Theory of Gas Ionization by Intense Electromagnetic Fields

(Invited Paper)
B. Shokri

Physics Department and Laser-Plasma Institute of Shahid Beheshti University, Tehran, Iran and Institute for Studies in Theoretical Physics and Mathematics, P.O. Box 19395-5531, Tehran, Iran.
E-mail: b-shokri@sbu.ac.ir

Abstract- The distribution function of the electrons produced in the interaction between an intense electromagnetic wave and a neutral gas is derived and is shown to be nonequilibrium and anisotropic. By assuming that the time scale of gas ionization is much greater than the field period, it is shown that the electron distribution function formed in microwave and optical discharges has sharp anisotropy affecting the discharge plasma. The anisotropy stimulated by the inequality of average longitudinal and transverse energy of electrons is investigated. Furthermore, the parametric instability stimulated by scattering of incident wave on the density oscillation of plasma under the positive slope of EDF with respect to velocity is studied.

I. INTRODUCTION

Progress in modern high-power microwave electronics and laser pulse techniques has drawn attention to many new aspects in the field-matter interaction. With appearance of intense laser pulses with intensity larger than $10^{14}$ W/cm$^2$, the mechanism of gas ionization due to intense laser radiation and correspondent electron distribution function (EDF) has been studied by a number of researchers [1]-[3]. Recently developed high-power lasers capable of generating pico and femto-second pulses open new possibilities for studying the fundamental properties of matter [4]-[7] and hold great promise for practical applications [8]-[10]. Furthermore, there is a strong mutual interaction between the plasma and electromagnetic fields. One of the key physical factors determining the evolution of high-intensity electromagnetic radiation in a medium and the dynamics of a field-created plasma is the ionization processes. In the strong fields, ionization-induced nonlinear mechanisms are practically inertialless and already manifest themselves at the initial stages of plasma production process [11]-[13].

In typical electromagnetic fields, almost all of the present-day sources generate microwave electric fields far weaker than the atomic field $E_{at} = 5.1 \times 10^9$ V/cm. Thus the atoms cannot be ionized directly by the tunneling ionization mechanism. In this case, gas atoms are primarily ionized by the impact of oscillatory electrons in the strong fields and the plasma eventually is produced by the development of the electron avalanche. The electron impact ionization mechanism has been considered theoretically in many works so far and it has been demonstrated that after several ionizing collisions the electron concentration of the plasma, produced by this mechanism, increases exponentially in time, $\exp(\gamma t)$, while the shape of the electron distribution function doesn't change any more [14]-[15]. Here $\gamma$ as the ionization frequency is usually assumed much smaller than the ionizing wave frequency $\omega_0$ [14]-[19]. But in the present paper it may approach to $\omega_0 (\gamma \approx \omega_0)$ too. We can observe this situation in the ionization of a gas at a moderate pressure by the super-strong MW fields. For example for air at pressure 10 Torr the ionization frequency can be estimated about $4 \times 10^{10}$ S$^{-1}$.

In the other hand, the oscillation energy of electrons in the presence of strong oscillating fields $e_0 = e\omega_0^2/2m_0$ is obviously greater than the ionization potential of atoms $\sim 10eV$. The amplitude of the radiation field $E_0$ with frequency $\omega_0$ is comparable with the atomic filed $E_0 \approx 5 \times 10^9$ V/cm. In applying optical radiation fields with $\omega_0 \approx 2 \times 10^{15} - 10^{16}$ s$^{-1}$ to the gas atoms, direct ionization mechanism will be important due to field tunneling. Although the tunneling time is small, but if radiation field is smaller than the atomic field it will be greater than the period of incident wave. So the gas ionization could be assumed as a slow phenomenon. In this article it was supposed that this condition is satisfied. Many theoretical investigations have been carried out on gas breakdown by a strong electromagnetic (EM) field and the EDF of generated plasma and its instability were considered by a number of researchers [1]-[3]. Many investigations have been performed on the ionization mechanisms of atoms by impact of oscillating electrons. However, the specific mechanism of gas ionization can only specify temporal electron density whereas it could not affect the EDF. The order of maximum power of MW pulse and its power density are $\approx 10^7 - 10^{10}$ W and $\leq 10^4$ W/cm$^2$, respectively. So the magnitude of related electric field is about $10^9$ V/cm which is much smaller than the atomic field. On the contrary the oscillation energy of electrons $E_0$ in these fields is much greater than the ionization energy $I_0 = 10 eV$. Since achievable maximum density of a laser pulse is about $\approx 10^{21}$ W/cm$^2$ and its electric field intensity has the same order of the atomic fields, then another
situation takes places in the optical frequency regime. So tunneling gas ionization mechanism occurs in the strong optical laser field with electron oscillation energy \( \varepsilon_o > 10^{-10}\) KeV as order as electron rest energy. Taking account of both relativistic and nonrelativistic criteria, and using condition
\[
I_o \leq \varepsilon_o = \frac{e^2 E^2_o}{2 m_o \gamma^2}, \tag{1}
\]
One may consider gas breakdown in the presence of the strong optical fields. The inequality
\[
I_o < \varepsilon_o \ll m c^2, \tag{2}
\]
is equivalent with the field \( E_o \leq 10^4 - 10^6 \) V/cm and power density \( \sim 10^4 - 10^6 \) W/cm\(^2\) in the MW range and field \( E_o \leq 3 \times 10^5 - 3 \times 10^3 \) V/cm and power density \( \sim 10^{14} - 10^{13} \) W/cm\(^2\) in the optical frequency range. Moreover, we consider the EDF of produced plasma, gas characteristics, instantaneous dynamics of electron density. Finally, we discuss the subject using a comparison between experimental and theoretical data.

