

# Unique Solution of Short Pulse Propagation in Nonlinear Fiber Bragg Grating

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**ABSTRACT**—In this study, a new numerical method is introduced to obtain the exact shape of output pulse in the chalcogenide fiber Bragg grating (FBG). A Gaussian pulse shape with 173 ps width is used as an input pulse for launching to a 6.6 mm nonlinear FBG. Because of bistable and hysteresis nature of nonlinear FBG the time sequence of each portion of pulse is affected the shape of output pulse. So we divide the pulse to leading and trailing portion in time. By using bistability curve and Fourier transformation, the exact shape of output pulse is simulated. In comparison of non-unique solution for output pulse in the previous papers, the results of this study have an optional merit.

**KEYWORDS:** Fiber Bragg Gratings, Fourier Transformation, Nonlinear Optics, Optical Bistability.

## I. INTRODUCTION

Fiber Bragg grating plays a crucial role in future of all optical communication systems. It has many applications in linear regime like: division wavelength multiplexer (DWM), optical filters, optical compensations, optical sensors [1, 2], and also in nonlinear regime such as optical switching, optical bistability and all optical transistors [3, 4].

Many works have been done for obtaining the exact shape of output pulse in nonlinear FBGs by using inverse Fourier transformation method [5, 6]. But their results show that the output pulse is not unique for a given input pulse. They consider two states (On-state and Off-state) for reaching the output while the relation between them is ignored in their simulation process [5-7]. In this paper our aim

is to introduce a new and simple method for overcoming such problems by suitable splitting the input pulse and using inverse Fourier transformation.

## II. THEORY

The refractive index along the length of FBG, which schematically is shown in Fig. 1, can be written as [6]:

$$n(z) = n_0 + n_1 \cos\left(\frac{2\pi}{\Lambda} z\right) + n_2 |E|^2, \quad (1)$$

where  $E$  is the electric field in FBG,  $\Lambda$  is the grating period,  $n_0$ ,  $n_1$  and  $n_2$  denote the linear refractive index, refractive index modulation amplitude and third order nonlinear refractive index coefficient, respectively.

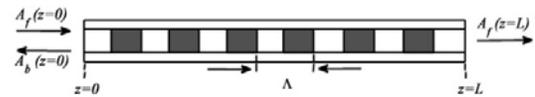


Fig. 1 The schematic of the FBG with length  $L$  and period of  $\Lambda$ . The forward and backward waves are shown.

The electric field in the FBG can be described as a combination of the forward and backward waves:

$$E(r, t) = \frac{1}{2} F(x, y) \left[ A_f(z, t) e^{i\beta_n z} + A_b(z, t) e^{-i\beta_n z} \right] e^{-i\omega_0 t} + c.c., \quad (2)$$

where,  $\omega_0$  is the carrier frequency,  $A_f$  and  $A_b$  represent the slowly varying amplitude of

forward and backward waves respectively, and  $F(x, y)$  is the transverse variations for the two counter propagating waves [8].

Substituting Eqs. (1) and (2) into the Maxwell's wave equations and using slowly varying amplitude approximation, the nonlinear coupled mode equations for both forward and backward wave amplitudes in constant frequency are obtained as follows:

$$\frac{\partial A_f}{\partial z} + \frac{\alpha}{2} A_f = i\delta A_f + i\kappa A_b + i\gamma \left( |A_f|^2 + 2|A_b|^2 \right) A_f, \quad (3)$$

$$-\frac{\partial A_b}{\partial z} + \frac{\alpha}{2} A_b = i\delta A_b + i\kappa A_f + i\gamma \left( |A_b|^2 + 2|A_f|^2 \right) A_b, \quad (4)$$

where  $\alpha$  is FBG loss that is neglected,  $\delta$ ,  $\kappa$  and  $\gamma$  account for the detuning from the Bragg wavelength, coupling coefficient and nonlinearity coefficient, respectively.

### III. NUMERICAL CALCULATIONS AND RESULTS

The numerical solution of nonlinear coupled mode equations is derived using predictor corrector and Runge-Kutta methods. We did this calculation from the ending of the FBG to its beginning. Therefore, this method requires initial values in  $z=L$  instead of boundary conditions as used in [6]:

$$A_f(z=L) = A_{fL}, \quad A_b(z=L) = 0. \quad (5)$$

The typical data which is used in this paper are  $\lambda_B=1550$  nm,  $\lambda=1550.015$ nm,  $L=0.0066$  m,

$$n_0=2.54, \quad n_1=1.5 \times 10^{-4}, \quad n_2=220 \times n_{2\text{silicon}} \quad [9],$$

where  $n_{2\text{silicon}}=0.273 \times 10^{-19} \text{m}^2/\text{W}$  [10].

