Quantization of electromagnetic fields in the presence of a spherical semiconductor quantum dot and spontaneous decay of an excited atom doped in this nanostructure

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ABSTRACT- In this paper we consider electromagnetic field quantization in the presence of a dispersive and absorbing semiconductor quantum dot. By using macroscopic approach and Green's function method, quantization of electromagnetic field is investigated. Interaction of a two-level atom , which is doped in a semiconductor quantum dot, with the quantized field is considered and its spontaneous emission rate is calculated. Comparing with the same condition for an excited atom inside the bulk, it is shown that the spontaneous emission rate of an atom will decrease.

KEYWORDS: Semiconductor Quantum Dot, Quantization, Spontaneous Emission

I. INTRODUCTION

In recent years, semiconductor nanostructures have drawn a lot of attentions. The rapid progress of nano-fabrication technologies making it possible to grow zero-dimensional semiconductor systems that are not spoiled by fabrication-induced damage. These zero-dimensional semi-conductors are known as quantum dots and they have so many useful applications in science and technology. Quantum dot applications proposed so far range from lasers to memory devices and from single-photon emitters to quantum bits for quantum optics and quantum information. In quantum optical applications of quantum dots, there are two approaches for considering interaction of QD with electromagnetic field. In one approach electromagnetic field modes in free space interact with QD. So, in this manner presence of QD does not have any effect on the process of field quantization. This approach is used in semiclassical and quantum considerations [1, 2].

In other approach, presence of the QD will have some effects in the process of the field quantization. This method is called the method of Green's function expansion [3], which can be regarded as a natural extension of the familiar method of mode expansion for arbitrary Kramers-Kronig media. This approach, which resembles, in a sense, the method of operator Langevin forces [4], directly starts with the Maxwell's equations for the macroscopic electromagnetic field, including the dielectric displacement vector and a phenomenologically given permittivity. The quantization of the radiation field is based on the classical Green's function representation of the vector potential. In this approach the external sources therein are identified with noise sources that are necessarily associated to the losses in the medium. By using these quantized fields, spontaneous emission of an atom in such a medium can be considered.

Spontaneous emission is known as a consequence of the action of the vacuum fluctuations on a physically measurable process. Einstein [5], already pointed out that in order to obtain the Planck radiation law, a process as spontaneous emission must necessarily be include in the theory of atomic decay. Later on, the radiation properties of an excited atom located in free space have been a subject of many studies [6]. The vacuum field is modified by the local environment where boundaries alter modes of vacuum field around the atom and affect the spontaneous emission rate. In many studies it has been shown that the spontaneous emission rate can be changed

by embedding the atom inside a dielectric host[6] and other cavities [7, 8].

In this paper we use Green's function approach to quantize radiation field in the presence of a semiconductor quantum dot. Then using this quantized field Hamiltonian, spontaneous emission of a two-level atom due to its interaction with the field inside a spherical quantum dot is calculated. The paper is organized as follows. In Sec.II, the quantization of electromagnetic fields in the presence of a semiconductor quantum dot is prepared. In Sec.III, by using quantization scheme gained, the spontaneous decay rate of an excited two-level atom doped at the center of a spherical quantum dot is investigated. Finally the results are discussed in Sec.IV.

II. QUANTIZATION OF ELECTOMAGNETIC FIELD IN THE PRESENCE OF A SEMICONDUCTOR QUANTUM DOT

A. *Quantization scheme for arbitrary dielectric*

Our analysis of quantization of the field in the presence of an absorbing medium is based on the scheme for quantization of the fields in linear Kramers-Kronig dielectrics developed in Refs.[3, 9, 10]. The start point of this approach is quantum Maxwell equations with constitutive relations

$$\hat{\mathbf{D}}(\mathbf{r},\omega) = \varepsilon_0 \hat{\mathbf{E}}(\mathbf{r},\omega) + \hat{\mathbf{P}}(\mathbf{r},\omega) = \varepsilon_0 \varepsilon(\mathbf{r},\omega)\hat{\mathbf{E}}(\mathbf{r},\omega), \tag{1}$$

$$\nabla \cdot \hat{\mathbf{B}}(\mathbf{r},\omega) = \mu_0 \nabla \cdot \hat{\mathbf{H}}(\mathbf{r},\omega). \tag{2}$$

