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Propagation and Interaction of Electrostatic and Electromagnetic Waves in Two Stream Free Electron Laser in the Presence of Self-Fields

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ABSTRACT— A relativistic theory for two-stream free electron laser (FEL) with a one-dimensional helical wiggler and ion-channel guiding in the presence of self-fields are presented. A dispersion relation (DR) which includes coupling between the electromagnetic and the electrostatic waves is derived from a fluid model, with all of the relativistic terms related to the transverse wiggler motion. This DR is solved numerically to study many unstable couplings among all possible modes. Numerical calculations are made to illustrate the effects of the self-fields on the unstable couplings. It is shown that the self-fields can produce large effects on the growth rate of the couplings.

KEYWORDS: Free-electron laser, self-fields, ion-channel, instability, two-tream

I INTRODUCTION

The free electron laser (FEL) is a device that uses a relativistic beam of electrons passing through a transverse periodic magnetic field (wiggler) to produce and amplify electromagnetic radiation. The wavelength of the emitted radiation depends on the period of the wiggler, the strength of its magnetic field and the electron energy [1]. In a FEL, ion channel guiding is applied to collimate the intense relativistic electron beam. The ion channel is an ion column produced by the head of the beam when the peak beam density exceeds the plasma density. The radial electric field of the beam blows out the plasma electrons, transversely, creating an ion channel. This ion channel confines the electron in transverse direction against self-fields effects. This technique was first proposed by Takayama and Hiramatsu [2] for use in FEL and was tested first by Ozaky *et al.* [3], and was evaluated by a numerical simulation [4-5]. The equilibrium trajectory of an electron and growth rate of waves in a combined helical wiggler and ion channel guiding were studied [6-12].

In FEL in the Raman regime, due to the low energy and high density of the beam, self-electric and self-magnetic fields can have considerable effects on gain and equilibrium orbits [13]. Mirzanejhad *et al.* found the self-fields effects on the dispersion relation and growth rate [14]. These studies are based on a single beam FEL with ion-channel guiding.

Two-stream free electron lasers (TSFEL), FELs in which two beams copropagate with different beam velocities, have been studied during the last few decades [15-22]. Two-stream was first proposed by Bekefi and Jacobs for use in FELs [23]. They have shown that the growth rate of electromagnetic and electrostatic waves are considerably affected by two-stream instability (TSI). The TSFEL, including its operation principle, design schemes, and mathematical descriptions were studied by Kulish *et al.* [24-25]. The effects of two-stream instability on the growth rate of waves in a helical wiggler free electron laser have also been investigated [26-27].

In this paper, we aim to derive a dispersion relation for the interaction of all the waves in a FEL with two-stream and helical wiggler and ion-channel guiding. The self-fields of electron beams and all of relativistic effects are included in the dispersion relation. This DR is solved numerically to study the effects of the self-fields on unstable couplings. In Sec. II, basic equations for the relativistic theory are introduced and self-fields are calculated. In Sec. III, three coupled equations are derived and a formula for the general DR is obtained. In Sec. IV, a numerical analysis is carried out to investigate the self-fields effects on the unstable couplings between the waves. In Sec. V, the paper is concluded.

II SELF-FIELDS CALCULATION

The configuration we employ is that of two electron beams (each of uniform density n_{0s} and velocity v_{0s} with $s=1$ and 2) propagating in the z-direction through a helical magnetic described by

$$\mathbf{B}_w = B_w\left(\hat{\mathbf{x}}\cos k_w z + \hat{\mathbf{y}}\sin k_w z\right), \tag{1}$$

B_w is the wiggler amplitude and $k_w(=2\pi/\lambda_w)$ is the wiggler wave number. In the presence of an ion-channel, and while its axis is coincident with the wiggler axis, the following transverse electrostatic field is acted upon the electron:

$$\mathbf{E}_i = 2\pi e n_i\left(x\hat{\mathbf{x}} + y\hat{\mathbf{y}}\right), \tag{2}$$

where n_i is the number density of positive ions with charge e.

The self-electric field induced by steady-state charge density of the electron beam can be obtained by solving Poisson's equation

$$\mathbf{E}_{ss} = -2\pi n_{bs} e r\,\hat{\mathbf{r}} = -2\pi n_{bs} e\left(x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}}\right). \tag{3}$$

The self magnetic field induced by transverse and axial velocity may be obtained by Ampere's law,

$$\nabla\times\mathbf{B}_{ss} = \frac{4\pi}{c}\mathbf{J}_{bs}\,, \tag{4}$$

where $\mathbf{J}_s = -en_{0s}\left(\mathbf{v}_{\perp s} + v_{\parallel s}\hat{\mathbf{z}}\right)$ is the beam current density and $\mathbf{v}_{\perp s}$ is transverse velocity, generated by the wiggler magnetic field. By the method of Ref. [13] B_s may be found as

$$\mathbf{B}_{ss} = \frac{\omega_{ps}^2\left(v_{\parallel s}/c\right)^2}{\omega_i^2 - \omega_{ps}^2\left(1+v_{\parallel s}^2/c^2\right)/\sqrt{2} - k_w^2 v_{\parallel s}^2}\mathbf{B}_w - 2\pi n_{0s} e\frac{v_{\parallel s}}{c}\left(y\,\hat{\mathbf{x}} - x\,\hat{\mathbf{y}}\right), \tag{5}$$

where $\omega_{ps} = \left(4\pi n_{0s} e^2/\gamma_{0s} m_0\right)^{1/2}$ and $\omega_{is}^2 = 2\pi n_i e^2/(\gamma_{0s} m_0)$.