For simulation we take a Gaussian pulse as input which implies:

$$A \exp\left(-\left(t/\sqrt{2}t_0\right)^2\right), \quad (6)$$

where,  $A$  is the peak of pulse,  $t_0$  is the half width of Gaussian pulse which is taken equal to 173 ps in this paper. The intensity of the input is sketched in Fig. 2.

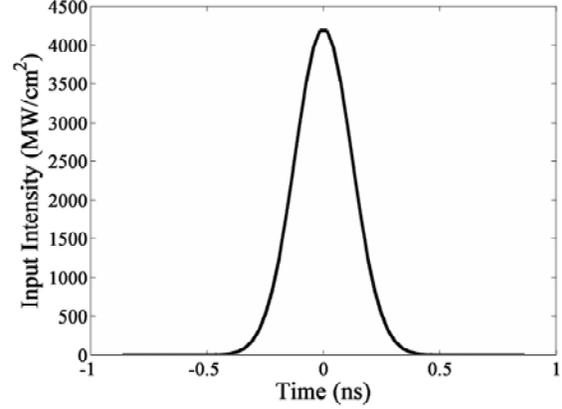


Fig. 2. Input intensity vs. time.

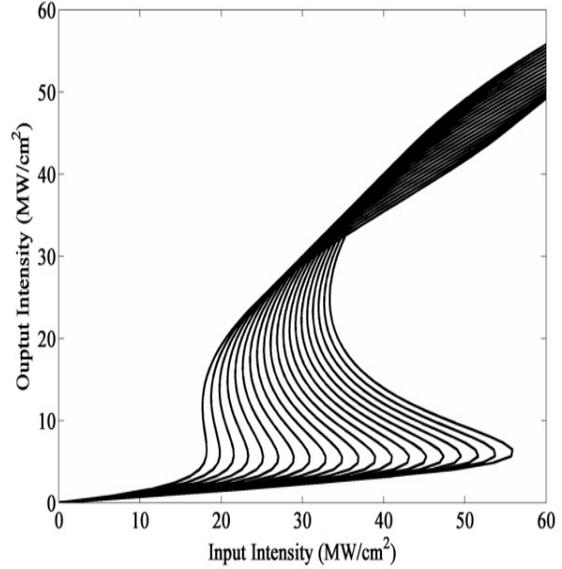


Fig. 3. Bistability diagrams for 10 frequency values.

Considering the bistability curve and hysterical behavior of nonlinear FBGs, each point of the output pulse depends on when and where the corresponding point of the input pulse was. In Fig. 3, ten bistability diagrams related to ten typical frequencies are plotted.

If the peak intensity of input pulse is smaller than the switching threshold intensity, the

output pulse can be calculated easily because of linear and non-hysteresis behavior of FBG in mentioned assumption. So, in this paper we assume that the peak input intensity is larger than the switching threshold intensity.

For taking into account these effects, we divided the Gaussian input pulse into two parts in time domain. One lies in negative side of time axis and the other lies in the positive side which are shown in Fig. 4-a and 4-b respectively. The pulse portion which lies in

the negative time is called leading part, and positive time called trailing part. First the leading part enters to the fiber while the trailing portion enters at latter time in the nonlinear FBG.

We calculate the Fourier transform (FT) of the leading and trailing parts of input pulse which are shown in Fig. 4-c and 4-d respectively. Because of the linearity of FT transformation, the FT of sum of two pulses is equal to sum of the FT of them.