where, $\varepsilon(\mathbf{r},\omega) = \varepsilon_R(\mathbf{r},\omega) + i\varepsilon_I(\mathbf{r},\omega)$ is the permittivity satisfying the kramers-Kronig relations and vector operator $\underline{\hat{\mathbf{P}}}(\mathbf{r},\omega)$, noise polarization vector, is related to $\underline{\hat{\rho}}(\mathbf{r},\omega)$ and $\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega)$, the noise charge and noise current density operators, respectively:

$$\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega) = -i\,\omega \underline{\hat{\mathbf{P}}}(\mathbf{r},\omega), \tag{3}$$

$$\underline{\hat{\rho}}(\mathbf{r},\omega) = -\nabla \cdot \underline{\hat{\mathbf{P}}}(\mathbf{r},\omega). \tag{4}$$

It is worth to note that the equation of continuity is satisfied by these density operators. Quantum Maxwell equations and constitutive relations (1) and (2) lead to the wave equation for $\underline{\hat{\mathbf{E}}}$ operator as follows

$$\hat{\nabla} \times \hat{\nabla} \times \underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) = i\,\omega\mu_0 \underline{\hat{\mathbf{j}}}(\mathbf{r},\omega). \tag{5}$$

This equation can be solved in terms of Green tensor as

$$\underline{\hat{\mathbf{E}}}_{\mathbf{k}}(\mathbf{r},\omega) = i\,\mu_0 \int d^3\mathbf{r}'\omega \mathbf{G}_{\mathbf{k}\mathbf{k}'}(\mathbf{r},\mathbf{r}',\omega)\underline{\hat{\mathbf{j}}}_{\mathbf{k}'}(\mathbf{r}',\omega), \tag{6}$$

where $\mathbf{G}_{kk'}(\mathbf{r},\mathbf{r}',\omega)$ is classical dyadic Green's function satisfying the partial differential equation

$$\hat{\nabla} \times \hat{\nabla} \times \mathbf{G}(\mathbf{r},\mathbf{r}',\omega) - \frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) = \delta(\mathbf{r}-\mathbf{r}'). \tag{7}$$

Using the Green's function and quantum Maxwell equations, one can find electric and magnetic field operators. Hamiltonian of the system can be written as

$$\hat{H}_{field} = \int d^3\mathbf{r} \int_0^\infty d\,\omega\hbar\omega \hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\hat{\mathbf{f}}(\mathbf{r},\omega), \tag{8}$$

This is the Hamiltonian of the field in the presence of medium. In this Hamiltoninan $\hat{\mathbf{f}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)$, are annihilation and creation field operators in the presence of the medium, respectively. Therefore, $\hat{\mathbf{f}}(\mathbf{r},\omega)$ is annihilation operator of a photon with energy $\hbar\omega$ in a medium. It is worth to note that in this approach, the effects of medium (quantum dot) have been included because of the existence of the noise operator in Maxwell's equations. This is due to the fact that absorption in the medium is modeled by the noise operator. On the other hand, the classical dyadic Green's function is determined by the boundary condition. Therefore, all physical effects associated to medium (absorption and

boundary conditions) are included in the configuration. It can be proved that quantization scheme is fully consistent with QED for arbitrary linear dielectrics [10], i.e.,

$$\varepsilon_0\left[\hat{E}_k(\mathbf{r}),\hat{B}_k(\mathbf{r}')\right]=-i\hbar\varepsilon_{klm}\partial^r_m\delta(\mathbf{r}-\mathbf{r}'), \qquad (9)$$

$$\left[\hat{E}_k(\mathbf{r}),\hat{E}_l(\mathbf{r}')\right]=\left[\hat{B}_k(\mathbf{r}),\hat{B}_l(\mathbf{r}')\right]=0. \qquad (10)$$

As is seen, the quantization in the presence of an arbitrary medium that satisfies Kramers-Kronig conditions leads to the problem of calculating the Green's function of that configuration. Then by using Eq. (6) to find the electric field operator in terms of creation and annihilation operators $\hat{\mathbf{f}}^{\dagger}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}(\mathbf{r},\omega)$, and using Maxwell equations, other field operators can be derived.