The steady-state solution of the equation of motion with constant axial velocity in the presence of the self-fields an be written as:

$$\mathbf{v}_{0s} = v_{ws}\left(\hat{\mathbf{x}}\cos k_w z + \hat{\mathbf{y}}\sin k_w z\right) + v_{\parallel s}\hat{\mathbf{z}}\,, \tag{6}$$

where

$$v_{ws} = \frac{k_w c\,\Omega_{ws}\left(v_{\parallel s}/c\right)^2}{\omega_{is}^2 - \omega_{ps}^2\left(1+v_{\parallel s}^2/c^2\right)/2 - k_w^2 v_{\parallel s}^2}\,. \tag{7}$$

The steady-state trajectories may be divided into two classes, group I with $v_{ws}<0$ and group II with $v_{ws}>0$. With the assumption that during the time the electron moves through one wiggler wavelength it also rotates through one complete turn, the relation $R_{0s} = v_{ws}/k_w v_{\parallel s}$ can be obtained. For lower values of normalized ion-channel frequency, for group II orbits, the quantity

$$\Phi_s = 1 - \Omega_{ws}^2 k_w^2 v_{\parallel s}^2\left[\left(\omega_{is}^2 - \omega_{ps}^2\right)\left(1+\gamma_{\parallel s}^2\right) - \gamma_{\parallel s}^2\omega_{ps}^2\right] \left\{\left[\omega_{is}^2 - \omega_{ps}^2\left(1+v_{\parallel s}^2/c^2\right) - v_{\parallel s}^2/c^2\right]^3 + 2\Omega_{ws}^2 \left(\omega_{is}^2 - \omega_{ps}^2\right)\left(v_{\parallel s}/c\right)^2\right\}, \tag{8}$$

is negative. This implies the existence of a negative-mass regime in which the axial velocity will decrease with increasing energy.

III DISPERSION RELATION

An analysis of the wave propagation in the two-stream FEL with ion-channel guiding will be based on the electron continuity equation, the cold-electron relativistic momentum equation and the wave equation. The perturbed state can be considered as the unperturbed state $n_{0s} = const.$, $\mathbf{v}_{0s}$, $\mathbf{B}_0$, and $\mathbf{E}_0$ plus small perturbations δn_s, $\delta \mathbf{v}_s$, $\delta \mathbf{E}$, $\delta \mathbf{B}$, and $\delta \mathbf{R}_s$ where $\mathbf{R}$ is the radial position of electrons. By substituting these quantities, linearized equations for the continuity equation, relativistic momentum equation, and the wave equation may be derived as

$$\frac{\partial \delta n_s}{\partial t} + n_{0s}\nabla \cdot \delta \mathbf{v}_s + \mathbf{v}_{0s} \cdot \nabla \delta n_s = 0\,, \tag{9}$$

$$\frac{\partial \delta \mathbf{v}_s}{\partial t} + \mathbf{v}_{0s} \cdot \nabla \delta\mathbf{v}_s + \delta\mathbf{v}_s \cdot \nabla \mathbf{v}_{0s} = -\frac{e}{m_0\gamma_{0s}} \times \Bigg[\delta \mathbf{E} - \frac{1}{c^2}\mathbf{v}_{0s}\mathbf{v}_{0s} \cdot \delta \mathbf{E} - \frac{1}{c^2}\mathbf{v}_{0s}\delta \mathbf{v}_s \cdot \mathbf{E}_0 - \frac{1}{c^2} \times \delta \mathbf{v}_s \mathbf{v}_{0s} \cdot \mathbf{E}_0 + \frac{1}{c}\delta \mathbf{v}_s \times \mathbf{B}_0 + \frac{1}{c}\mathbf{v}_{0s} \times \delta \mathbf{B} - \frac{\gamma_{0s}^2}{c^2} \times (\mathbf{E}_0 - \frac{1}{c^2}\mathbf{v}_{0s}\mathbf{v}_{0s} \cdot \mathbf{E}_0 + \frac{1}{c}\mathbf{v}_{0s} \times \mathbf{B}_0)(\mathbf{v}_{0s} \cdot \delta\mathbf{v}_s)\Bigg], \tag{10}$$

$$\nabla \times (\nabla \times \delta \mathbf{E}) + \frac{1}{c^2}\frac{\partial^2 \delta \mathbf{E}}{\partial t^2} = \frac{\partial}{\partial t}\frac{4\pi e}{c^2}\left(\delta n_s \mathbf{v}_{0s} + n_{0s}\delta \mathbf{v}_s\right). \tag{11}$$