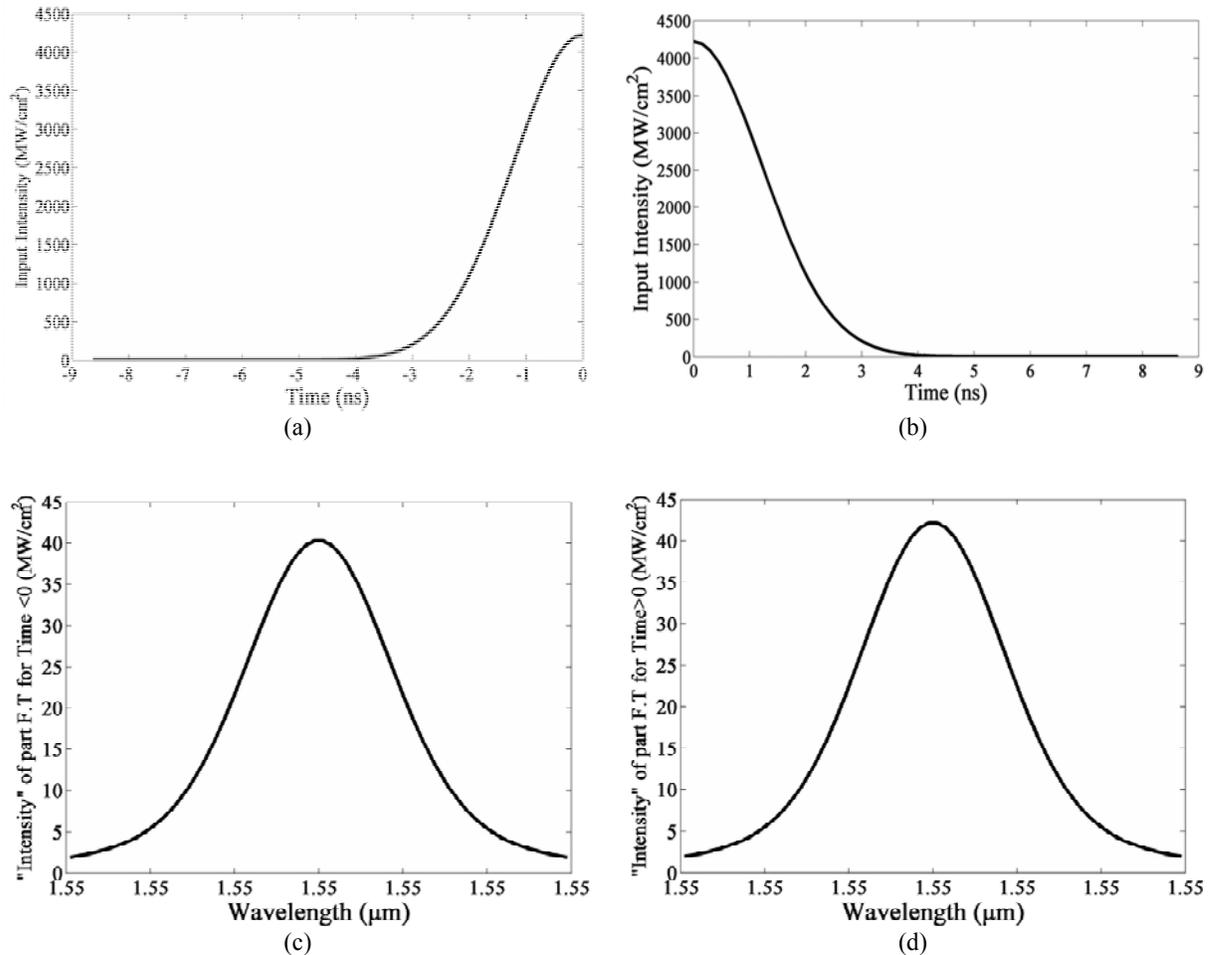


Fig. 4. (a) Leading part of input pulse (part 1), (b) trailing part of input pulse (part 2) (c) FT of leading part, (d) FT of trailing part.

To investigate the propagation of pulse in nonlinear FBG first we simulate leading part by using path1 of bistable diagram (Fig. 5-a), then we simulate trailing part by using path 2

of bistable diagram [11] (Fig. 5-b). We use FT of leading part of pulse (Fig. 4-c) and divided it to the same frequency interval which is obtained from Fig 3. Path 1 of each bistable

curve corresponding to each sample of frequency is used to obtain output intensity in frequency domain. In this stage the output intensity of leading portion of pulse is obtained which shown in Fig 6-a. A corresponding procedure is repeated for trailing part of the pulse. The FT of trailing of

the pulse (Fig. 4-d) is sampled in frequency domain corresponding to frequency divisions in Fig 3. By using path 2 of each bistable curve the corresponding output intensity is obtained in frequency domain. In the second stage the trailing portion of output pulse is obtained in frequency domain (Fig. 6-b).