B. *Dielectric constant and dyadic Green's function*

In previous subsection, we have found that the problem of quantization of electromagnetic fields in the presence of an arbitrary dielectric leads to calculate the classical Green's function of the system. Also, the dielectric function of that medium should satisfy the Kramers-Kronig relations. So, for quantizing the electromagnetic fields in the presence of the semiconductor quantum dot there are two stages: first of all, we should introduce a dielectric function of the semiconductor quantum dot and in the next stage we have to find the appropriate Green's function for the system. A schematic structure of this system is shown in Fig.1. The dielectric constant of a quantum dot can be defined as a Lorentzian model [11]:

$$\varepsilon(\omega)=\varepsilon_\infty+\frac{F}{\omega_{ex}^2-\omega^2-i\gamma\omega} \qquad (11)$$

where ε_∞ is the dielectric constant of the background, ω_{ex} is the first excitonic transition, γ is the width of the peak of the first excitonic transition and F is the parameter related to the excitonic oscillator strength and the dimensions of quantum dot [12].

Now we can solve classical Green equation (7), using boundary conditions on the surface of quantum dot

$$\mathbf{r}\times\mathbf{G}^{fs}=\mathbf{r}\times\mathbf{G}^{(f+1)s},$$
$$\frac{1}{\mu_0}\mathbf{r}\times\vec{\nabla}\times\mathbf{G}^{fs}=\frac{1}{\mu_0}\mathbf{r}\times\vec{\nabla}\times\mathbf{G}^{(f+1)s}, \qquad (12)$$

($f=1,2$ refers to different mediums). By calculating the Green's function, all field operators will be obtained. The Green's function related to the spherical quantum dot in free space can be written as a superposition of two parts

$$\mathbf{G}^{fs}(\mathbf{r},\mathbf{r}')=\mathbf{G}^{V}(\mathbf{r},\mathbf{r}')+\mathbf{R}^{\mathbf{fs}}(\mathbf{r},\mathbf{r}'), \qquad (13)$$

that $\mathbf{G}^{V}(\mathbf{r},\mathbf{r}')$ is vacuum Green tensor relevant to dielectric without boundary and R^{fs} is relevant to all multiple transmission and reflection because of the boundaries of quantum dot. Using $f=1,2$ one can find the field outside of quantum dot and inside it, respectively (see Fig.1).

$\mathbf{R}(\mathbf{r},\mathbf{r}',\omega)$ can be divided in two parts: $\mathbf{R}^{12}(\mathbf{r},\mathbf{r}',\omega)$ is the field in the free space and $\mathbf{R}^{22}(\mathbf{r},\mathbf{r}',\omega)$ is the field inside a quantum dot:

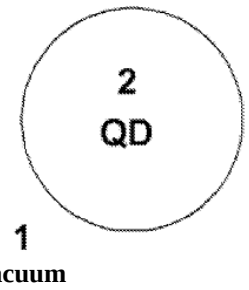

Fig 1. A spherical quantum dot in the free space.

$$R^{12}(\mathbf{r},\mathbf{r}',\omega)=\frac{ik_2}{4\pi}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(2-\delta_m^0\right)\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\times$$
$$\left[A_M^{12}M_{\substack{e\\o\\nm}}(\mathbf{r},k_1)\otimes M_{\substack{e\\o\\nm}}(\mathbf{r}',k_2)+\right.$$
$$\left.A_N^{12}N_{\substack{e\\o\\nm}}(\mathbf{r},k_1)\otimes N_{\substack{e\\o\\nm}}(\mathbf{r}',k_2)\right], \qquad (14)$$

$$R^{22}(\mathbf{r},\mathbf{r}',\omega)=\frac{ik_2}{4\pi}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(2-\delta_m^0\right)\frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!}\times$$
$$\Big[C_M^{22}M_{\substack{e\\o\\nm}}(\mathbf{r},k_2)\otimes M_{\substack{e\\o\\nm}}(\mathbf{r}',k_2)+$$
$$C_N^{22}N_{\substack{e\\o\\nm}}(\mathbf{r},k_2)\otimes N_{\substack{e\\o\\nm}}(\mathbf{r}',k_2)\Big], \quad (15)$$

where

$$M_{\substack{e\\o\\nm}}(k)=\hat{\nabla}\times\left[\psi_{\substack{e\\o\\nm}}(k)\mathbf{r}\right], \quad (16)$$

$$N_{\substack{e\\o\\nm}}(k)=\frac{1}{k}\hat{\nabla}\times\hat{\nabla}\times\left[\psi_{\substack{e\\o\\nm}}(k)\mathbf{r}\right], \quad (17)$$

with

$$\psi_{\substack{e\\o\\nm}}(k)=j_n(kr)P_n^m(\cos\Theta)\begin{pmatrix}\cos\\ \sin\end{pmatrix}(m\varphi), \quad (18)$$

that

$$k_1=\frac{\omega}{c},\quad k_2=\sqrt{\varepsilon(\omega)}\frac{\omega}{c}, \quad (19)$$

and $j_n(kr)$ is spherical Bessel function of the first kind; $P_n^m(\cos\Theta)$ is associated Legendre polynomial; For the rather lengthy expressions of the generalized reflection coefficients $A_{M(N)}^{12}, C_{M(N)}^{22}$, see [13].