Many features of the discharge of a gas by the strong EM fields have been already investigated. Gas ionization mechanisms mainly affect the manner of the plasma density evolution in time. In this case, the interaction between the EM pulses and the plasma produced gives rise to a number of interesting phenomena, such as nonlinear instabilities [20], wakefield excitation [21]-[23] and laser frequency upshift during the propagation of a pulse through a plasma [24]-[26].

In a plasma produced in the interaction of the electromagnetic field of a laser pulse with a gas, the EDF is nonequilibrium and may give rise to various plasma instabilities [27]. The most important features of this interaction are the high rate of gas ionization and the relatively minor role of hydrodynamic processes during the laser pulse. The key role in the plasma processes is played by the kinetic effects associated with the specific features of the electron distribution, because of the short interaction times. These features are governed completely by the EM pulse parameters. In calculating the EDF, we restrict our study on the nonrelativistic case. A similar problem has been treated in connection with breakdown in a low-pressure gas and the electron-impact ionization of a gas [28]-[34].

Relativistic effects come into play when the kinetic energy of the electron oscillations in an electromagnetic field becomes comparable with the electron rest mass energy. For the laser pulses with a wavelength of about \( \lambda \), this condition holds at laser intensities of about \( 10^{18} \) W/cm\(^2\). This circumstance gives rise to one of the challenges in determining the properties of a laser-produced plasma. Specifically, since the electronic field of such laser pulses is much stronger than the atomic field \( E_o = 5.1 \times 10^4 \) V/cm, the ionization rate of gas atoms cannot be calculated rigorously. The Keldysh formula for the tunneling ionization rate [35]-[36] was derived perturbatively and fails to produce the desired result. Additionally, it is necessary to take into account the effects of multiple ionization, because multiply ionized gases with high ion charge numbers have already been observed in laser experiments [37-38]. The relativistic EDF of a plasma produced in the interaction of EM pulses of intensity \( \geq 10^{18} \) W/cm\(^2\), and field amplitude \( E_o \geq 2 \times 10^6 \) V/cm, with a monoatomic gas has already been investigated. Here, we use a somewhat different approach to solving the kinetic equation and describe the effects associated with the generation of the longitudinal current in a laser-produced plasma. To do this, we derive a model expression for the ionization rate without allowance for the reversion effect of the produced plasma on the EM pulse.

We organize this paper as follows: In Section 2 we will obtain the EDF generated by the interaction of the EM fields with a neutral gas. In Section 3 by making use of the dielectric tensor element of the produced plasma, we study the stability of produced plasma. Finally, a discussion and summary is presented.