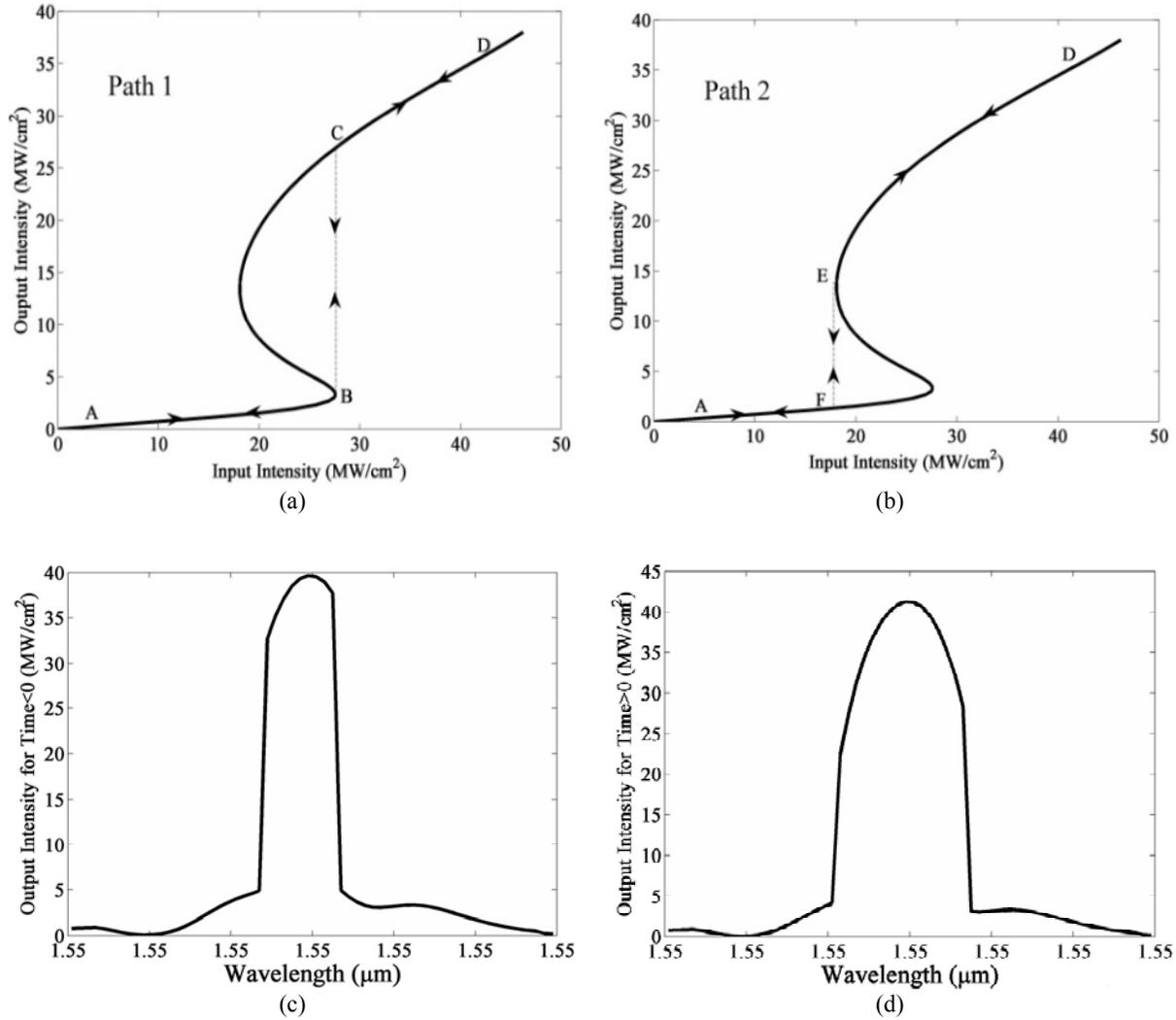


Fig. 5. (a) path 1 for negative time which shows in a typical bistability diagram, (b) path 2 for positive time which shows in a typical bistability diagram, (c) the output pulse for part 1 trajectory evolution, (d) the output pulse for part 2 trajectory evolution.

