

Investigation of Normalized Intensity of Laser Field and Self-focusing of the Hermite-Gaussian Laser Pulse Radiation in Underdense and Collisionless Plasma

Mostafa Sadeghzadeh* and Davood Raoufi

Department of Physics, University of Bu-Ali Sina, Hamedan, Iran.

Corresponding author email: m.sadeghzadeh2012@gmail.com

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ABSTRACT— This paper investigates the phenomena of Normalized intensity of laser field (NILF) and self-focusing in a collisionless underdense plasma using Hermite-Gaussian laser pulse. NILF is an effective method to generate stronger self-focusing and high self-focusing length. This study focuses on the utilization of Hermite-Gaussian laser pulse to investigate NILF and self-focusing effectively. The influences of different hermite mode index value and initial amplitude of electric field of Hermite-Gaussian are examined on NILF and self-focusing. NILF has been plotted against propagation distance in plasma. Our research reveals that the NILF grows as the Hermite mode index value increases, regardless of the initial amplitude of Hermite-Gaussian electric field. Under different conditions, we observe that NILF also increases with the initial amplitude of Hermite-Gaussian electric field increases, when the Hermite mode index value is fixed. As a result, the self-focusing becomes stronger as the NILF increases, leading to a longer self-focusing length and an earlier onset of self-focusing.

KEYWORDS: Hermite-Gaussian laser pulse, laser intensity, NILF, plasma frequency, Self-focusing

I. INTRODUCTION

The interaction of high-intensity lasers with plasma is a subject of global research because of its wide-ranging applications. Here, this is required that the laser beam propagates over several Rayleigh lengths in the plasma and that too without divergence [1]. This encompasses a range of phenomena, spanning from stimulated

scattering to various self-action effects such as self-focusing. Due to their intricate and physics-rich nature, these nonlinear phenomena have the potential to captivate researchers for many years. Hence, to enhance the comprehension of the physics of laser-matter interaction, numerous researchers are exploring various facets of these nonlinear phenomena [2].

Self-focusing is a strongly nonlinear optical phenomenon that occurs because of the intensity dependent refractive index of the medium. In this way, even in the absence of any lens-like structure, the medium exhibits behavior akin to a convex lens [2]. Self-focusing of laser beam in plasma is one of the important nonlinear optical effects due to its applications in laser-plasma interaction [3] such as high harmonic generation [4]–[6], charged particle acceleration [7]–[10], inertial confinement fusion (ICF) [11],[12], etc. However, the wave property of light, diffraction, poses a challenge to the features of the aforementioned applications by negating them. Hence, endeavors are underway to discover methods that can mitigate the diffraction of laser beams. The diffraction broadening of laser beams can be avoided by employing waveguides or by leveraging the phenomenon of self-focusing. Self-focusing, a nonlinear optical effect, occurs when the intensity of the laser beam causes a local increase in refractive index, leading to the concentration of beam energy over a confined

region of space [13]. Self-focusing of short pulses arises due to plasma's nonlinear permittivity being an increasing function of laser intensity. This occurs in view of relativistic increase of electron mass due to the laser field and depression in the electron density owing to the transverse ponderomotive force [14],[15] (along the z-axis) to have Gaussian intensity profile in the transverse coordinator. In this case, paraxial ray approximation is quite reasonable, and one obtains analytically tractable results in certain limits [1]. In most of the cases, due to the symmetric intensity distribution around the axis of the beams, paraxial theory makes them a little bit easier to characterize their propagation dynamics [3]. Using the equation of laser beam propagation in plasma and assuming the paraxial and Wentzel–Kramers–Brillouin (WKB) approximations, we obtain an equation for the laser beam intensity in the direction of propagation [16].

For general scientific applications with the advent of laser technology, lasers need to be utilized in different capacities and configurations. Due to this, various non-Gaussian beams were adopted in the gymnastics of laser–plasma interaction [17]. Recently, considerable interest is exploited in the self-focusing of non-Gaussian beams such as Hermite–Gaussian [18],[19], Quadruple-Gaussian [20]–[22], Cosh-Gaussian [23]–[27], Bessel-Gaussian [28]–[31], q-Gaussian [2, 13, 32–36], Laguerre-Gaussian [3],[37],[38] and Lorentz–Gaussian [39].