By introducing a new set of basis vectors $\hat{\mathbf{e}} = (\hat{\mathbf{x}} + i\,\hat{\mathbf{y}})/\sqrt{2}$, $\hat{\mathbf{e}}^* = (\hat{\mathbf{x}} - i\,\hat{\mathbf{y}})/\sqrt{2}$, and $\hat{\mathbf{e}}_z = \hat{\mathbf{z}}$, the unperturbed quantities can be written as

$$\mathbf{R}_{0s} = i\sqrt{2}\, R_{0s}\left[\exp(-ik_w z)\,\hat{\mathbf{e}} + \exp(ik_w z)\hat{\mathbf{e}}^*\right], \tag{12}$$

$$\mathbf{E}_{is} = i\sqrt{2}\,\pi e\left(n_i - n_{bs}\right)R_{0s}\left[\exp(-ik_w z)\,\hat{\mathbf{e}} + \exp(ik_w z)\hat{\mathbf{e}}^*\right], \tag{13}$$

$$\mathbf{B}_0 = \left(\frac{1}{\sqrt{2}}\right)\left(-2\pi e n_{bs}R_{0s}\frac{v_{\parallel s}}{c} + \lambda\, B_w\right) \times \left[\exp(-ik_w z)\hat{\mathbf{e}} + \exp(ik_w z)\hat{\mathbf{e}}^*\right], \tag{14}$$

$$\mathbf{v}_{0s} = \left(v_{ws}/\sqrt{2}\right)\left[\exp(-ik_w z)\hat{\mathbf{e}} + \exp(ik_w z)\hat{\mathbf{e}}^*\right] + v_{\parallel s}\hat{\mathbf{e}}_z. \tag{15}$$

The perturbed state is assumed to consist of a longitudinal space-charge wave and right and left circularly polarized electromagnetic waves with all perturbed waves propagating in the positive z direction,

$$\delta \mathbf{E} = \left[2\pi\left(n_i - n_{bs}\right)e\delta R_R + \delta E_R\right]\hat{\mathbf{e}} + \left[2\pi e\,\delta R_L \times \left(n_i - n_{bs}\right) + \delta E_L\right]\hat{\mathbf{e}}^* + \delta E_z \hat{\mathbf{e}}_z, \tag{16}$$

$$\delta \mathbf{B} = \left(-i\,2\pi e n_{bs}\beta_{\parallel s}\delta R_R + \delta B_R\right)\hat{\mathbf{e}} + \left(-i\,2\pi e \times n_{bs}\beta_{\parallel s}\delta R_L + \delta B_L\right)\hat{\mathbf{e}}^*, \tag{17}$$

$$\delta \mathbf{v}_s = \delta v_{Rs}\hat{\mathbf{e}} + \delta v_{Ls}\hat{\mathbf{e}}^* + \delta v_{zs}\hat{\mathbf{e}}_z\,, \tag{18}$$

$$\delta \mathbf{R} = \delta R_R \hat{\mathbf{e}} + \delta R_L \hat{\mathbf{e}}^*, \tag{19}$$

$$\delta n_s = \tilde{n}_s \exp[i(kz - \omega t)], \tag{20}$$

$$\delta v_{Rs} = \tilde{v}_{Rs} \exp[i(k_R z - \omega t)], \tag{21}$$

$$\delta v_{Ls} = \tilde{v}_{Ls} \exp[i(k_L z - \omega t)], \tag{22}$$

$$\delta v_{zs} = \tilde{v}_{zs} \exp[i(kz - \omega t)]. \tag{23}$$

δv_{zs} and δE_z are analogous to δn_s; δE_R and δB_R are analogous to δv_{Rs}; δE_L and δB_L are analogous to δv_{Ls}; the wave numbers are related to $k_R = k - k_w$ and $k_L = k + k_w$.

By substituting the perturbed quantities in the relativistic momentum equation, after some algebra operations, components of the perturbed velocity $(\tilde{v}_{Rs}, \tilde{v}_{Ls}, \tilde{v}_{zs})$ versus components of the perturbed electric field are found. Substituting the perturbed quantities into the linearized wave equation gives three equations for the amplitude of the perturbed quantities, with eliminating $\tilde{v}_{Rs}, \tilde{v}_{Ls}$, and $\tilde{v}_{zs}$ the system of the equation will reduce to

$$\begin{cases} K_1\tilde{E}_R + K_2\tilde{E}_L + K_3\tilde{E}_z = 0 \\ K_4\tilde{E}_R + K_5\tilde{E}_L + K_6\tilde{E}_z = 0 \\ K_7\tilde{E}_R + K_8\tilde{E}_L + K_9\tilde{E}_z = 0 \end{cases} \tag{24}$$

$$K_1 = k_R^2 c^2 - \omega^2 + \sum_s \left(B_{5s}\kappa_{1zs} + \omega\omega_{ps}^2\,\kappa_{1Rs}\right), \tag{25}$$

$$K_2 = \sum_s \left(- B_{5s}\kappa_{2zs} - \omega\omega_{ps}^2\,\kappa_{2Rs} \right), \tag{26}$$

$$K_3 = \sum_s \left(- B_{5s}\kappa_{3zs} + \omega\omega_{bj}^2\,\kappa_{3Rs} \right), \tag{27}$$

$$K_4 = \sum_s \left(B_{5s}\kappa_{1zs} - \omega\omega_{ps}^2\,\kappa_{1Ls} \right), \tag{28}$$

$$K_5 = k_L^2c^2 - \omega^2 + \sum_s \left(- B_{5s}\kappa_{2zs} + \omega\omega_{ps}^2\,\kappa_{2Ls} \right), \tag{29}$$

$$K_6 = \sum_s \left(- B_{5s}\zeta_{3zs} + \omega\omega_{ps}^2\,\kappa_{3Ls} \right), \tag{30}$$

