Study of the Parameters Affecting Cylindrical Cloak Using Dispersive FDTD

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ABSTRACT— **In this paper, an ideal cylindrical metamaterial invisibility cloak with infinite-length which its electric permittivity and magnetic permeability mapped to the Drude dispersion model is simulated. The sinusoidal plane waves with microwave frequencies used as sources. To this end, the dispersive finite-difference time-domain method (FDTD) used with Convolutional Perfectly Matched Layered (CPML) absorbing boundaries conditions. A comparison performed between scattering of cloaked and non-cloaked PEC cylinder. And finally, the influence of incident wave frequency, thickness of cloak and observer angle relative to the propagation line to performance of cloak, was surveyed.**

KEYWORDS: FDTD, Finite-Difference Time-Domain, CPML, Invisibility Cloak, Metamaterial.

I. INTRODUCTION

In recent years, metamaterial which is an artificial geometric composite material, attracts wide attention because of its novel features, such as negative refractive index [1, 2], sub-diffraction imaging [3], and electromagnetic cloaking [4, 5] and so on.

Scientists showed much interest to cloaking devices after experimental verification of a negative index of refraction [6]. Pendry *et al.* proposed an inhomogeneous and anisotropic electromagnetic invisibility cloak using trans-formation optics [7]. Their theory verified experimentally in microwave frequencies based on Split Ring Resonators (SRRs) at 2006 [4]. It should be noted that the SRRs which are often dispersive and lossy mediums, are the basic building block for many metamaterials [8, 6].

There are various numerical methods for simulation of electromagnetic interaction of wave and materials exist. The Finite-different time-domain (FDTD) is one of the relatively simple, accurate and powerful numerical algorithms for solving Maxwell's equations [9]. Because of dispersion nature of proposed in-visibility cloak in [4, 7], a radially dependent dispersive FDTD algorithm that offered in [10], was employed in this work for simulation of an infinite-length cylinder metamaterial cloak. It is used for study of various parameters effects to cloaking performance such as: incident wave frequency, cylinder radius and cloak thickness. Also, the efficient Convolutional Perfectly Matched Layers (CPML) absorbing boundary conditions is employed in order to avoid unwanted reflections from computational boundaries [9, 11].

After designing the problem and presentation of basic formula in section 2, the results of simulation will be discussed in section 3.

II. DESIGN AND THEORY

Consider an infinite length Perfect Electric Conductor (PEC) cylinder along z-axis, which coated with an ideal metamaterial cloak and placed in the free space, as shown in Fig.1. The full set of electromagnetic parameters of the ideal cloaking structure, in cylindrical co-ordinates, is given by:

$$\varepsilon_r = \mu_r = \frac{r - R_1}{r} \tag{1}$$

$$\varepsilon_\phi = \mu_\phi = \frac{r}{r - R_1} \tag{2}$$

$$\varepsilon_z = \mu_z = \left(\frac{R_2}{R_2 - R_1}\right)^2 \frac{r - R_1}{r} \tag{3}$$

where R_1 and R_2 are the inner and outer radius of the cloak, respectively [7]. The values of ε_r, μ_r, ε_z, and μ_z are less than one, thus they should be mapped into Drude lossless dispersion model for FDTD simulation [10]. for example in the case of ε_r can be write:

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \tag{4}$$

where ω_p is plasma frequency of the medium. For the case of other electromagnetic parameters that always are greater than one, the conventional FDTD method with constant material parameters will be performed.

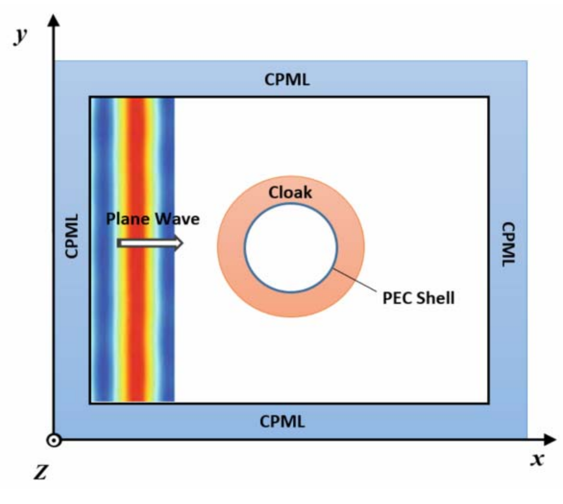

Fig. 1 Calculation scheme for cylindrical cloak structure with PEC core and CPML boundaries conditions

In this paper, considered the dispersive FDTD method for a two-dimensional (2-D) case for simplicity, therefore only E_x, E_y and H_z are non-zero and can be set $\mu_r = \mu_\phi = \varepsilon_z = 0$.