There are many significant mechanisms, viz., ponderomotive force and relativistic which contribute to the changes in the plasma's dielectric constant during laser-plasma interactions [40]. Intense laser beams can induce nonlinear responses in plasmas primarily through two mechanisms. These mechanisms are (1) Relativistic mass nonlinearity (2) nonlinearity due to Ponderomotive force. In the relativistic mechanism, alterations in the optical properties of plasma occur without any changes in electron density. Conversely, in the other mechanism, this modification takes place through the redistribution of plasma electrons.

Relativistic nonlinearity becomes prominent when the power of the incident laser surpasses the threshold power needed for self-focusing, and as a result, it exhibits no transient behavior. In this mechanism, the optical properties of plasma vary as a function of the intensity of the electromagnetic beam. This change occurs because of the oscillation of electrons at a velocity comparable to that of light, leading to a modification in electron mass. In the ponderomotive force mechanism, plasma electrons are depleted from the high-intensity region [22].

In 1962 Askaryan [41] recommended the possibility of happening the self-focusing phenomenon and highlighting the important role of the special effects of nonlinear refraction in the propagation of intense electromagnetic radiation in a material medium. Self-focusing or defocusing of intense laser beams in nonlinear media has been well studied by Akhmanov *et al.* [42] and Sodha *et al.* [43]. Walia *et al.* (2024) have investigated a self-focusing of high power q-Gaussian laser beam in collisional magnetized plasma [44]. They studied the influence of change in known laser-plasma parameters including laser intensity, plasma density, cyclotron frequency, radius of beam and 'q' parameter on beam width of input wave with dimensionless distance of propagation. Patil *et al.* (2024) have studied self-focusing of Laguerre-Gaussian laser beams in collisionless plasma: paraxial-like approach [3]. They conclude that with increase in intensity parameter of LGBs and plasma electron density, the self-focusing of LGBs increases. Aggarwal *et al.* (2021) have used Gaussian laser beam to study the effect of relativistic-ponderomotive self-focusing in magnetized cold quantum plasma [45]. They studied the role of the magnetic field in amplifying the ponderomotive effect has been thoroughly investigated. Hassan *et al.* (2024) have investigated the self-focusing of hollow Gaussian laser beam in magnetized plasma with transverse magnetic field by operating relativistic nonlinearity [46]. They studied the effect of order and initial intensity of hollow Gaussian laser beam on the self-focusing process and the impact of the applied magnetic

field and plasma density on the self-focusing process. Gupta *et al.* (2024) have investigated a self-focusing of rippled elliptical q-Gaussian laser beam in plasma with axial density ramp [47]. Nikam *et al.* (2023) have investigated a combined effect of relativistic and ponderomotive nonlinearities on self-focusing of asymmetric finite Airy-Gaussian laser beams in plasma [17]. They demonstrated that self-focusing of finite Airy-Gaussian beams in plasma under relativistic-ponderomotive regime of interaction can be controlled by tuning the asymmetric modulation parameters associated with the Airy function part of intensity distribution of the beam.

In this paper, we have chosen Hermite-Gaussian laser pulse to investigate the role of NILF on self-focusing beam in cold, underdense and collisionless plasma. Expression for NILF, the stronger of self-focusing, self-focusing length and start of self-focusing by each pulse is derived analytically. The effects of Initial amplitude of electric field of Hermite-Gaussian and Hermite mode index value on NILF, the stronger of self-focusing, self-focusing length and start of self-focusing is examined. Curves are illustrated to compare the effects produced by each pulse profile. Analytical solution is given in section 2. Curves are illustrated and outcomes are discussed in result and discussion section 3. Conclusions are given section 4. The paper concludes with the references.

II. ANALYTICAL SOLUTION OF NORMALIZED INTENSITY OF LASER FIELD (NILF)

The propagation of a Hermite-Gaussian laser beam along the z direction could be expressed in the cartesian coordinate system. The waves equation of the laser beam propagating in homogenous underdense collisionless plasma are:

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi e}{c} n_e \mathbf{v}, \quad (1)$$

$$\nabla \cdot \mathbf{E} = 4\pi e(n_e - n_{0e}), \quad (2)$$

where, $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$, e is charge of electron, n_e is electron density perturbation, n_{0e} is electron density non-perturbation, $\mathbf{v}$ is velocity of electron. The following fundamental equation of motion is used to create laser-plasma interaction [48].

