), coherent ( $g^{(2)}(\tau=0) = 1$ ), and antibunching ( $g^{(2)}(\tau=0) < 1$ ) (quantum features of light) light

emission. If the arrival of one photon is detected then there is an increased possibility of another photon arriving soon afterward. This phenomenon is called photon bunching. In other words, photon antibunching is the tendency of photons to gather together randomly in time rather than appear in groups. The important difference between the classical and quantum mechanical description is that, in the latter case, the detection of a photon at  $t$  reduces the number of photons at  $t+\tau$ . In the following, we investigate (7) two transmitted  $M$ -photon number states and coherent states through the structure of Fig. 1.

### A. Number State

The general state of the system is represented by the product state of the form

$$|\psi\rangle \equiv |M, \xi\rangle_R |0\rangle_L |F\rangle. \quad (8)$$

in which  $|0\rangle_L$  and  $|M, \xi\rangle_R$  are the left-going vacuum state and right-going number state, respectively, and  $|F\rangle$  is associated with the noise contribution of the slab arising from amplifying slab at zero temperature. The number state can be generated with the use of a quantum operator acting on the vacuum of the form [21]

$$|R\rangle \equiv |M, \xi\rangle = \frac{1}{\sqrt{M!}} \left[ \int_0^\infty d\omega \xi^*(\omega) \hat{a}_R^{(1)\dagger}(\omega) \right]^M |0\rangle, \quad (9)$$

where  $\xi(\omega)$  describes the frequency distribution of the photon-number wave packet, whose form is determined by how the photon state is prepared such as the nature of the incident light and any subsequent filtering process. Here, we consider a Gaussian wave packet distribution centered on the frequency  $\omega_{PT}$  and the mean-square spatial length  $L^2$  as [21]

$$\xi(\omega) = \left( L^2 / 2\pi c^2 \right)^{1/4} \exp \left[ -L^2 (\omega - \omega_{PT})^2 / 4c^2 \right] \quad (10)$$

After some lengthy mathematical manipulations using (3) and (7)-(10), one can obtain a compact formula for the second-order

correlation function (7) in region 4 for the transmitted number state at  $t=z/c$  as

$$\begin{aligned} g^{(2)}(\tau) = & \left\{ M(M-1) |I_1(0)|^2 |I_1(\tau)|^2 \right. \\ & + MI_2(0) \left[ |I_1(0)|^2 + |I_1(\tau)|^2 \right] \\ & + 2M \operatorname{Re} \left[ I_1(0) I_1^*(\tau) I_2(\tau) \right] + |I_2(\tau)|^2 + I_2^2(0) \left. \right\} \\ & \times \left\{ [M |I_1(0)|^2 + I_2(0)] [M |I_1(\tau)|^2 + I_2(0)] \right\}^{-1}. \end{aligned} \quad (11)$$

where in the explicit forms of the  $I_1$  and  $I_2$  are,

$$I_1(\tau) \equiv \sqrt{\hbar / 4\pi\epsilon_0 c A} \int_0^\infty d\omega e^{-i\omega\tau} \omega^{1/2} t(\omega) \xi(\omega), \quad (12a)$$

$$I_2(\tau) \equiv \frac{\hbar}{4\pi\epsilon_0 c A} \int_0^\infty d\omega e^{-i\omega\tau} \omega \left\langle \hat{F}_R^\dagger(\omega) \hat{F}_R(\omega) \right\rangle \quad (12b)$$

where  $\left\langle \hat{F}_R^\dagger(\omega) \hat{F}_R(\omega) \right\rangle$  is the average flux of the noise photons, which is given by (B7) in [15] for the exact multilayer theory. The dependency of the second-order coherence (11) on the dimensionless time delay,  $\tau \omega_{PT}$ , is plotted in Fig. 2 for a two-photon Gaussian wave packet transmitting through the proposed structure. For the sake of clarity, we focus on four special values of loss coefficients, i.e.,  $|\alpha_g| = \alpha_l = 24, 114$  (anisotropic transmission resonance), 52 (accidental degeneracy), and 890 (exceptional point) which their significances are given in [15].

The results show that the transmitted light is antibunched for  $|\alpha_g| = \alpha_l = 24, 52$ , and 114. Then, by increasing  $\tau$ , each plot for the given angles first increases sharply and reaches a maximum value, approaching a near-unity value, saturating for  $\tau \omega_{PT} > 1.5$ . While for  $|\alpha_g| = \alpha_l = 890$  and  $\tau = 0$ ,  $g^{(2)}(0) = 2$  due to the noise dominating the pulse contribution at elevated  $\alpha_l$ . Our results show that despite the apparent compensation of the losses within the bilayer in the PT-symmetry phase, the outgoing light is no longer antibunched for  $|\alpha_g| = \alpha_l = 890$ . Because the gain

layer adds noise to a beam of light at zero temperature, having detrimental effects on the antibunching feature of the quantum light. Therefore, we see that at elevated values of  $\alpha_1$  the output photons are bunched.