Now the Green's function of the system is calculated and according to the previous section we can write quantized electromagnetic field operators inside the quantum dot and in free space:

$$\hat{E}_k^{fs}(\mathbf{r},\omega)=i\mu_0\int d^3\mathbf{r}'\omega\mathbf{G}_{kk'}^{fs}(\mathbf{r},\mathbf{r}',\omega)\hat{j}_{k'}(\mathbf{r}',\omega), \quad (20)$$

$$\hat{B}_k^{fs}(\mathbf{r},\omega)=(i\omega)^{-1}\nabla\times\hat{E}_k^{fs}(\mathbf{r},\omega) \quad (21)$$

In the next section we will consider the effects of the quantization on the spontaneous emission of a two level atom centered in a semiconductor quantum dot.

III. Spontaneous Emission of an Excited Atom Doped Inside a Semiconductor Quantum Dot

The electric and magnetic field operators, (20) and (21), can be expressed in terms of vector $\hat{\mathbf{A}}$ and scalar $\hat{\varphi}$ potentials. In the following we will work with Coulomb Gauge. This gauge condition implies that both the transverse and longitudinal electric fields are obtained from the vector potential

$$\hat{\mathbf{A}}(\mathbf{r})=\hat{\mathbf{A}}^{(+)}(\mathbf{r})+\hat{\mathbf{A}}^{(-)}(\mathbf{r}), \quad (22)$$

$$\hat{\mathbf{A}}^{(+)}(\mathbf{r})=\mu_0\int_0^{\infty}d\omega\int d^3\mathbf{r}'_{kk'}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\hat{j}_{k'}(\mathbf{r}',\omega). \quad (23)$$

Now, we consider the case which there is an external (two-level) atomic system in the position $\mathbf{r}_A$. The atom is located inside a QD and interacts with the quantized field inside the QD. Treating its interaction with electromagnetic field in dipole and rotating wave approximations, the Hamiltonian of the total system can be given by

$$\hat{H}=\int d^3\mathbf{r}'\int_0^{\infty}d\omega\hbar\omega\hat{\mathbf{f}}^{\dagger}(\mathbf{r},\omega)\hat{\mathbf{f}}(\mathbf{r},\omega)+\sum_{\alpha=1}^{2}\hbar\omega_\alpha\hat{A}_{\alpha\alpha}-$$
$$\left[i\omega_{21}\hat{A}_{21}\hat{\mathbf{A}}^{(+)}(\mathbf{r}_A)\cdot\hat{\mathbf{d}}_{21}+H.c.\right]. \quad (24)$$

Here the atomic operators $\hat{A}_{\alpha\alpha'}=|\alpha\rangle\langle\alpha'|$ are introduced, with $|\alpha\rangle$ s being the atomic energy eigenstates ($\alpha=1,2$). The energies of the two states are assumed to be $\hbar\omega_1$ and $\hbar\omega_2$ ($\hbar\omega_2>\hbar\omega_1$), $\omega_{21}=\omega_2-\omega_1$ and $\hat{\mathbf{d}}_{21}$ are the atomic transition frequency and dipole moment of the atom, respectively. Note that in the interaction term in Eq.(24), the $\hat{\mathbf{A}}^2$ term and the counter-rotating terms are dropped. Following the approach introduced in [6], for the macroscopic dielectric medium, decay rate of an excited atom can be written as:

$$\gamma=\frac{2\omega_A^2\mu_k\mu_{k'}}{\hbar\varepsilon_0c^2}\mathbf{Im}R_{kk'}^{22}(\mathbf{r}_A,\mathbf{r}_A,\omega_A), \quad (25)$$