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \left(\frac{n_e \mathbf{P}}{\gamma m} \right) = 0, \quad (3)$$

$$\frac{\partial \mathbf{P}}{\partial t} + \mathbf{v} \cdot \nabla (\mathbf{P}) = -e \left[\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right], \quad (4)$$

where $\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}}$ is relativistic factor,

$\mathbf{P} = \gamma m_0 \mathbf{v}$ is relativistic momentum of electron.

Let us define new variables $a = \frac{e|\mathbf{A}|}{mc^2}$ as normalized intensity of laser field (NILF),

$\phi = \frac{e\varphi}{mc^2}$ as generated normalized laser wake potential (NLWP), $\xi = (z - v_g t)$ depending

on space and time, and $\beta_g = \frac{v_g}{c}$ for comparison of group velocity v_g and free space speed of light, c . By solving Eq. 1 to Eq. 4 under quasi-static approximation and Wentzel-Kramers-Brillouin (WKB) approximations, we obtain two coupled general second-order differential equation for NILF and NLWP as:

$$\frac{d^2 a}{d\xi^2} = \left\{ \frac{(\beta_g)^2}{\left[\sqrt{1+(a_0)^2} + \phi \right]^2 - \left[1 - (\beta_g)^2 \right]^2 (1+a^2)} \right\}^{\frac{1}{2}} \times \left[\frac{a}{1 - (\beta_g)^2} \right] - \omega^2 a \quad (5)$$

$$\frac{d^2 \phi}{d\xi^2} = \left[\frac{\beta_g}{1 - (\beta_g)^2} \right] \times$$

$$\times \left\{ \left[\sqrt{1 + (a_0)^2} + \phi \right]^2 - \left[1 - (\beta_g)^2 \right] (1 + a^2) \right\}^{\frac{-1}{2}} \times \quad (6)$$

$$\times \left[\sqrt{1 + (a_0)^2} + \phi \right] - \left[1 - (\beta_g)^2 \right]^{-1}$$

Here, $\omega = \frac{\omega_0}{\omega_p}$ is comparing the laser frequency, ω_0 , and plasma frequency, ω_p ,

$$\beta_g = \frac{v_g}{c} = \left[1 - \left(\frac{\omega_p}{\omega_0} \right)^2 \right]^{\frac{1}{2}} \text{ and } a_0 \text{ is the initial}$$

intensity of laser field (IILF) which depends on initial electric field of Hermite-Gaussian as

$$a_0 = \frac{e|\mathbf{E}|}{m\omega_0 c}. \text{ In our current study, we have}$$

chosen the profile of field distribution of Hermite-Gaussian laser pulse as:

$$\mathbf{E} = \hat{\mathbf{x}}E_0 \exp \left[-\frac{\left(\xi - \frac{L}{2} \right)^2}{r_0^2} \right] \times H_s \left[\frac{\sqrt{2} \left(\xi - \frac{L}{2} \right)}{r_0} \right], \quad (7)$$

Here, s can have any positive integer number. Pulse shape varies with s values. In this study, we have chosen 3 different values of s as $s = 0, 1, 2$. E_0 is Initial amplitude of electric field of Hermite-Gaussian, L is the laser pulse length, r_0 is the laser beam waist.

$$\text{For } s = 0, \quad \mathbf{E}_0 = \hat{\mathbf{x}}E_0 \exp \left[-\frac{\left(\xi - \frac{L}{2} \right)^2}{r_0^2} \right] \times 1, \quad (7a)$$

For $s = 1$,

$$\mathbf{E}_1 = \hat{\mathbf{x}}E_0 \exp \left[-\frac{\left(\xi - \frac{L}{2} \right)^2}{r_0^2} \right] \times 2 \left[\frac{\sqrt{2} \left(\xi - \frac{L}{2} \right)}{r_0} \right], \quad (7b)$$

For $s = 2$

$$\mathbf{E}_2 = \hat{\mathbf{x}}E_0 \exp \left[-\frac{\left(\xi - \frac{L}{2} \right)^2}{r_0^2} \right] \times 4 \left[\frac{\sqrt{2} \left(\xi - \frac{L}{2} \right)}{r_0} \right]^2, \quad (7c)$$

Substitution of Eqs. 7(a)–7(c) into Eqs. 5 and 6 yields the NILF, together with the key parameters characterizing the self-focusing of the Hermite-Gaussian laser pulse, namely its strength, self-focusing length, and onset.