$$K_7 = -\sum_s \left(B_{6s}\kappa_{1zs} \right), \tag{31}$$

$$K_8 = \sum_s \left(B_{6s}\kappa_{2zs} \right), \tag{32}$$

$$K_9 = 1 + \sum_s \left(B_{6s}\kappa_{3zs} \right). \tag{33}$$

Equation (24) shows that the DR for the right circularly polarized electromagnetic wave in the absence of the other two waves and the wiggler, is

$$\left(k_R^2c^2 - \omega^2\right)\left[\omega - k_R v_{\|s} - \frac{\omega_{is}^2 - \omega_{ps}^2}{2\gamma_{\|s}^2\left(\omega - k_R v_{\|s}\right)}\right] + \omega_{ps}^2\left(\omega - k_R v_{\|s}\right) = 0, \tag{34}$$

and the DR for the left circularly polarized electromagnetic wave in the absence of the other two waves and the wiggler, is

$$\left(k_L^2c^2 - \omega^2\right)\left[\omega - k_L v_{\|s} - \frac{\omega_{is}^2 - \omega_{ps}^2}{2\gamma_{\|s}^2\left(\omega - k_L v_{\|s}\right)}\right] + \omega_{ps}^2\left(\omega - k_L v_{\|s}\right) = 0. \tag{35}$$

It should be noted that DRs, in the absence of the wiggler, for the right and left circularly polarized electromagnetic waves are similar. Each DR for the electromagnetic waves consists of three modes, i.e. right-betatron plus (R_{b+}), right-betatron minus (R_{b-}), and right-escape (R_e) for the right wave; and left-betatron plus (L_{b+}), left-betatron minus (L_{b-}), and left-escape (L_e) modes for the left circularly polarized electromagnetic wave. The DR for the space-charge wave in the absence of the wiggler, the right and left waves is

$$\frac{\omega_{p1}^2}{\gamma_{\|1}^2(\omega - kv_{\|1})^2} + \frac{\omega_{p2}^2}{\gamma_{\|2}^2(\omega - kv_{\|2})^2} = 1. \tag{36}$$

Dispersion relation (36) consists of four physical modes, i. e., slow and fast (negative and positive energy) space-charge modes of the faster beam $(Sc_{2\pm})$ and slow and fast space-charge modes of the slower beam $(Sc_{1\pm})$. The TSI can be described in terms of the coupling of the slow mode carried by a faster beam and the fast mode carried by a slower beam.

The necessary and sufficient condition for a nontrivial solution consists of the determinate of coefficients in Eqs. (24)-(26) to be equal to zero. Imposing this condition yields the dispersion relation

$$K_1(K_5K_9 - K_6K_8) + K_2(K_6K_7 - K_4K_9) + K_3(K_4K_8 - K_5K_7) = \circ, \tag{37}$$

Equation (37) is the DR for coupled electrostatic and electromagnetic waves propagating along with two relativistic electron beams in the presence of self-fields and an ion-channel guiding.

IV NUMERICAL ANALYSIS

In this section, numerical results are presented for a two-stream free electron laser with a helical wiggler and ion-channel guiding in the presence of the self-fields. In order to investigate self-fields effects, we dispersion relation will be solved numerically. The parameters are $\omega_{p1}/k_wc = 0.5436$, $\omega_{p2}/k_wc = 0.7444$, $B = 1kG$, $k_w = 2\,cm^{-1}$, $\gamma_{01} = 6$, $\gamma_{02} = 5.6$. The roots of the DR (37) are found numerically for group I orbits with $\omega_{is}/k_wc = 0.2$. The positive and negative space-charge waves ($Sc_{1\pm}$ and $Sc_{2\pm}$) supported by each electron beam and the escape branch of

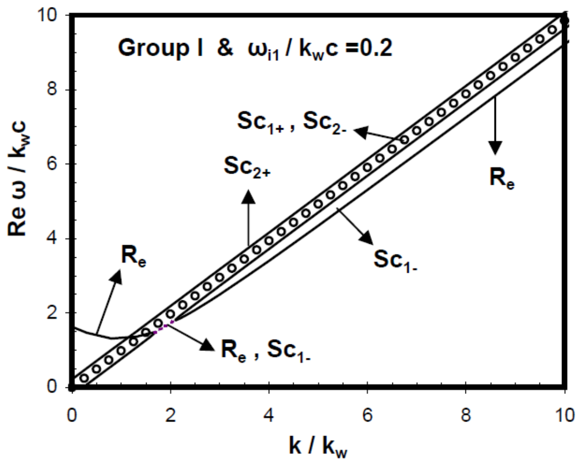


Fig. 1. Couplings of the FEL resonance and two-steam instability for group I orbits. Dotted line and circles line denote complex-conjugate roots.