The auxiliary differential equation (ADE) FDTD is a relatively simple method that can be used for solving the Maxwell equations in dispersive media and hence employed by [10]. In this work, where the electromagnetic parameters of medium are greater than one, the conventional ADE FDTD updating equations:

$$\mathbf{B}^{n+1} = \mathbf{B}^n - \Delta t.\breve{\nabla} \times \mathbf{E}^{n+1/2} \tag{5}$$

$$\mathbf{D}^{n+1} = \mathbf{D}^n + \Delta t.\breve{\nabla} \times \mathbf{H}^{n+1/2} \tag{6}$$

will be used as well as the constitutive relations $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$ where ε and μ are expressed by Eq. (1-3), $\breve{\nabla}$ is the discrete curl operator, Δt is the discrete FDTD time step and n is the number of the time steps. Also, where the electromagnetic parameters of medium are less than one, the following equations will be used for electric fields calculations [10]:

$$\begin{aligned} E_x^{\ n+1} &= [c_1 D_x^{\ n+1} + c_2 D_x^{\ n} + c_1 D_x^{\ n-1} + c_5 \bar{D}_y^{\ n+1} \\ &+ c_6 \bar{D}_y^{\ n} + c_5 \bar{D}_y^{\ n-1} - \left(c_7 E_x^{\ n} + c_8 E_x^{\ n-1}\right)] / c_8 \end{aligned} \tag{7}$$

$$\begin{aligned} E_y^{\ n+1} &= [c_3 D_y^{\ n+1} + c_4 D_y^{\ n} + c_3 D_y^{\ n-1} + c_5 \bar{D}_x^{\ n+1} \\ &+ c_6 \bar{D}_x^{\ n} + c_5 \bar{D}_x^{\ n-1} - \left(c_7 E_y^{\ n} + c_8 E_y^{\ n-1}\right)] / c_8 \end{aligned} \tag{8}$$

where field quantities $\bar{D}_x$ and $\bar{D}_y$ are locally averaged values of D_x and D_y like as [10]. The c_i coefficients in Eq. (7) and (8) are:

$$c_1 = \sin^2\phi \left[\frac{1}{(\Delta t)^2} + \frac{\omega_p^2}{4}\right] + \frac{\varepsilon_\phi \cos^2\phi}{(\Delta t)^2} \tag{9}$$

$$c_2 = \sin^2\phi \left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2}\right] - \frac{2\varepsilon_\phi \cos^2\phi}{(\Delta t)^2} \tag{10}$$

$$c_3 = \cos^2\phi \left[\frac{1}{(\Delta t)^2} + \frac{\omega_p^2}{4}\right] + \frac{\varepsilon_\phi \sin^2\phi}{(\Delta t)^2} \tag{11}$$

$$c_4 = \cos^2\phi \left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2}\right] - \frac{2\varepsilon_\phi \sin^2\phi}{(\Delta t)^2} \tag{12}$$

$$c_5 = \left\{ \frac{\varepsilon_\phi}{(\Delta t)^2} - \left[\frac{1}{(\Delta t)^2} + \frac{\omega_p^2}{4} \right] \right\} \sin\phi \cos\phi \tag{13}$$

$$c_6 = \left\{ \frac{-2\varepsilon_\phi}{(\Delta t)^2} - \left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2} \right] \right\} \sin\phi \cos\phi \tag{14}$$

$$c_7 = \varepsilon_\phi \varepsilon_0 \left[-\frac{2}{(\Delta t)^2} + \frac{\omega_p^2}{2} \right] \tag{15}$$

$$c_8 = \varepsilon_\phi \varepsilon_0 \left[\frac{1}{(\Delta t)^2} + \frac{\omega_p^2}{4} \right] \tag{16}$$

Notice that in the above relations, the ϕ is azimuth angle in cylindrical coordinate and ε_ϕ is driven from Eq. (2).