where $\mu_k \equiv (d_{21})_k$, $\omega_A \equiv \omega_{21}$ and $\mathbf{G}^{22}_{kk'}(\mathbf{r}_A, \mathbf{r}_A, \omega_A)$ is given in relation (13) by replacing $\mathbf{r}$ and $\mathbf{r}'$ with $\mathbf{r}_A$. In the coincidence limit ($\mathbf{r} \to \mathbf{r}' \to 0$) only the TM wave vector Debye potential $N_{e_{o}10}(k)$ and $N_{e_{o}11}(k)$ contribute to $R^{22}_{kk'}(\mathbf{r}_A, \mathbf{r}_A, \omega_A)$ and we find that

$$R^{22}_{kk'}(\mathbf{r}_A, \mathbf{r}_A, \omega_A)|_{r_A=0} = \frac{i\omega}{6\pi c}\left(C_N^{22}(\omega)\right)_{n=1}\delta_{kk'}, \quad (26)$$

Hence, for $\omega = \omega_A$, we obtain the decay rate in the form of

$$\gamma = \gamma_0\left[1 + \mathbf{Re}\left(C_N^{22}(\omega_A)\right)_{n=1}\right], \quad (27)$$

where the generalized reflection coefficient $\left(C_N^{22}(\omega)\right)_{n=1}$ is given by

$$\left(C_N^{22}\right)_{n=1} = \frac{T^V_{F1}R^V_{P1}}{T^V_{P1}}, \quad (28)$$

with

$$T^V_{F1} = \frac{k_2\left(\Im_2\partial\hbar_2 - \partial\Im_2\hbar_2\right)}{k_2\Im_2\partial\hbar_1 - k_1\partial\Im_2\hbar_1}, \quad (29)$$

$$R^V_{P1} = \frac{k_2\hbar_2\partial\hbar_1 - k_1\hbar_1\partial\hbar_2}{k_2\partial\Im_1\hbar_2 - k_1\Im_1\partial\hbar_2}, \quad (30)$$

$$T^V_{P1} = \frac{k_2\left(\partial\Im_2\hbar_2 - \Im_2\partial\hbar_2\right)}{k_2\partial\Im_1\hbar_2 - k_1\Im_1\partial\hbar_2}, \quad (31)$$

and

$$\Im_f = j_n(k_f R), \quad (32)$$

$$\hbar_f = h_n^{(1)}(k_f R), \quad (33)$$

$$\partial\Im_f = \frac{1}{\rho}\frac{d\left[\rho j_n(\rho)\right]}{d\rho}\bigg|_{\rho=k_f R}, \quad (34)$$

$$\partial\hbar_f = \frac{1}{\rho}\frac{d\left[\rho h_n^{(1)}(\rho)\right]}{d\rho}\bigg|_{\rho=k_f R}, \quad (35)$$

and R being the radius of quantum dot.

The decay rate of an excited atom as a function of the atomic transition frequency ω_A/ω_{ex} and $r = \lambda/R$ has been depicted in Fig.2, according to the Eq.(27).

As is clear, the less cavity radius R, the more maximum value of spontaneous decay rate, which occurs near the medium resonance.

In Fig. 3, we have plotted the atom decay rate Γ/Γ_0 doped in the center of the quantum dot (solid line) and the atom decay rate Γ_b/Γ_0 of an atom in the spherical cavity of radius R_b in a surrounding medium with the single resonance model permittivity $\varepsilon(\omega) = \omega_p^2/\left\{\omega_0^2 - \omega^2 - i\gamma\omega\right\}$, (dashed line)[1].

These decay rates are plotted as a function of their atomic transition frequency with $R = R_b = 0.02\lambda$. As is seen in this plot, Γ_b/Γ_0 exceeds Γ/Γ_0 while two decay rates have the same behavior near the resonance of their mediums, qualitatively.

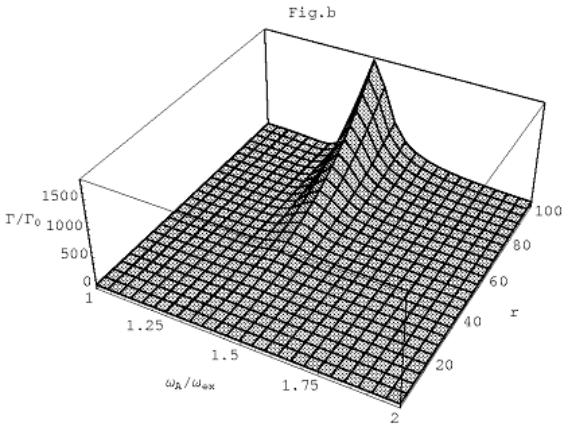

Fig. 2. The spontaneous decay rate Γ/Γ_0, Eq. (27), of a two level atom at the center of a quantum dot as a function of $r = \lambda/R$ and the atomic transition frequency ω_A near the first exciton resonance at ω_{ex}, using permittivity (11) for $\gamma = 0.2\omega_{ex}$, $R = 0.02\lambda$, $F = 66$.

