Phase Properties of One-Dimensional Quaternary Photonic Crystals

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ABSTRACT— In this paper, properties of reflection phase in one-dimensional quaternary photonic crystals combining dispersive metamaterials and positive index materials are investigated by transfer matrix method. Two omni-directional band gaps are located in the band structure of considered structure. However, we limit our studies to the frequency range of the second wide band gap. We observe that the value of the reflection phase difference between TE and TM waves can be controlled by changing the incident angle and frequency. Also, the results show that the reflection phase difference in the second band gap increases by increasing the incident angle, and remains almost unchanged in a broad frequency band. Furthermore, at two points near to the edges of the gap the reflection phase difference keeps almost zero in spite of the change of incident angle. Based on these properties, phase compensators and omni-directionally synchronous reflectors and also polarizers can be designed.

KEYWORDS: Band Gap, Omni-Directionally Synchronous Reflectors, One-Dimensional Quaternary Photonic Crystals, Phase, Phase Compensators, Polarizers, Reflection

I. INTRODUCTION

Photonic crystals (PCs) have attracted considerable attention in recent years owing to their unique ability to control and manipulate light [1]. Conventional PCs are composed of periodic dielectric or metallo-dielectric nanostructures in which photonic band gaps (PBGs) exist due to the interference of multiple Bragg scattering [2]. Double-negative (DNG) or left handed (LH) materials with both negative electric permittivity and magnetic permeability in the same range of wavelength, exhibit different physical properties [3],[4] from the conventional right handed materials (RHMs). Besides DNG materials, single-negative (SNG) materials include the epsilon-negative (ENG) media with negative permittivity but positive permeability and mu-negative (MNG) media with negative permeability but positive permittivity, are also different from double-positive (DPS) materials [5]. It is demonstrated that stacking alternating layers of DPS and DNG media leads to a zero-$\bar{n}$ band gap in which the average refractive index of the structure becomes zero [6]-[10]. Also, it has been found that one-dimensional photonic crystals (1D-PCs) constituted by a periodic repetition of MNG and ENG layers can possess another type of photonic band gap with zero effective phase φ_{eff}, called the SNG band gap or the zero-φ_{eff} band gap [11]-[12]. Although the zero-n band gap and the zero-φ_{eff} band gap result from different mechanisms, they share many properties, which are quite distinct as compared to those of Bragg gaps. So far, most studies of PCs are based on the properties of the amplitude and frequency of transmitted and reflected light. By comparison, the investigations on phase properties are relatively fewer. Phase properties are representative properties of the light. Many important phenomena are related to the phase properties, such as superluminal phenomena and slow light in PCs [13]-[15]. So, the study of phase properties is as important as the properties of amplitude. In one study in 2008, Wu *et al.* studied phase engineering of 1D defective photonic crystals and their applications. They found two disadvantages that restricted the development and application of phase devices. The main problems were the rapid change of light intensity around the defect mode and influence of the substrate. They showed that many new sensitive phase PC devices will be developed, if some existing problems can be overcome [16]. In other study in 2011, the properties of phase of reflected waves from 1D-PCs composed of SNG materials were investigated. The results show that within two omni-directional gaps, the phase difference between reflected waves changes smoothly and increases with increasing the incident angle. Based on these studies, phase compensators can be designed [17]. In this paper, we show that 1D-PCs with a unit cell consisting of four layers made of MNG, ENG, DNG, and DPS materials can possess another wide gap which the phase properties of reflected waves from this gap and their potential applications are studied.

II. STRUCTURE MODEL AND CALCULATION METHOD

In this work we consider a 1D quaternary PC with the periodic structure of $(ABCD)^{10}$, where A, B, C, and D represent four layers made of MNG, ENG, DNG, and DPS materials, respectively. The permittivity and permeability of the constituents are denoted respectively by ε_j and μ_j, where the subscript *j*=1, 2, 3, 4 stands for the layer index. Layer thickness is denoted by d_j. We suppose that relative permittivity and permeability are given by: $\mu_j = 1 (j = 2,4)$; $\varepsilon_j = 4 (j = 1,4)$

$$\varepsilon_j(\omega) = 1 - \frac{\omega_{ep}^2}{\omega^2} \ (j = 2,3)\,, \tag{1}$$

$$\mu_j(\omega) = 1 - \frac{\omega_{mp}^2}{\omega^2} \ (j = 1,3)\,. \tag{2}$$

where ω is the angular frequency, ω_{ep} and ω_{mp} are the electric plasma frequency and the magnetic plasma frequency, respectively. Both ω_{ep} and ω_{mp} are set to be 10×109 rad/s [17]. Let a wave be incident from the vacuum onto the considered structure. The tangential (to the interfaces) components of the electric and the magnetic fields across the jth layer of width d_j are related by the following transfer matrix [18]:

$$M_j[\mathrm{d}_j] = \begin{pmatrix} \cos\gamma_j & -\frac{i}{p_j}\sin\gamma_j \\ -ip_j\sin\gamma_j & \cos\gamma_j \end{pmatrix} \tag{3}$$