Also, for the case of magnetic field, the permeability mapped into the lossless Drude model as:

$$\mu_z(\omega) = A\left(1 - \frac{\omega_{pm}^2}{\omega^2}\right) \tag{17}$$

$$A = R_2 / (R_2 - R_1) \tag{18}$$

where ω_{pm} is magnetic frequency of material. Therefore the H_z updating equation can be derived as [10]:

$$\begin{aligned} H_z^{n+1} = \frac{1}{A}\Big(&\alpha_1 B_z^{n+1} + \alpha_2 B_z^n + \alpha_1 B_z^{n-1} \\ &+ A\alpha_3 H_z^n - A\alpha_4 H_z^{n-1}\Big) / \alpha_4 \end{aligned} \tag{19}$$

$$\alpha_1 = \frac{1}{\mu_0 (\Delta t)^2} \tag{20}$$

$$\alpha_2 = \frac{-2}{\mu_0 (\Delta t)^2} \tag{21}$$

$$\alpha_3 = \frac{2}{(\Delta t)^2} - \frac{\omega_{pm}^2}{2} \tag{22}$$

$$\alpha_4 = \frac{1}{(\Delta t)^2} + \frac{\omega_{pm}^2}{4} \tag{23}$$

To ensure accuracy and stability of calculations, the following rules must be done:

A) Using $\Delta x \le \lambda / 35$ for more modeling accuracy and avoiding numerical errors [10]. Where, λ is the wavelength at the operating frequency.

B) Using the Courant-Friedrich-Levy conditions for numerical stability as [9]:

$$\Delta t \le \frac{1}{c_{\max}\sqrt{(\Delta x)^{-2} + (\Delta y)^{-2}}} \tag{24}$$

where c_{max} is the maximum speed of light in medium.

C) Using the numerical corrected plasma frequency for interception the modeling instability as [10]:

$$\breve{\omega}_p^2 = \frac{4(1-\varepsilon_r)}{(\Delta t)^2} \tan^2 \frac{\omega \Delta t}{2} \tag{25}$$

Finally, for performance analysis of simulation results, the Radar Cross Section (RCS) and Total Cross Section (TCS) as:

$$\mathrm{RCS}(\phi) = \lim_{\mathrm{r}\to\infty} 2\pi \mathrm{r} \frac{|\mathrm{H_s}(\phi)|^2}{|\mathrm{H_0}|^2} \tag{26}$$

$$TCS = \int_0^{2\pi} RCS(\phi)\, d\phi \tag{27}$$

were used [12]. Where H_s is scattered field at far and H_0 is the incident wave.

III. PROBLEM DESIGN AND THEORY

The split Perfectly Matched Layers (PML) and Periodic Boundary Condition (PBC) was employed in [10] for simulation of cloak, but the CPML relatively is superior to other methods like Uniaxial PML (UPML), PML and Absorption Boundaries Conditions (ABC) [9, 11].

Fig.2 shows the H_z distribution of a TE_z propagating plane wave in free space, with $\lambda = 5\mu m$. It illustrates excellent influence of CPML on wave absorption in calculation boundaries.

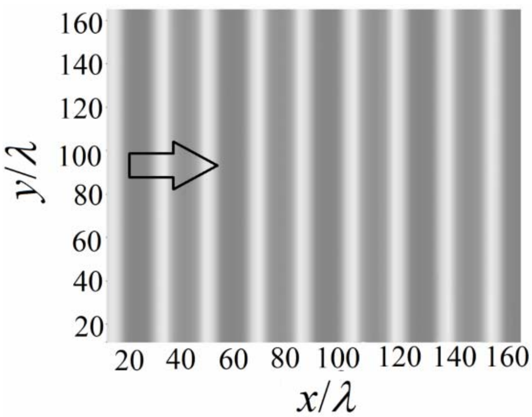

Fig. 2 H_z distribution of TE_z plane-wave propagating along x-direction, with applying the CPML boundaries conditions