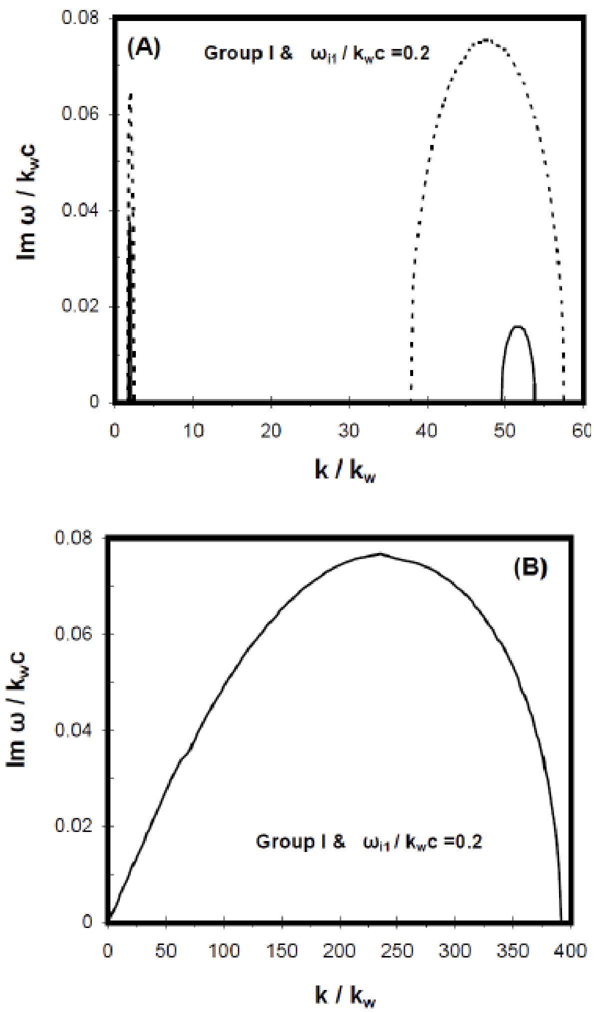


Fig. 2. Normalized growth rate $\mathrm{Im}\,\omega/k_w c$ versus k/k_w for group I orbits. (A) for FEL coupling, (B) for two-stream instability.

the right escape wave (R_e) are shown in Fig. 1. Circles line shows that the negative-energy space-charge wave of the fast electron beam (Sc_{2-}) couples with the positive-energy apace-charge wave of the slow electron beam (Sc_{1+}), this coupling is called the two-stream instability.

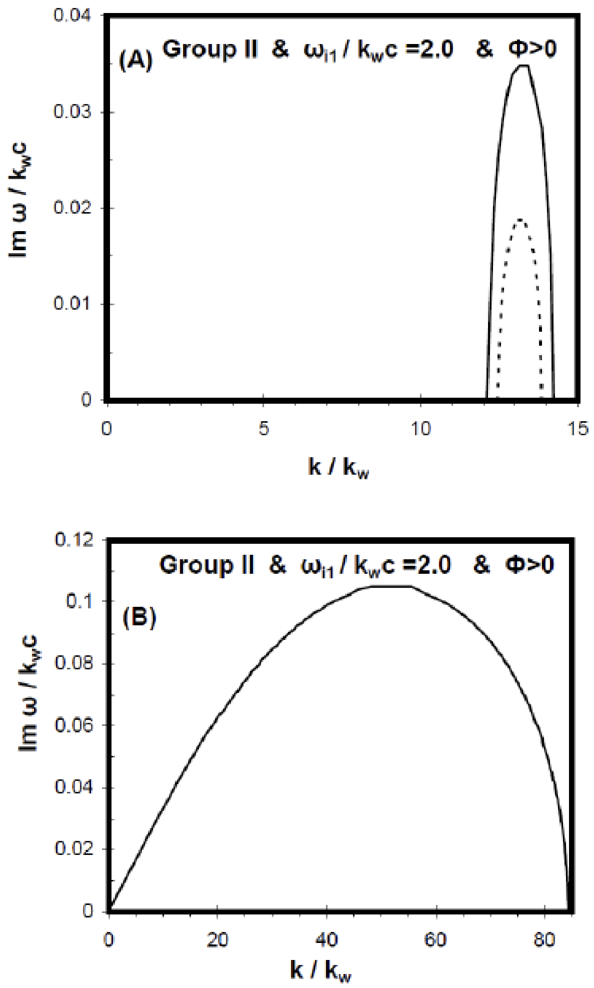
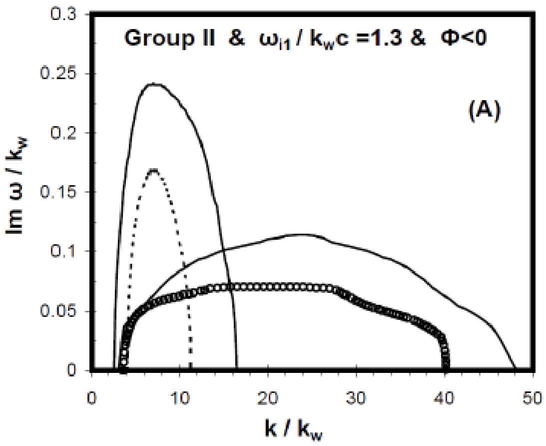


Fig. 3. Normalized growth rate $\mathrm{Im}\,\omega/k_w c$ versus k/k_w for group II orbits with $\Phi > 0$. (A) for FEL coupling, (B) for two-stream instability.