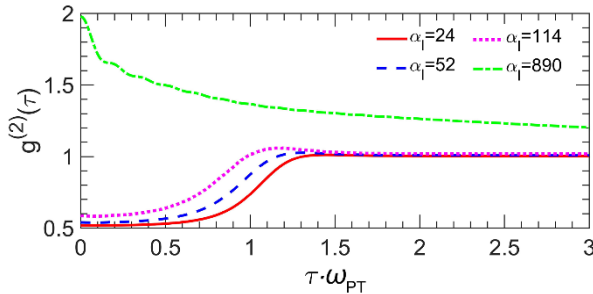


Fig. 2. The second-order coherence  $g^{(2)}(\tau)$  versus  $\tau \omega_{PT}$  for two-photon wave packet transmitted through the bilayer structure for  $\alpha_1=24, 52, 114,$  and  $890$ .

### B. Coherent State

One of the most important classes of states in quantum optics is coherent states. The coherent state  $|\alpha\rangle$  is generally introduced as the eigenstate of the non-Hermitian annihilation operator associated with the complex eigenvalue [27]

$$\hat{a}_R^{(1)}|\alpha\rangle = \alpha|\alpha\rangle. \tag{13}$$

This state can also be represented easily by operating the unitary displacement operator on the vacuum state  $|0\rangle$ ,

$$|\alpha\rangle = \exp[\alpha \hat{a}_R^{(1)\dagger} - \alpha^* \hat{a}_R^{(1)}]|0\rangle. \tag{14}$$

The coherent state possesses a better-defined phase than the number state. Accordingly, it is described as a state most closely to the classical kind of behavior.

In this section, we consider that the incident rightward and leftwards fields on the slab are the monochromatic coherent state and conventional vacuum state respectively. Using the method outlined in [21], and making use of the input-output relation (4), and after some algebraic calculations using (3) and (13)-(14), we obtain the second-order correlation function (7) in the region 4 for transmitted coherent state at  $t=z/c$  as:

$$g^{(2)}(\tau) = \left\{ |I_1(0)|^2 |I_1(\tau)|^2 + 2 \text{Re} [I_1(0) I_1^*(\tau) I_2(\tau)] + |I_2(0)| [ |I_1(0)|^2 + |I_1(\tau)|^2 ] \right\} \times \left\{ [ |I_1(0)|^2 + I_2(0) ] [ |I_1(\tau)|^2 + I_2(0) ] \right\}^{-1} \tag{15}$$

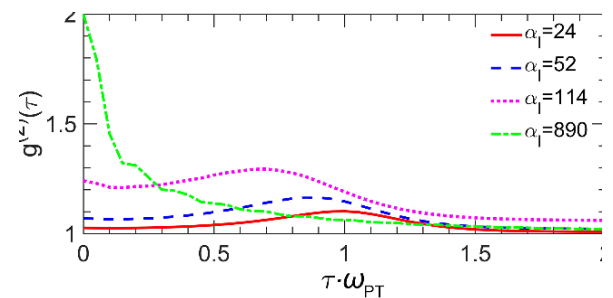


Fig. 3. The second-order coherence  $g^{(2)}(\tau)$  versus  $\tau \omega_{PT}$  for coherent state transmitted through the bilayer structure for  $\alpha_1=24, 52, 114,$  and  $890$ .

The dependency of the second-order coherence (11) on the dimensionless time delay,  $\tau \omega_{PT}$ , for the coherent states transmitting through the proposed structure with  $|\alpha_g| = \alpha_1 = 24, 52, 114,$  and  $890$ . The results show that as  $\tau \rightarrow 0, g^{(2)} > 1$  for all cases of  $|\alpha_g| = \alpha_1 = 24, 52, 114,$  and  $890$  due the noise dominating the pulse contribution. Therefore, for each loss coefficient, we find that the output photons are bunched, and this bunching effect enhances slightly as the loss/gain coefficient inside the bilayer increases. Moreover, for large values of  $\tau$ , the second-order coherence progressively tends to unity — i.e.,  $g^{(2)}(\tau \omega_{PT} \gg 1) \sim 1$ , because photon arrival times are not correlated if the photons are detected in a larger time intervals.

### IV. CONCLUSION

In this paper, we have investigated the dispersion and medium effects of a bilayer non-Hermitian structure on the antibunching property of a transmitted number and coherent states of normally-incident light at zero temperature, particularly at a specific frequency for which the bilayer holds PT-symmetric. We have calculated the second-order coherence at the output of bilayer versus dimensionless time delay for different values of loss coefficients, keeping the incident signal frequency fixed at the corresponding frequency where PT-

symmetry holds. Although we observe the compensation of loss effect within the PT-symmetric bilayer in the exact phase regime, the transmitted light is no longer antibunched for large (all) values of loss coefficients for the incident number (coherent) state. One may attribute this effect to the contribution of the quantum noise within the PT-symmetric structures at  $\omega=\omega_g$ , originating from the gain nanolayers at zero temperature.

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