A. *Indication of the Ideal Cloak Efficiency*

At first, an uncloaked PEC cylinder and then a cloaked PEC cylinder with an ideal metamaterial was simulated. In this implementation, the geometric parameters were R_1=10 *cm* (PEC shell radius) and R_2=22.5 *cm* and incident plane wave frequencies was between 1.5 to 2.25 *GHz*. Also square mesh with $\Delta x = \Delta y = \lambda / 35$ was applied. As shown in Fig.3, the cloaked PEC can decrease the wave scattering about 6 dB relative to uncloaked PEC. Also, Fig.4 displays the H_z distribution, which constructed from z-component of incident and scattered **H** field. It can be seen that the crossed wave front has only a slight numerical distortions which is related to the mesh size value. With smaller amounts of Δx and Δy, the simulation becomes more accurate and decrease the numerical errors, but it makes computation memory and CPU time more expensive.

B. *Cloak Thickness and Frequency Effects*

In the previous subsection, invisibility efficiency of ideal cloak was indicated. Here, effect of cloak thickness to invisibility was studied. Thus, an ideal metamaterial cloak with 5, 7.5, 10 and 12.5 *cm* thicknesses which coated a PEC shell with R=10 *cm*, was simulated. To this end, the dispersive FDTD algorithm was implemented with a plane wave in 1.5-2.25 GHz frequency range. For calculations stability and avoiding numerical anomalies, the mentioned A, B and C laws in Section 2 were applied to simulation.

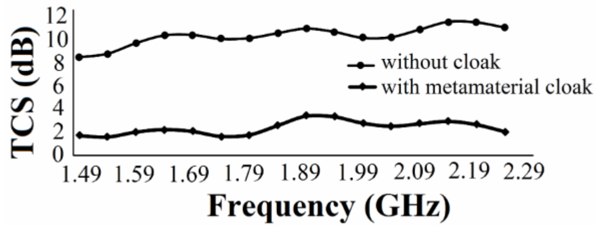

Fig. 3 Total scattering of cloaked and un-cloaked PEC shell with metamaterial in various frequencies.

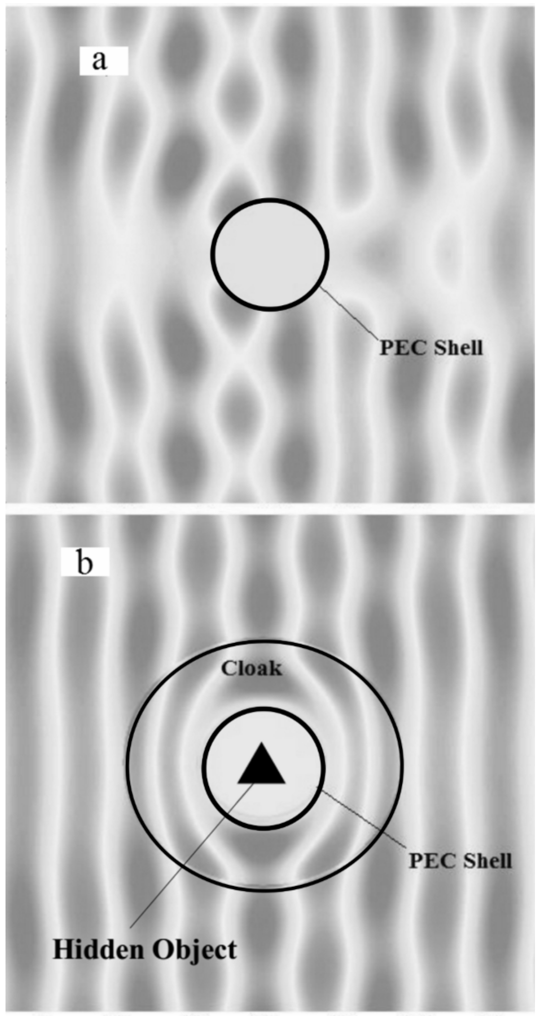

Fig. 4 H_z distribution of wave around a) PEC Shell and b) coated PEC shell by ideal metamaterial cloak.