III. RESULTS AND DISCUSSION

In this paper, we have chosen plasma of electron density $3 \times 10^{23} \text{ m}^{-3}$, plasma frequency corresponding to this plasma density is $2.49 \times 10^{13} \text{ rad/s}$. Laser pulse of wavelength $10.64 \mu\text{m}$, laser frequency $1.76 \times 10^{14} \text{ rad/s}$ and pulse length $48.09 \mu\text{m}$ are chosen for numerical investigation [38]. Initial amplitude of electric field of Hermite-Gaussian is selected as $2.13 \times 10^{11} \text{ V/m}$, $4.28 \times 10^{11} \text{ V/m}$ and $8.58 \times 10^{11} \text{ V/m}$. We have chosen the initial conditions as $a = a_0$ at $\xi = 0$ and $\frac{da}{d\xi} = 0$ at $\xi = 0$.

A. Effect of hermite mode index value (s) on NILF and self-focusing

Figure 1 illustrates the variation of NILF as a function of the propagation length for cases (a) $s=0$, (b) $s=1$, and (c) $s=2$. The initial amplitude of the Hermite-Gaussian electric field is set to $E_0 = 4.28 \times 10^{11} \text{ V/m}$. The maximum NILF values are 4, 6.5, and 1000 for cases (a), (b), and (c), respectively. The onset of self-focusing occurs after a short propagation distance of $\xi = 10.5 \mu\text{m}$, $\xi = 5 \mu\text{m}$, and $\xi = 4 \mu\text{m}$, with the corresponding self-focusing lengths of

$$\Delta\xi = 24 \mu\text{m} - 10.5 \mu\text{m} = 13.5 \mu\text{m},$$

$$\Delta\xi = 28 \mu\text{m} - 5 \mu\text{m} = 23 \mu\text{m}, \text{ and}$$

$$\Delta\xi = 29 \mu\text{m} - 4 \mu\text{m} = 25 \mu\text{m}$$

for cases (a), (b), and (c), respectively.

B. Effect of initial amplitude of electric field of Hermite-Gaussian (E_0) on NILF and self-focusing

Figures 2(a-c) present the variation of NILF vs. propagation length for cases (a) $E_0 = 2.13 \times 10^{11} \text{ V/m}$, (b) $E_0 = 4.28 \times 10^{11} \text{ V/m}$, and (c) $E_0 = 8.58 \times 10^{11} \text{ V/m}$, respectively. The peaks of the three curves correspond to the Hermite mode with index value $s = 0$. The maximum NILF values are 3, 4, and 5.8 for cases (a), (b), and (c), respectively.