In Fig. 1, also the wide spectrum coupling between the escape mode and the slow mode for slow beam, is called the FEL coupling, are shown by dotted line. Fig. 2 shows the normalized imaginary part of frequency $\mathrm{Im}\,\omega/k_w c$ as a function of the normalized k/k_w for the FEL coupling and the two-stream instability with $\omega_{i1}/k_w c = 0.2$. The downshifted

FEL coupling begins at $k/k_w = 1.69$ and ends at $k/k_w = 2.08$ that the maximum growth rate happens at $k/k_w = 1.87$ with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.0375$. The upshifted FEL coupling begins at $k/k_w = 49.5$ and ends at $k/k_w = 53.8$. Coupling between Sc_{1+} and Sc_{2-} is at $0 \le k/k_w \le 392.04$ in Fig. 2(B) with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.0767$ at $k/k_w = 235.62$.



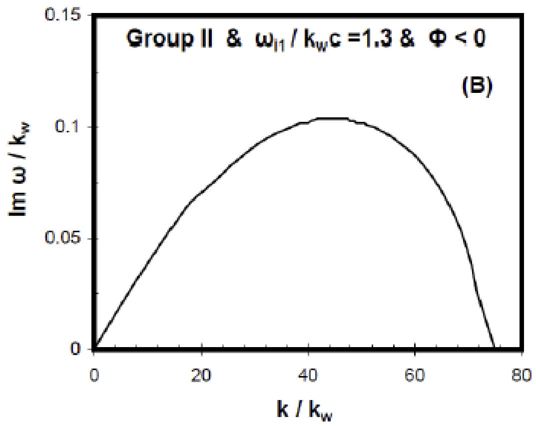


Fig. 4. Normalized growth rate $\mathrm{Im}\,\omega/k_w c$ versus k/k_w for group II orbits with $\Phi < 0$. (A) for FEL coupling, (B) for two-stream instability.

In Fig. 2(A), the FEL coupling in the absence of self-fields for comparison is shown with dotted line. For the group I orbits in the presence of self-fields, the maximum growth rate for upshifted FEL coupling happens at $k/k_w = 51.6$ with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.0158$ but in the absence of self-fields it happens at $k/k_w = 47.6$ with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.0753$. In the absence of self-fields, two curves (upshifted and downshifted FEL coupling) coalesce. In group I orbits, self-fields reduce the growth about 79%.

In the upper part of group II orbits, in which Φ_j is positive, Dispersion relation (37) are solved numerically. The parameters are $\omega_{p1}/k_w c = 0.9415$, $\omega_{p2}/k_w c = 0.9870$, $B = 1kG$, $k_w = 2\,cm^{-1}$, $\gamma_{01} = 3$, $\gamma_{02} = 3.2$. The coupling between R_{b+} and Sc_{1-} modes and between Sc_{2-} and Sc_{1+} modes for group II orbits with $\omega_{i1}/k_w c = 2.0$ and $\Phi_s > 0$ are shown in Fig. 3. The FEL coupling begins at $k/k_w = 12.10$ and ends at $k/k_w = 14.24$, and two-stream instability begins at $k/k_w = 0$ and continues to $k/k_w = 84.46$ that the maximum growth rate happens at $k/k_w = 52.53$ with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.105$. The FEL coupling in the absence of self-fields for comparison is shown with dotted line in Fig. 3(A). In the presence of self-fields for the group I orbits with $\Phi_j > 0$, the maximum growth rate of FEL coupling happens at $k/k_w = 13.29$ with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.0348$ but in the absence of self-fields it happens at $k/k_w = 13.20$ with $\mathrm{Im}(\omega/k_w c)_{\max} = 0.0188$. In group II orbits, self-fields increase the growth rate by 85% and widens the width of the unstable spectrum.

For the group II orbits, when Φ_s is negative, the wiggler induced velocity v_w is large in negative mass regime. The parameters are $\omega_{p1}/k_w c = 0.9415$, $\omega_{p2}/k_w c = 0.9870$, $B = 1kG$, $k_w = 2\,cm^{-1}$, $\gamma_{01} = 3$, $\gamma_{02} = 3.2$, with $\omega_{i1}/k_w c = 1.3$. The coupling between the cyclotron branch of the right betatron plus wave (R_{b+}) and the slow mode (Sc_{2-}) of the slower electron beam are shown in Fig. 4(A) for group orbits with $\omega_{i1}/k_w c = 1.3$ and $\Phi_s < 0$. Also, in this Fig., coupling between the right and the left waves are shown. This coupling does not exist, for the group II orbits, when Φ_s is positive. This coupling starts at $k/k_w = 3.06$ and continues to $k/k_w = 48.25$ with

$\mathrm{Im}\left(\omega/k_w c\right)_{\max}=0.114$ at $k/k_w=24.37$ in Fig. 4(A). The $R_{b+}-Sc_{1-}$ coupling starts at $k/k_w=2.53$ and continues to $k/k_w=16.44$ that the maximum growth rate occurs at $k/k_w=6.96$ with $\mathrm{Im}\left(\omega/k_w c\right)_{\max}=0.242$ in Fig. 4(A). In Fig. 6(B) the two-stream instability starts at $k/k_w=0$ and continues to $k/k_w=74.97$ with $\mathrm{Im}\left(\omega/k_w c\right)_{\max}=0.104$ at $k/k_w=44.47$. The FEL coupling and coupling of the right and the left waves in the absence of self-fields for comparison is shown with dotted line and circles line in Fig. 4(A), respectively. In group II orbits with $\Phi_s>0$, self-fields increase the growth rate of FEL coupling and the coupling of the right and the left waves by 43% and 61%, respectively.