Fig.5 shows the total scattering (TCS) wave of simulated cloaks. As illustrated in the charts, invisibility of ideal metamaterial becomes better with increasing the cloak thickness. Also, it is obvious that success of cloak depends on incident wave frequency. Actually, there is a frequency limitation to invisibility with ideal cloak parameters. Also, the ideal metamaterial invisibility cloak gives the best response in 1.55 GHz in surveyed frequency ranges for all cloak thicknesses.

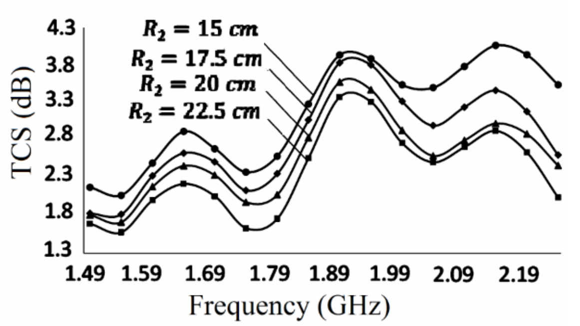

Fig. 5 total scattering of cloaked PEC with different thicknesses in various frequencies

C. *The Effect of Observer Angle*

In the simulated cases, the cloaks with 5 and 12.5 cm thicknesses have the worst and the best invisibility, respectively. Fig.6 shows RCS of these two cases at 1.55 GHz, in comparison of uncloaked PEC shell scattering.

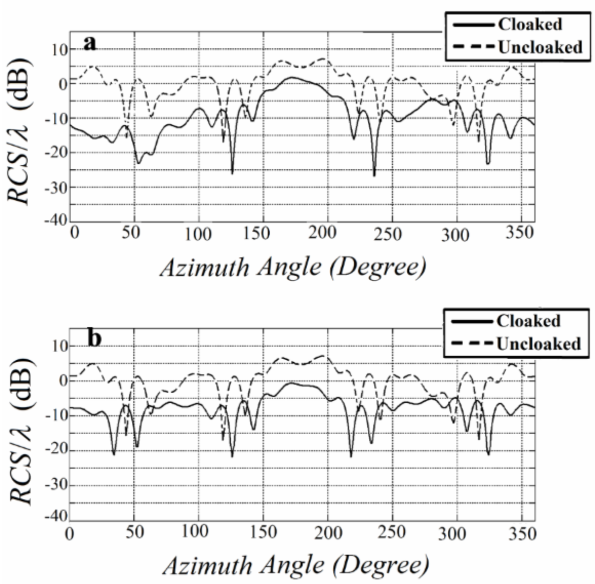

Fig. 6 RCS diagram of bare and cloaked with metamaterial PEC with a) t=5 cm b) t=12.5 cm, thicknesses in 1.55 GHz

It is obvious from diagrams that the ideal cloak decrease scattering in most angles, but in some cases the cloaked PEC shell scattering is equal or greater than uncloaked PEC shell scatter. This means that the ideal metamaterial cloaks performance depends on angle of observer. Therefore, there is an angle limitation to invisibility success, moreover to frequency restriction. The RCS chars asymmetry relative to 0 or 180 degree and those little anomalies related to meshing size amount. If Δx and Δy become smaller, the RCS charts will have complete symmetry.

IV. CONCLUSION

After introducing the basic algorithm that was employed in this work, performance of the ideal metamaterial cloak with various thicknesses in 1.5-2.25 GHz frequency range was surveyed. The CPML absorbing boundary condition as an efficient method used in calculations. At first, a comparison performed between total scattering (TCS) of uncloaked and cloaked PEC shell by ideal metamaterial and illustrated efficiency of cloak in decreasing the TCS. Then the role of cloaks thickness at different incident wave frequencies was analyzed and was indicated that the cloaks with greater thickness have more efficiency. In addition, results show that invisibility performance of ideal cloak, depends on incident wave frequency. The results of simulation were showed that the cloak gives the best response in 1.55 GHz, relative to other studied frequencies in this work. Also, with analysis of RCS diagram for uncloaked and cloaked PEC shell, illustrated that invisibility depends on observer angle relative to incident wave direction.

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