with $\gamma_j = \frac{\omega}{c}\sqrt{\varepsilon_j}\sqrt{\mu_j}\, d_j \sqrt{1 - \frac{\sin^2\theta}{\varepsilon_j\mu_j}}$. Here c is the light speed in the vacuum. For TE wave, $p_j = \sqrt{\varepsilon_j}\big/\sqrt{\mu_j}\sqrt{1 - \frac{\sin^2\theta}{\varepsilon_j\mu_j}}$; for TM wave, $p_j = \sqrt{\mu_j}\big/\sqrt{\varepsilon_j}\sqrt{1 - \frac{\sin^2\theta}{\varepsilon_j\mu_j}}$. The whole transfer matrix of the structure i.e. $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ is the product of the matrices of each layer. Thus the reflection coefficient $r(\omega)$ is given by

$$r(\omega) = \frac{(m_{22} + m_{12}\cos\theta)\cos\theta - (m_{21} + m_{11}\cos\theta)}{(m_{22} + m_{12}\cos\theta)\cos\theta + (m_{21} + m_{11}\cos\theta)} \tag{4}$$

From $r(\omega)$, we can calculate the total phase shift φ of light wave reflected from the structure [19]:

$$\varphi = \tan^{-1}\frac{\mathrm{Im(r)}}{\mathrm{Re(r)}} \pm m\pi\,, \tag{5}$$

where *m* is an integer and is determined by the requirement that φ is a uniform function of frequency.

III. RESULTS AND DISCUSSION

In Fig. 1 (a) we plot the band structure of the mentioned structure. Here, the PBGs are denoted by blue (dark) areas. Also, in Fig. 1 (b) the reflectance of the structure is shown in the plane of incident angle and frequency. Here, the white areas are high reflectance regions (correspond to the PBGs). The left part of this figure shows the reflectance for the TE polarization and the right part shows the one for the TM polarization. Here, we consider the thicknesses of the layers as $d_1=d_4=10$ mm and $d_2=d_3=5$ mm. It is seen from both figures that there are two omni-directional band gaps in the band structure of our 1D-PC. Due to the wide width of the second band gap, we limit our investigations to the frequency range of this wide band gap.

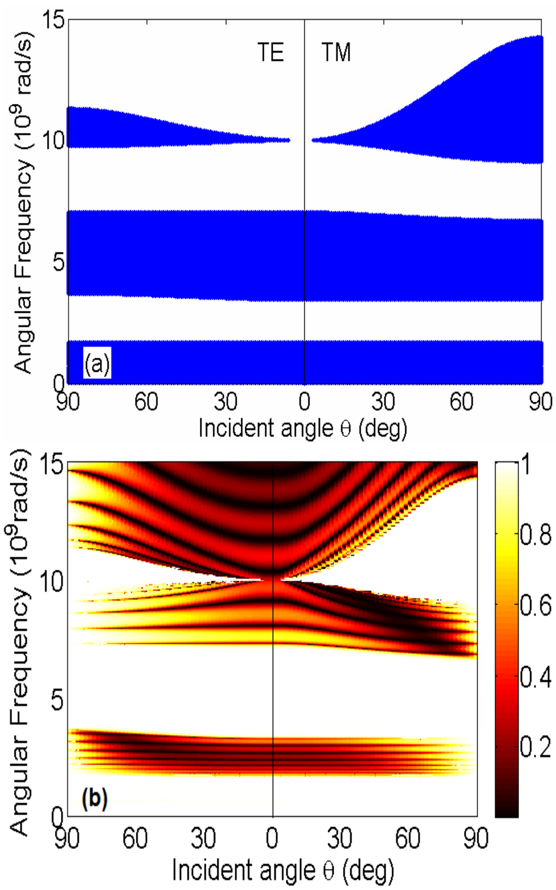

Fig. 1. (a) Photonic band structure of the structure $(ABCD)^{10}$ as a function of the angular frequency and incident angle for TE and TM waves. The blue (dark) areas correspond to the forbidden band gaps, and the white areas are the allowed bands. (b) Reflectance of this structure on the plane of angular frequency and incident angle. The white areas are high reflectance regions. Also, the left part of (b) shows the reflectance for the TE polarization and the right part show the one for the TM polarization. Here, we assumed that $d_1=d_4=10$ mm and $d_2=d_3=5$ mm.

Now, we calculate the dependence of the phase shift φ of the reflection waves on the frequency and incident angle for both TE and TM polarizations. The structural parameters are the same as those of Fig. 1. The results are shown in Fig. 2. The figure shows that the phase shift φ increases by increasing the incident angle for TE mode (see the inset), and decreases by increasing the incident angle for TM mode. Interestingly, at two points near the edges of the second omni-directional gap, the phase shift φ is invariant (apart from the mπ phase jumping) upon the change of incident angle for both polarizations. Now, what happens to the phase difference?