V CONCLUSION

The purpose of the present paper is to study the self-fields effects on a two-stream FEL with helical wiggler and ion-channel guiding. A general DR is derived for a FEL with a helical wiggler and ion-channel guiding in the presence of self-fields when two relativistic electron beams propagate in parallel to each other. In order to investigate the self-fields effects on unstable couplings, dispersion relation (37) is solved numerically. The results that take into account the self-fields are compared with Ref. 24 that investigated two-stream FEL in the absence of the self-fields. The FEL resonance is supported by the unstable coupling of the negative energy space-charge mode on the faster beam with the right circularly polarized electromagnetic wave.

In order to analyze self–field effects, the effective wiggler magnetic field has been defined as $(B_w)_{eff}=\lambda B_w$, where λ is given by equation 22. In the absence of the self-fields, λ is equal to 1. For group I orbits, λ is smaller than 1, thus the effective wiggler magnetic field, which is the cause of the waves' coupling, will be smaller than B_w, and growth rate will reduce in comparison with the case where there are no self-fields. As for the group II orbits, λ is larger than 1, thus the coupling cause is further strengthened, and growth rate increases. The increase or decrease of growth rate in the presence of self-fields can be described by the change in the rate of transverse velocity, since beam energy is introduced by $d\gamma/dt=-\left(e/m_0c^2\right)\mathbf{v}\cdot\mathbf{E}$. Considering self-fields, transverse velocity of the electron beam decreases for the orbits of group I. However, when considering self-fields for group II orbits, transverse velocity of the electron beam increases, which causes an increase in the energy transfer from the beam to the wave. Consequently, for the orbits of group I and II, transverse velocity of the electron beam in the presence of self-fields, decreases and increases, respectively, resulting in growth rate decrease for the former and increase for the latter.

Two-stream instability is due to the interaction of two longitudinal motions associated with the negative energy space-charge mode on the faster beam with the positive energy space-charge mode on the slower beam. Therefore, we should expect the wiggler induced transverse motion and the resulting self-fields not to influence the two-stream instability.

APPENDIX

The following quantities are used in Eqs. (25)-(30):

$$\kappa_{1Rs}=\left\{\left[\left(1-\frac{v_{ws}^2}{2c^2}-\frac{k_R v_{\parallel s}}{\omega}\right)B_{1Ls}-\frac{v_{ws}^2}{2c^2}B_{2Ls}\right]-\left[B_{2s}\left(B_{1Ls}+B_{2Rs}\right)\right]\kappa_{1zs}\right\}B_{4s}^{-1}, \tag{38}$$

$$\kappa_{2Rs}=\left\{\left[\left(1-\frac{v_{ws}^2}{2c^2}-\frac{k_L v_{\parallel s}}{\omega}\right)B_{2Rs}+\frac{v_{ws}^2}{2c^2}B_{1Ls}\right]-\left[B_{2s}\left(B_{1Ls}+B_{2Rs}\right)\right]\kappa_{2zs}\right\}B_{4s}^{-1}, \tag{39}$$

$$\kappa_{3Rs}=\left\{-\left[B_{2s}\left(B_{1Ls}+B_{2Rs}\right)\right]\kappa_{3zs}+\frac{v_{ws}v_{\parallel s}}{\sqrt{2}c^2}\times\left(B_{1Ls}-B_{2Rs}\right)\right\}B_{4s}^{-1}, \tag{40}$$

$$\kappa_{1Ls} = \left[B_{1Rs}\kappa_{1Rs} - \left(1 - \frac{v_{ws}^2}{2c^2} - \frac{k_R v_{\|s}}{\omega}\right) + B_{2s}\kappa_{1zs} \right] B_{2Rs}^{-1}, \tag{41}$$

$$\kappa_{2Ls} = \left[B_{1Rs}\kappa_{2Rs} - \frac{v_{ws}^2}{2c^2} + B_{2s}\kappa_{2zs} \right] B_{2Rs}^{-1}, \tag{42}$$

$$\kappa_{3Ls} = \left[B_{1Rs}\kappa_{3Rs} - \frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} + B_{2s}\kappa_{3zs} \right] B_{2Rs}^{-1}, \tag{43}$$

$$\kappa_{1zs} = \left\{ \left[\left(1 - \frac{v_{ws}^2}{2c^2} - \frac{k_R v_{\|s}}{\omega}\right) B_{1Ls} - \frac{v_{ws}^2}{2c^2} B_{2Ls} \right] \times \left(B_{4zs}B_{2Ls} - B_{3z}B_{1Ls}\right) + \left(\frac{v_{ws}^2}{2c^2} B_{4zs} - B_{1Ls}B_{1zs} \right) \times \left(B_{1Rs}B_{1Ls} - B_{2Rs}B_{2Ls}\right) \right\} B_{3s}^{-1}, \tag{44}$$

$$\kappa_{2zs} = \left\{ \left[\left(1 - \frac{v_{ws}^2}{2c^2} - \frac{k_L v_{\|s}}{\omega}\right) B_{2Rs} + \frac{v_{ws}^2}{2c^2} B_{1Ls} \right] \times \left(B_{4zs}B_{2Ls} - B_{3zs}B_{1Ls}\right) + \left[\left(1 - \frac{v_{ws}^2}{2c^2} - \frac{k_R v_{\|s}}{\omega}\right) \times B_{4zs} + B_{2zs}B_{1Ls} \right] \left(B_{1Rs}B_{1Ls} - B_{2Rs}B_{2Ls}\right) \right\} B_{3s}^{-1}, \tag{45}$$