[1] It has been depicted according to Green's function derived in Ref. [6]

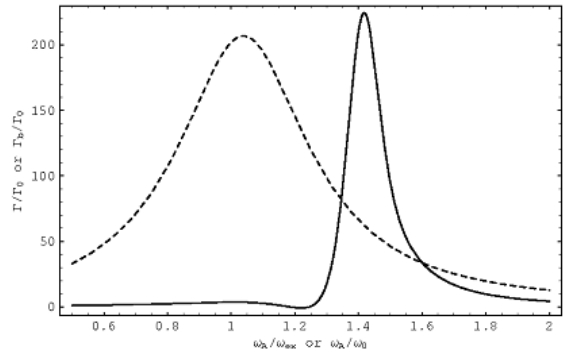

Fig. 3. The spontaneous decay rate Γ/Γ_0, Eq. (27), of a two level atom at the center of a quantum dot as a function of the atomic transition frequency ω_A near the first exciton resonance at ω_{ex}, using permittivity of Eq. (11) for $\gamma = 0.2\omega_{ex}$, $R = 0.02\lambda$ and $F = 66$ (solid line). The spontaneous decay rate Γ_b/Γ_0 of a two level atom inside a microcavity in the bulk as a function of the atomic transition frequency ω_A near the medium resonance ω_0, using a model permittivity of Lorentz type and for $\gamma = 0.5\omega_0$ $R = 0.02\lambda$, $\omega_p = 0.46\omega_0$ (dashed line).

As another interesting aspect, an excited atom in an absorbing medium undergoes both radiative and nonradiative damping, and in dense media nonradiative decay can be much faster than radiative one. Since the radiationless decay typically happens at longitudinal frequency ω_L, one observes, for sufficiently small values of γ and $R\,(R_b)$, a shift of the maximum of the decay rate from ω_{ex} to longitudinal frequency $(\omega_L)_{ex} = \sqrt{\omega_{ex}^2 + (\omega_p)_{ex}^2}$ for Γ/Γ_0 and from ω_0 to longitudinal frequency $\omega_L = \sqrt{\omega_0^2 + \omega_p^2}$ for Γ_b/Γ_0 $(\omega_p)_{ex}$ and ω_p to longitudinal frequency $\omega_L = \sqrt{\omega_0^2 + \omega_p^2}$ for Γ_b/Γ_0 ($(\omega_p)_{ex}$ and ω_p are plasma frequency related to quantum dot and bulk, respectively), which $(\omega_L)_{ex}/\omega_{ex} > \omega_L/\omega_0$. By introducing $(\omega_p)_{ex}$ as a plasma frequency of quantum dot, and by comparing with a plasma frequency of a bulk, we can conclude that $(\omega_p)_{ex}/\omega_{ex} > \omega_p/\omega_0$.

IV. CONCLUSION

In this paper the quantization of electromagnetic fields in the presence of a semiconductor quantum dot is discussed. Our approach is based on the Green's function that is applicable only for Kramers-Kronig dielectrics. In this approach by calculating the Green's function of the wave equation that its sources are considered as noises due to absorption in medium, field operators are obtained. Then we use the Green's function of the system to calculate the spontaneous emission of a two level atom doped in the center of a quantum dot. The results show that the maximum value of spontaneous decay rate occurs near the medium resonance and increases by decreasing value of quantum dot radius. If one, wrongly, uses simple Lorentz model for permittivity of quantum dot, negative values of decay rate will be appeared which is not correct.

In comparison with decay rate of an atom inside a spherical microcavity in the bulk [6], it is shown that the decay rate of spontaneous emission for a two level atom doped in the center of a semiconductor quantum dot is smaller than the decay rate of an atom in the center of the cavity inside a bulk but it is yet much greeter than Γ_0. Also, it is shown that the shift of the maximum of the decay rate from transition frequency of the quantum dot is larger than the same one for a bulk. By introducing $(\omega_p)_{ex}$ as a plasma frequency of nanostructure quantum dot it means that $(\omega_p)_{ex}/\omega_{ex} > \omega_p/\omega_0$.

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