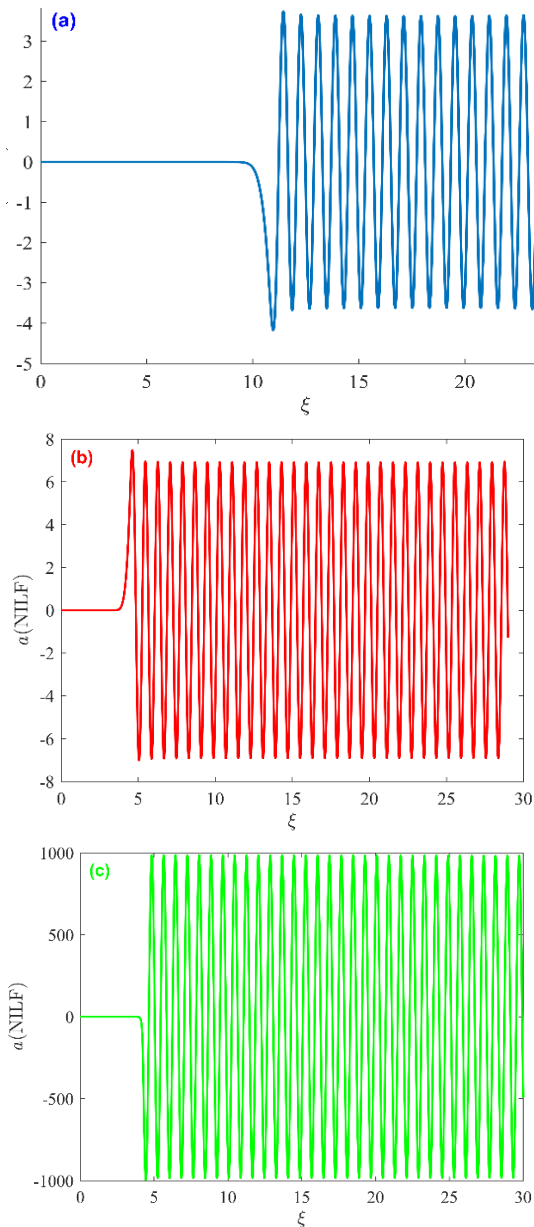


Fig. 1. Variation of NILF vs. $\xi(\mu\text{m})$ for (a) $s = 0$, (b) $s = 1$, and (c) $s = 2$ for $E_0 = 4.28 \times 10^{11} \text{ V/m}$ and $r_0 = 32.94 \mu\text{m}$.

The onset of self-focusing is observed after propagation distances of $\xi = 11 \mu\text{m}$, $\xi = 10.5 \mu\text{m}$ and $\xi = 5 \mu\text{m}$, while the associated self-focusing lengths are

$$\Delta\xi = 22 \mu\text{m} - 11 \mu\text{m} = 11 \mu\text{m},$$

$$\Delta\xi = 24 \mu\text{m} - 10.5 \mu\text{m} = 13.5 \mu\text{m}, \text{ and}$$

$$\Delta\xi = 24 \mu\text{m} - 5 \mu\text{m} = 19 \mu\text{m}$$

for cases (a), (b), and (c), respectively.

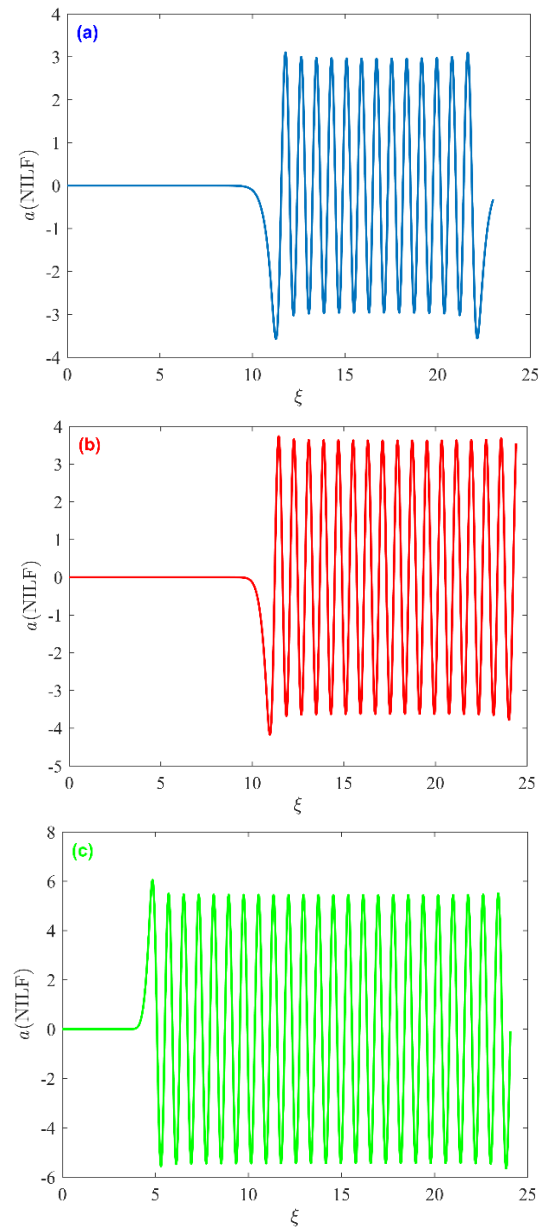


Fig. 2. Variation of NILF vs. $\xi(\mu\text{m})$ for (a) $E_0 = 2.13 \times 10^{11} \text{ V/m}$, (b) $E_0 = 4.28 \times 10^{11} \text{ V/m}$, and (c) $E_0 = 8.58 \times 10^{11} \text{ V/m}$ for $s = 0$ and $r_0 = 32.94 \mu\text{m}$.