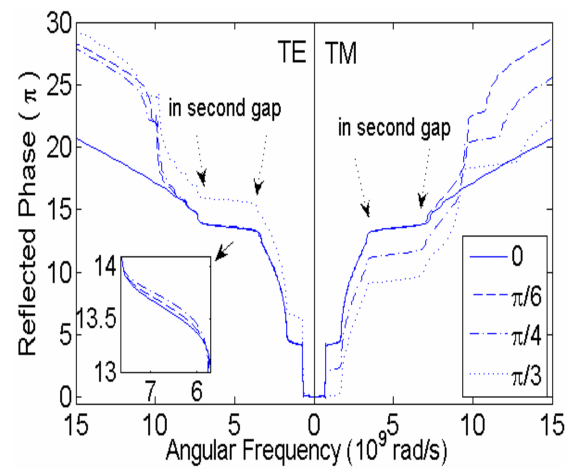

Fig. 2. The dependence of the phase shift of the reflection waves on the frequency and incident angle for TE and TM mode. The structural parameters are the same as those of Fig. 1.

To answer this question, we consider the dependence of the phase difference between two TE and TM polarized reflected waves $(\Delta\varphi = \varphi_{TM} - \varphi_{TE})$ on the frequency and the incident angle. The results are illustrated in Figs. 3 and 4. The Fig. 3 represents the dependence of the phase difference on the frequency for different incident angles θ= π/6, π/4, and π/3. One can see from this figure that within the second omni-directional gap the value of Δφ increases as the incident angle increases. In the central parts of the gap, the value of Δφ is almost constant. Moreover, there are two points near the edges of the band gap which the value of $\Delta\varphi$ is nearly zero.

These properties can be used to design a wide-band phase compensator and an omni-directionally synchronous reflector and also phase controllers.

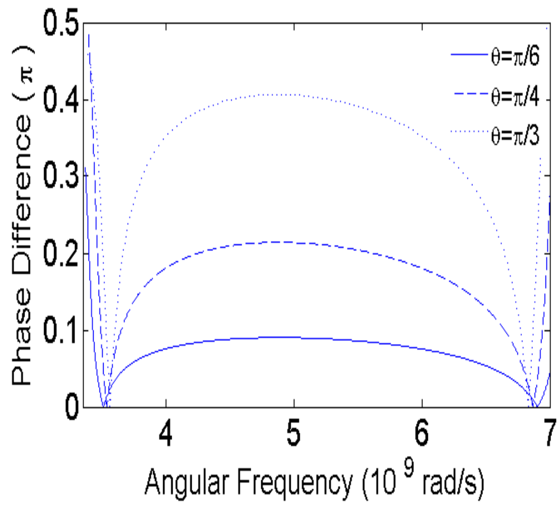

Fig. 3. The dependence of the phase difference on the frequency for different incident angles. The parameters are the same as those of Fig. 1.

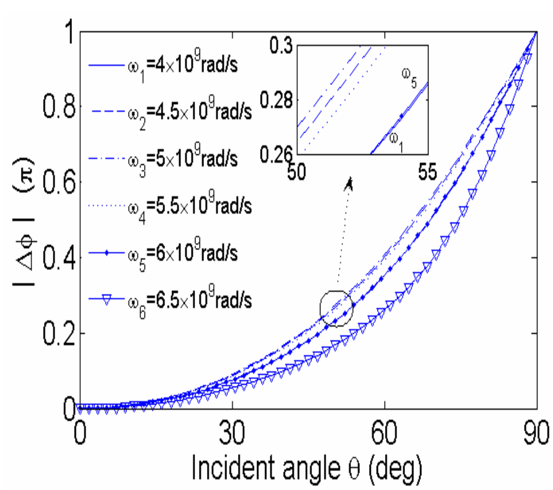

Fig. 4. The curves of the phase difference ($|\Delta\varphi|$) vs. the incident angles for different frequencies within the second omni-directional gap. The parameters are the same as those of Fig. 1.

For further inspection, we plot the dependence of Δφ on the incident angle at different frequencies in Fig. 4. As it can be seen from this figure, in the frequency range of 4×10^9rad/s<ω<6×10^9rad/s the curves almost overlap. Especially, in the interval 4.5×10^9rad/s<ω<5.5×10^9rad/s, the value of Δφ is nearly independent from the frequency. In other words, in this range, Δφ changes only with the incident angle. As a result, this structure can be used as wide-band phase compensators. In addition, at both edges of the band gap the value of Δφ is almost zero for all the incident angles. So, this structure can be also used as omni-directionally synchronous reflectors. Also, in the previous studies the phase properties of the binary PCs have been investigated [17, 18]. However, the main advantage of our used quaternary PC is its wide width band gap. As a result, one can use this structure as a phase compensator in a wide frequency range.

IV. Conclusion

In summary, using the transfer matrix method, the phase properties of the reflected waves from the second band gap of 1D quaternary PC composed of SNG, DNG, and DPS materials are investigated. The results show that the value of phase difference between two TE and TM polarized reflected waves (Δφ) increases by increasing the incident angle. In the central part of the gap, the value of Δφ is almost constant. At the edges of the second band gap the values of Δφ is nearly zero for all the incident angles. These properties can be engineered for designing a wide-band phase compensator, a synchronous reflector and phase controllers.

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