Finally the output pulse in frequency domain is obtained by combination of two leading and trailing part of pulse in frequency domain at output which is shown in Fig. 6-c. The output pulse in time domain is obtained by inverse

Fourier transform (IFT) of output pulse in frequency domain. Fig. 6-c show the unique solution of output pulse in time domain.

Finally, we obtain the unique solution for the output intensity in time domain by a simple

method. This method is very exact and simple comparison of last work [7] which overcome the problems that seen in previous papers [5, 6].

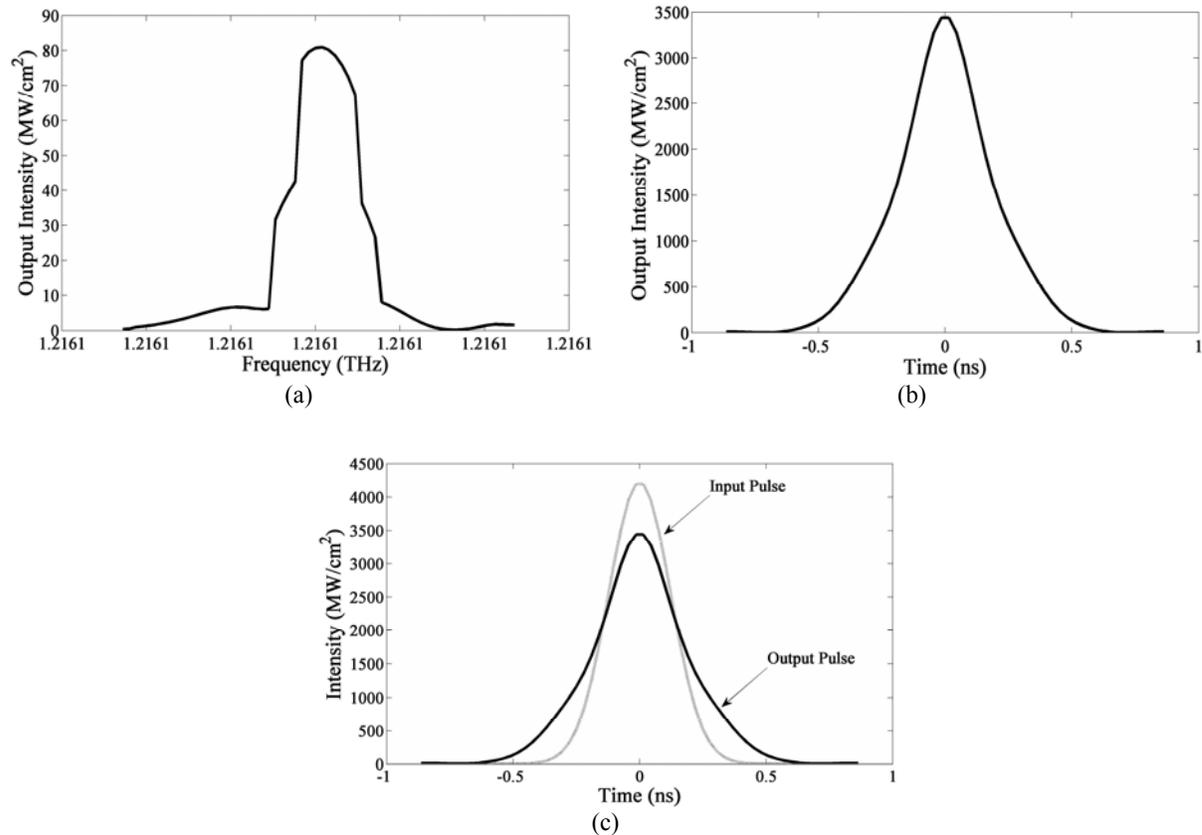


Fig. 6. (a) Output pulse in frequency domain, (b) output pulse in time domain and (c) comparison between input and output pulses.

#### IV. CONCLUSION

In this study, we have introduced a new and simple numerical method for obtaining the unique shape of output pulse nonlinear bistable fiber Bragg grating. Initially the input pulse is divided in to two parts in time domain. The Fourier transformation of each part is taken and the frequency response of each part is obtained in the output by using bistability frequency response of FBG. Finally the inverse Fourier transform is used to obtain the output pulse in time domain. This method can be used for pulse propagation in bistable systems. We have used this method for

propagation of a Gaussian pulse through the nonlinear bistable chalcogenide FBG.

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