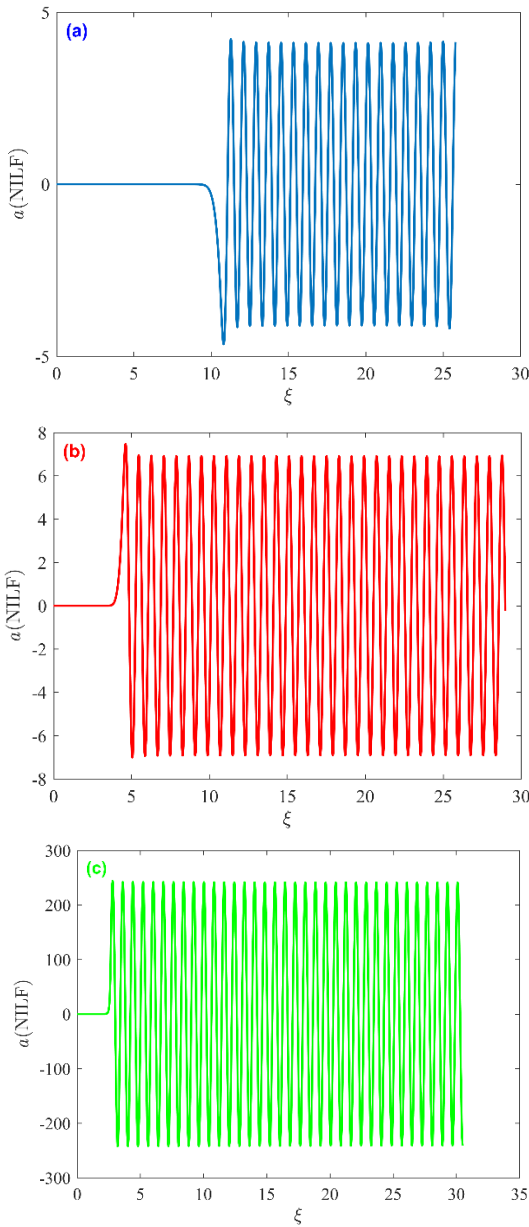


Fig. 3. Variation of NILF with $\xi(\mu m)$ for (a) $E_0 = 2.13 \times 10^{11} V/m$, (b) $E_0 = 4.28 \times 10^{11} V/m$ and (c) $E_0 = 8.58 \times 10^{11} V/m$ for $s=1$ and $r_0 = 32.94 \mu m$

Figures 3(a-c) depict the evolution of NILF as a function of propagation length for cases (a) $E_0 = 2.13 \times 10^{11} V/m$, (b) $E_0 = 4.28 \times 10^{11} V/m$, and (c) $E_0 = 8.58 \times 10^{11} V/m$, respectively. The peaks of all three curves correspond to the Hermite mode with index value $s=1$. The maximum NILF values are obtained as 4, 6.5, and 250 for cases (a), (b), and (c), respectively. The onset of self-focusing occurs after propagation distances of $\xi = 11 \mu m$, $\xi = 5 \mu m$,

and $\xi = 3 \mu m$, with the corresponding self-focusing lengths of

$$\Delta \xi = 26 \mu m - 11 \mu m = 15 \mu m,$$

$$\Delta \xi = 28 \mu m - 5 \mu m = 23 \mu m, \text{ and}$$

$$\Delta \xi = 30 \mu m - 3 \mu m = 27 \mu m$$

for cases (a), (b), and (c), respectively.