$$\kappa_{3zs} = \left\{ \left(B_{1Ls} - B_{2Rs}\right)\left(B_{4zs}B_{2Ls} - B_{3zs}B_{1Ls}\right) \times \frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} - \left[\frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} B_{4zs} - B_{1Ls}\gamma_{\|s}^{-2} \right] \left(B_{1Rs}B_{1Ls} - B_{2Rs}B_{2Ls}\right) \right\} B_{3s}^{-1}, \tag{46}$$

where

$$B_{1Rs} = \omega - k_R v_{\|s} + B_{1s} - \frac{N\omega_{is}^2}{\omega - k_R v_{\|s}}\left(1 - \frac{v_{wj}^2}{2c^2}\right) - \frac{N\omega_{is}^2 v_{ws}^2}{2k_w c v_{\|s}}, \tag{47}$$

$$B_{2Rs} = \frac{N\omega_{is}^2}{\omega - k_L v_{\|s}} \frac{v_{ws}^2}{2c^2} + \frac{N\omega_{is}^2 v_{ws}^2}{2k_w c v_{\|s}} + B_{1j}, \tag{48}$$

$$B_{1Ls} = \omega - k_L v_{\|s} - B_{1s} - \frac{N\omega_{is}^2}{\omega - k_L v_{\|s}}\left(1 - \frac{v_{wj}^2}{2c^2}\right) + \frac{N\omega_{is}^2 v_{ws}^2}{2k_w c v_{\|s}}, \tag{49}$$

$$B_{2Ls} = \frac{N\omega_{is}^2}{\omega - k_R v_{\|s}} \frac{v_{ws}^2}{2c^2} - \frac{N\omega_{is}^2 v_{ws}^2}{2k_w c v_{\|s}} - B_{1j}, \tag{50}$$

$$B_{1zs} = -\frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} + \frac{k_R}{\omega}\frac{v_{ws}}{\sqrt{2}c}, \tag{51}$$

$$B_{2zs} = -\frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} + \frac{k_L}{\omega}\frac{v_{ws}}{\sqrt{2}c}, \tag{52}$$

$$B_{3zj} = \frac{\omega_{is}^2}{k_w c}\frac{v_{ws}}{\sqrt{2}c} - \frac{\omega_{is}^2}{\omega - k_R v_{\|s}}\frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} - \frac{\Omega_{ws}}{\sqrt{2}}, \tag{53}$$

$$B_{4zj} = -\frac{\omega_{is}^2}{k_w c}\frac{v_{ws}}{\sqrt{2}c} - \frac{\omega_{is}^2}{\omega - k_L v_{\|s}}\frac{v_{ws}v_{\|s}}{\sqrt{2}c^2} + \frac{\Omega_{ws}}{\sqrt{2}}, \tag{54}$$

$$N = 1 - \frac{2n_{ps}}{n_i}, \tag{55}$$

$$B_{1s} = \frac{\gamma_{0s}^2 v_{ws}}{2c}\left(\frac{\omega_{is}^2 v_{ws}}{k_w v_{\|s}} - \frac{\Omega_{ws} v_{\|s}}{c} - \frac{\omega_{ps}^2 v_{ws}}{k_w v_{\|s}} + \frac{v_{ws}v_{\|s}}{c^2}\omega_{ps}^2 \right)\frac{v_{ws}}{c}, \tag{56}$$

$$B_{2s} = -\frac{\omega_{ps}^2}{k_w c}\frac{v_{ws}}{\sqrt{2}c} + \frac{\gamma_{0s}^2}{\sqrt{2}}\left[\frac{\left(\omega_{is}^2 - \omega_{ps}^2\right)}{k_w c} - \frac{\Omega_{wj} v_{\|s}^2}{c^2} + \frac{\omega_{ps}^2}{k_w c}\frac{v_{\|s}^2}{c^2} \right]\frac{v_{ws}}{c} + \frac{k_w v_{ws}}{\sqrt{2}c} + \frac{\Omega_{ws}}{\sqrt{2}}, \tag{57}$$

$$B_{3s} = \frac{B_{2s}\Omega_{ws}}{\sqrt{2}}\left(B_{3Ls} + B_{4Rs}\right)\left(B_{4Ls} + B_{3Ls}\right) + \left[\frac{B_{2s}\Omega_{ws}}{\sqrt{2}} + B_{3Ls}\left(\omega - kv_{\|s}\right) \right]\left(B_{3Rs}B_{3Ls} - B_{4Rs}B_{4Ls}\right), \tag{58}$$

$$B_{4s} = B_{3Rs}B_{3Ls} - B_{4Rs}B_{4Ls}, \tag{59}$$

$$B_{5s} = \frac{kv_{ws}}{\sqrt{2}}\frac{\omega\omega_{ps}^2}{\left(\omega - kv_{\|s}\right)}, \tag{60}$$

$$B_{6s} = \frac{\omega_{ps}^2}{(\omega - kv_{\|s})}. \tag{61}$$

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