II. INSTANTANEOUS DYNAMICS OF ELECTRON DENSITY AND RELATED EDF IN GAS BREAKDOWN BY INTENSE MW

A. MW Discharge

As mentioned, low-pressure gas breakdown by MW radiation fields occurs in condition
\[
\omega_0 > \gamma (E_0) \tag{3}
\]
where \( \gamma (E_0) \) is electron collision frequency, and \( \gamma (E_0) \) is the time scale of plasma density growth. In the MW radiation range, condition (3) is satisfied by pressure less then 10 Torr, where \( E_o \approx 10^{-10} \) V/cm and \( e_o \approx 10^3 - 10^5 \) ev. Kinetic equation integration method and ionization processes of atoms were used to consider the stability of equivalent plasma. It is supposed that the equivalent plasma is homogeneous under conditions (2) and (3) maintained by the nonrelativistic oscillating velocity, since
\[
r_v = (v_e / \omega_0) = (e E_o / m \omega_0) = (|/|) \sqrt{2 \varepsilon / m} \ll (|/|). \tag{4}
\]
The last condition permits us to neglect the initial energy of the generated electrons, electron energy losses in ionization of atoms and magnetic effects on breakdown processes. Taking previous assumptions into account, kinetic equation of produced plasma in the presence of intense electromagnetic wave with linear polarization \( E_o = E_o \cos \omega_o t \) could be written as follows:
\[
\frac{\partial \rho (v, t)}{\partial t} + \frac{e E_o}{m} \cos \omega_o \frac{\partial \rho (v, t)}{\partial v} = \frac{\partial N}{\partial \varepsilon} = \delta (v), \tag{4}
\]
\[
\frac{\partial N}{\partial t} = \gamma (E_0) N_e = \gamma (E_0) f (v, t) d v = N_x \sigma (v) f (v, t) d v, \tag{5}
\]
where \( \sigma (v) \) is the atomic ionization cross section of electron impact \( N_x \), is the density of atoms, and \( f (v, t) = \int d v d v \) is the electron distribution function; \( N_x = \int f (\hat{v}, t) \) is the plasma density. Changing variable \( v \) into \( v - v_k \) sin \( \omega_o t \) as a oscillating coordinate system and using Eqs. (4) and (5) we are led to
\[
\frac{df(v, t)}{dt} = N_N \delta(v' - \bar{v} \sin \omega t) \\
\times \sigma \left( v' - \bar{v} \sin \omega t \right) \left[ \delta(v' - \bar{v} \sin \omega t) f_0(v, t) \right] d\nu'.
\]

It is obvious that after passing of time \( T > 2 \pi / \omega \), a solution is established which satisfies the following relation

\[
f_0(v, t) = N_N \bar{f}_0(v, t), \quad \bar{f}_0(v, t) = \bar{f}_0(v + \frac{2m}{\omega}).
\]

Since \( \bar{f}_0(v', t) \) does not depend on initial conditions, and also it is normalized by unity, so only electron density \( N_e(t) \) depends on gas ionization mechanisms and the effect of ionization mechanisms on the distribution function \( \bar{f}_0(v', t) \) is negligible while \( \omega_0 \gg \gamma(E_0) \). This will be easily seen from:

\[
\bar{f}_0(v) = \frac{N_N}{\pi \omega_0^2} \frac{\theta(v - v_e)}{\sqrt{1 - (v/v_e)^2}} \sigma(v - \bar{v}) \bar{f}_0(v, t) d\nu'.
\]

where, \( \theta(v - v_e) \) is the Heaviside's step function. The above equation is obtained from Eq. (4) in the lowest order of \( \gamma / \omega_0 \). In fact smallness of \( \gamma / \omega_0 \) is necessary in our treatment in getting \( \bar{f}_0(v) \) and finding a solution for Eq. (4). It can be shown that this equation only has one positive answer \( \gamma_0 \) when \( \sigma(x) = \sigma_0 \sigma(x) \) where \( \sigma(x) > 0 \) (really all of ionization cross sections have such constructions), which correspond to an electron distribution function \( \bar{f}_0(v) \). Equation (8) is solved using numerical integration method by assumption

\[
\bar{f}_0(v/v_e) = \frac{\phi(v/v_e)}{\sqrt{1 - (v/v_e)^2}}.
\]

where the approximation \( \phi(\lambda) = \text{const} = 1 / \pi \) corresponds to an uniform distribution of generated electrons as a function of field phase. Of course, the real value of function \( \phi(\lambda) \) differs from \( 1 / \pi \). However, a good approximation could be obtained by making use of the better expression ionization constant for the avalanche ionization. For example for He atoms [14]-[15]

\[
\gamma(E_0) = 3.5 \times 10^{-3} N_N \nu_l/v_e
\]

\[
\times \left[ 2N_2 + 2 \ln(\nu_l/v_e) + (\nu_l/v_e)^2 - 2.25 \right]
\]

where \( \nu_l = \sqrt{2I_e/m} \) is the electron velocity while its energy is equals to ionization potential of He atom. It is worthwhile to point out that the distribution (10) conserves its form in the MW radiation with circular polarization. So in this situation \( E_0(t) \) has two nonzero components \( E_0(t) = E_0 \cos(\omega_0 t, \sin \omega_0 t) \). Thus oscillating velocity of electrons does not depend on time, therefore, changing of coordinate system into oscillating one is not necessary. The low-pressure EDF of generated electrons \( \omega_0 \gg \gamma(E_0) \) in the gas breakdown generated by ultra-intense MW radiations is described by an uniform distribution function as a function of phase field. This situation in \( H_2 \), \( H_2 \) and air occurs.