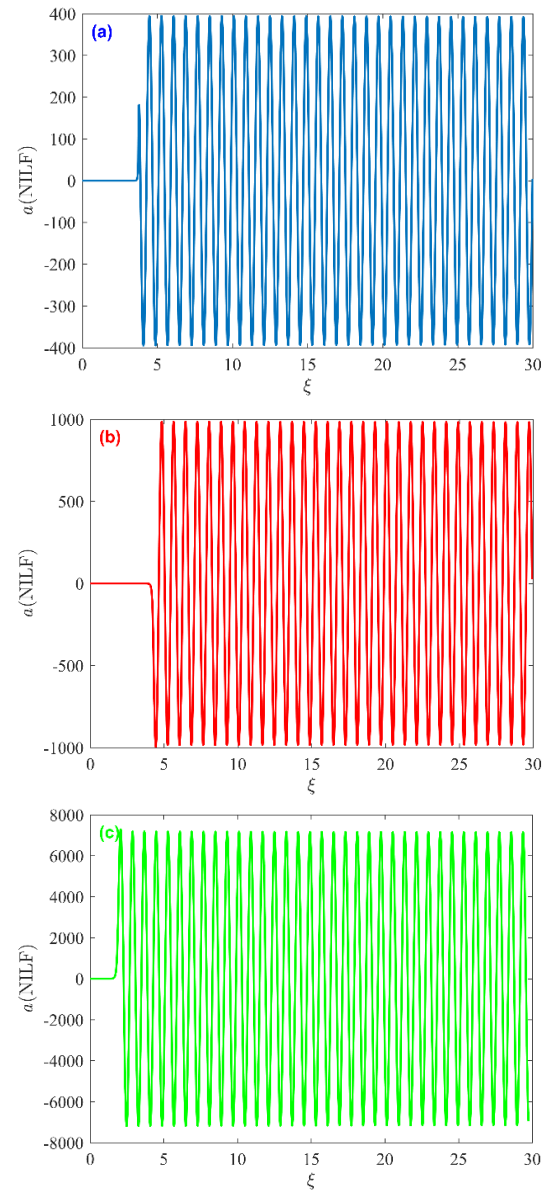


Fig. 4. Variation of NILF with $\xi(\mu m)$ for (a) $E_0 = 2.13 \times 10^{11} V/m$, (b) $E_0 = 4.28 \times 10^{11} V/m$, and (c) $E_0 = 8.58 \times 10^{11} V/m$ for $s=2$ and $r_0 = 32.94 \mu m$.

Figures 4(a-c) illustrate the variation of NILF with propagation length for cases (a) $E_0 = 2.13 \times 10^{11} V/m$, (b) $E_0 = 4.28 \times 10^{11} V/m$,

and (c) $E_0 = 8.58 \times 10^{11} \text{ V/m}$. The peaks of the curves correspond to the Hermite mode with index value $s = 2$. The maximum NILF values are 400, 1000 and 7000 for cases (a), (b), and (c), respectively. The onset of self-focusing is observed after propagation distances of $\xi = 4.5 \mu\text{m}$, $\xi = 4 \mu\text{m}$ and $\xi = 2.5 \mu\text{m}$, and the corresponding self-focusing lengths are

$$\Delta\xi = 30 \mu\text{m} - 4.5 \mu\text{m} = 25.5 \mu\text{m},$$

$$\Delta\xi = 29 \mu\text{m} - 4 \mu\text{m} = 25 \mu\text{m}, \text{ and}$$

$$\Delta\xi = 30 \mu\text{m} - 2.5 \mu\text{m} = 27.5 \mu\text{m}$$

for cases (a), (b), and (c), respectively.

This region represents the strongest nonlinear interaction among all investigated beam waist in $r_0 = 26 \mu\text{m}$. For larger beam radii ($r_0 \geq 20 \mu\text{m}$), strong oscillations appear, reflecting the increased sensitivity of the mode structure to the transverse coordinate and the reduced nonlinear strength at lower intensities. These oscillations indicate that the nonlinear response of the plasma becomes highly irregular for broader beams, leading to weaker or delayed self-focusing behavior. Overall, the plot demonstrates that the beam width directly governs the nonlinear interaction strength and determines the stability or instability of the propagation dynamics in plasma.

To further confirm the self-focusing effect observed from NILF, we have evaluated the evolution of the NILF with respect to beam waist radius r_0 .

Figure 5 shows the variation of the normalized intensity of the laser field (NILF) as a function of the beam waist radius, r_0 . For very small beam radii ($r_0 \leq 10 \mu\text{m}$), the NILF exhibits a pronounced peak around a few microns, which indicates a strong enhancement of the nonlinear response due to the high transverse confinement of the laser field. In this regime, the beam is tightly focused and the intensity is maximized, leading to a significant relativistic-ponderomotive contribution and consequently a sharp rise in the NILF value. In the intermediate range of ($10 \mu\text{m} \leq r_0 \leq 20 \mu\text{m}$), the NILF

becomes nearly constant, corresponding to a quasi-stable regime in which the Hermite–Gaussian profile is less sensitive to variations of the input beam radius.

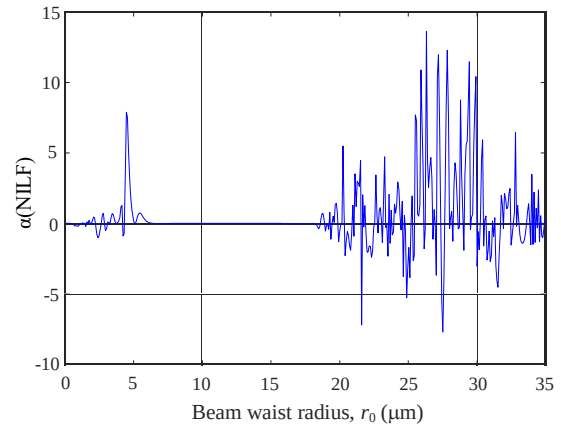


Fig. 5. Variation of NILF with beam waist radius, $r_0 (\mu\text{m})$ for $E_0 = 4.28 \times 10^{11} \text{ V/m}$, $s = 1$, and $L = 48.09 \mu\text{m}$.

Furthermore, Fig. 5 shows that the NILF attains a value of 7 at, confirming the $r_0 = 32.94$ consistency with Fig. 3(b).

IV. CONCLUSION

In this work, the nonlinear propagation and self-focusing behavior of Hermite–Gaussian laser beams in plasma were systematically investigated by examining the effects of the initial beam waist, initial field amplitude, and mode order through the normalized intensity of the laser field (NILF). The results demonstrate that smaller beam waists and higher initial field amplitudes significantly enhance the maximum NILF, leading to stronger nonlinear interactions and an earlier onset of self-focusing. Beams with intermediate waist sizes exhibit quasi-stable propagation, whereas larger beam waists produce oscillatory nonlinear responses associated with reduced intensity and increased transverse-mode sensitivity. Furthermore, increasing the mode order results in a pronounced increase in the maximum NILF and accelerates the self-focusing process, emphasizing the crucial role of the transverse mode structure. Overall, these findings provide a unified physical picture of how beam geometry, field strength, and mode structure

collectively govern the strength, stability, and localization of self-focusing in Hermite–Gaussian beams, offering valuable insights for the control and optimization of nonlinear laser–plasma interactions.

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Authors' contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by M. Sadeghzadeh and D. Raoufi. The first draft of the manuscript was written by M. Sadeghzadeh and all authors commented on previous versions of the manuscript. All authors reviewed and approved the final manuscript.

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Mostafa Sadeghzadeh was born in Borujerd, Iran in Dec. 15, 1991. He received his BSc., MSc., and PhD degrees in nuclear physics respectively from Urmia University of Urmia, West Azerbaijan in 2014, Atomic and molecular physics from University of Science and Technology, Tehran, Iran in 2016, and in Atomic and molecular physics from Bu-Ali Sina University, Hamedan, Iran in 2025.

He teaches at Ayatollah Borujerdi University, Boroujerd, Iran as tutor. He has published two journal papers. His research interest is optics and laser. He is a member of Iran Nanotechnology Innovation Council.



Davood Raoufi was born in Hamedan, Iran, in 1963. He received his PhD degree in Atomic and molecular physics from University of Isfahan, Isfahan, Iran, in 2007.

He is a faculty member of Bu-Ali Sina University, Hamedan, Iran. He has published 41 papers. His research interest is optics and laser.

Dr. Davood Raoufi is a member of Iran Nanotechnology Innovation Council and Iranian Physics Society.

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