## B. Optical discharge

Now we consider optical breakdown of a gas by an intense laser pulse. Taking account of condition (2) we limit our considerations in the nonrelativistic case. Under this condition, in the optical region, tunneling ionization mechanism takes place which its probability is defined by relation

\[
W_{\text{ionize}} = 4m_0 \omega_0 I / E_0 \exp \left[ - \frac{2\nu_0}{3} (\nu_0 / E_0)^3 \right]
\]

where \( \omega_0 = 5 \times 10^{15} \text S^{-1} \) is atomic frequency, \( E_0 = 5 \times 10^6 \text V/cm \) is the atomic field, \( I_0 = 1.36 \text eV \) is the ionization of Hydrogen atom. In the optical radiation field with intensity \( 10^{18} \text W/cm^2 \) and amplitude \( E_0 \leq E_0 \), the condition \( \omega_0 \gg \omega_0 \) is confirmed with sufficient high accuracy. This condition is analogous of condition (3) in MW region, and permits us to consider gas breakdown by the same method used for MW case. In the case of linear polarization of the radiation field the kinetic equation can be written as follows:

\[
\frac{df}{dt} = \frac{eE_0}{m} \frac{df}{dv} = \left( N_N - N_e \right) \omega_0 \left( E_0 \right) \delta(v)
\]

By averaging over the initial phase of generated electrons, the electron density will be obtained as follows:

\[
N_e = N_N \left[ 1 - \exp \left( - \omega_0 \right) \right]
\]

If \( \bar{E}_0(t) = E_0 \cos \omega_0 t \), then

\[
\bar{E}_0(t) = E_0 \cos \omega_0 t \sin \omega_0 t
\]

Further consideration of this problem is similar to MW gas breakdown to find the eigen value and function of the integral equation. Furthermore, one can solve Eq. (13) using numerical method. However, its solution does not differ essentially from EDF of electrons as a function of their initial phases.

\[
\bar{f}_0(v/v_e) = \frac{N_N}{\pi} \frac{1}{\sqrt{1 - (v/v_e)^2}}
\]

(compared with relation (10)). From Eqs. (14) and (1) it is found that \( \omega_0 \gg \omega_0 \). Therefore, in the case of optical gas breakdown, the EDF of the produced plasma is similar to MW case seemingly. The only difference is in the instant dynamics of the electron density. Threshold is determined by tunneling ionization mechanism of atoms and electron impact in optical discharge and microwave avalanche ionization, respectively.

## III. Plasma Instability in Gas Breakdown by Intense Wave Field

The EDF of the generated plasma in the previous section has sharp anisotropy along and transverse of the intense radiation field. At first, this case must appear in the form of the Wieble instability.

To see this behavior and the increment of the instability,
we consider dispersion equation for small oscillation by using quasi stationary approximation. This approximation will be valid in the breakdown of gas when instability increment is greater than the inverse of growth rate of the electron density. Thus adiabatic approximation is used for small perturbation of EDF in which perturbed quantities in EDF are \( \sim \exp[\imath k \cdot \vec{r} - \omega t] \). Moreover, if we assume that \( k v_o > \omega_o \) and \( \omega_p > \omega_o \), then only very small amplitude waves are excited. Anisotropic distribution functions (9) and (15), and conditions (3) and (7) are used to determine electric permittivity \( \varepsilon_i \) in the linear polarization field \( E_i \). So we have [39]:

\[
\varepsilon_i \left( \omega, k \right) = 4 \pi e^2 N \left( \frac{c}{m \omega^2} \right) \int d \omega \left[ \frac{\varepsilon \delta}{i \omega - \omega_j} + \frac{\varepsilon \delta}{i \omega - \omega_i} \right].
\]

(16)

Pole at \( \omega = -k \cdot \vec{c} \) in integrand of (16) indicates the mutual interaction of electrons and oscillating fields. Plasma stability is determined by dispersion equation [39]

\[
k^2 \varepsilon_{ij} - k_i k_j - \frac{\omega_p^2}{c^2} \varepsilon_{ij} \left( \omega, k \right) = 0.
\]

(17)

This equation will be considered only for the longitudinal and transverse propagating perturbation. By solving the dispersion equation (17) along with Eq. (16) and EDFs (9) and (15), one can find the dispersion relation between frequency \( \omega \) and wave vector \( k \) [28-33]. It shows that the transverse modes manifest growing periodic oscillations and the Weible-instability remains unchanged even in circular polarization, and it occurs when \( \omega_p \gg \omega_0, \left( c / \omega_0 \right), \gamma \left( E_0 \right) \left( i / \omega_0 \right), \frac{\alpha}{\alpha_{lonz}} \left( c / \omega_0 \right) \).

(18)

That is, gas breakdown may take place in dense plasmas. Now we consider region \( k v_o \gg \omega_o \) and \( \omega_p \ll \omega_o \), where the effects of the intense field perturbations are important. Here, the stability of a plasma in the high frequency intense field is investigated by the dispersion equation (17), whose all roots show that high frequency purely electron oscillation branch is always stable but the low frequency ion branch is always unstable. This case is due to this fact that the frequency of the perturbations becomes positively imaginary. Clearly, transverse electron perturbations are unstable only when the growth rate determined by Eq. (17) exceeds the avalanche ionization constant \( \gamma \left( E_0 \right) \) in the case of microwave breakdown or the ionization probability \( \alpha_{lonz} \) in the case of optical breakdown. It should be noticed that the instability under analysis is governed by the positive slope of distribution functions (9) and (15) along the energy axis and can be interpreted as stimulated Cherenkov excitation of the low-frequency electrons oscillation in the radiation fields. Of course, this instability can only grow at a rate faster than that at which the plasma density increases. Therefore, discharge plasmas in super-strong pulsed fields are subject to instabilities associated with such factors as the anisotropic nature of the electron distribution function (the Weibel instability), its positive (nonequilibrium) derivatives with respect to velocity (kinetic excitation of the ion oscillations), and periodic temporal variations of the mean electron energy (the parametric instability, or Stimulated Raman Scattering of the wave field by electron-density waves). It should be noted that in Maxwellian plasmas, placed in an external strong high frequency fields, when \( \omega_p \ll \omega_o \), all oscillations are stable. However, instability developed by negative slope of EDFs (9) and (15) appears in stimulated Cherenkov radiation as well.

### IV. DISCUSSION

The basic phenomenon which should be observed in experiments is the high average energy of electrons of produced plasma, which according to Eqs. (9) and (15) is in order of \( \approx m v^2 / 2 \). This energy specifies the time of all dissipative processes such as the time scale of gas ionization, plasma lifetime (deals with recombination), and development (diffusion or scattering) time. All of these times must be very retarding, since cross section of these processes with increasing the electron energy decreases intensively, except from scattering time which must be anomaly small. Also instability effects such as Weible instability must appear. These instabilities lead to generation of powerful static fields. However, these phenomena are not special for intense radiations and they are frequently observed in optical and MW breakdown. One of the basic phenomena with interesting result is the nonlinear effect such as harmonic generation of incident radiation. However, this appears in the limit of the relativistic fields. Thus we have studied the behavior of the plasma of MW discharge. In fact with the appearance of the first theoretical works in the gas breakdown by strong MW fields, experimental works begin to verify the theoretical results. But in that time the strong fields could not be produced and the theoretical results did not coincide with experimental results. The second effect taking place not only in breakdown by MW, but by optical fields is the recombination delay which is the result of high average energy of plasma electrons produced in breakdown. Making use of experimental data, it is clear that in the first 20µs the recombination coefficient is much less than in 30µs. Moreover, just after turn off, in the duration of some microseconds, plasma density increases and even exceeds critical density an order of magnitude. After turning off the strong radiations in duration of 60-100ns, plasma density increases.

When quiver (oscillation) energy of electrons in the intensive fields greatly exceeds the ionization energy of gas atoms, in the microwave frequency region, such fields are very small in comparison with atomic fields. Consequently in this case the ionization mechanism of gas atoms is determined by the development of avalanche ionization through electronic impact. In the optical frequency region, such field is comparable with the atomic field and as a result, gas atom ionization is determined principally by tunneling ionization taking place in a very short time. In both cases we assume that the time scale of gas ionization is much greater than the field period. In fact, in such cases, electron distribution function in the plasma of both discharge types is similar and has sharp anisotropy.
affecting the discharge plasma. The anisotropy stimulated by the inequality of average longitudinal and transverse energy of electrons is investigated. Moreover we investigated the parametric instability stimulated by scattering of incident wave on the density oscillation of plasma under the positive slope of EDF with respect to velocity. The distribution function of the relativistic electrons produced in the interaction between an intense electromagnetic wave and a neutral gas is derived and is shown to be nonequilibrium and anisotropic.

REFERENCES