The Effect of Linewidth Enhancement Factor on the Period-One Dynamics of Cavity Solitons

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ABSTRACT— Cavity Solitons (CSs) in an injected broad-area semiconductor laser with intracavity saturable absorber exhibit phase noise free and high-contrast intensity oscillations in period-one dynamical state. The continuous-wave periodic intensity oscillation of the CSs has a frequency well beyond the relaxation oscillation frequency and can be regarded as a photonic microwave source. Here we numerically investigate the effect of linewidth enhancement factor on the dynamical characteristics of CSs in the period-one regime. We show that in a fixed injection amplitude, it has a key role in shifting the oscillation frequencies of the period-one CSs and that its effects strongly depend on the cavity detuning value.

KEYWORDS: Cavity Solitons, microwave photonics, period-one state, linewidth enhancement factor

I. INTRODUCTION

Injected semiconductor lasers have a long history of complex and nonlinear dynamics [1-2]. They include stable locking [3], frequency locking [4], periodic and regular oscillations, and chaotic pulsations [5-12]. Period-one (P1) oscillation state is an intermediate state where the output intensity of the laser exhibits high-speed single-period oscillation beyond the relaxation oscillation frequency due to external injection. The state is typically found in a large region of the parameter space and is suitable for photonic microwave applications as they have minimal phase noise. Photonic microwave generation by the nonlinear dynamics of a semiconductor laser in period-one oscillation state has been applied for narrow-linewidth microwave signal generation, radio-over-fiber (RoF) subcarrier transmission, wavelength conversion, signal AM to FM conversion, and remote target detection [13-17].

Spatiotemporal dynamics in broad-area semiconductor lasers occur due to the presence of spatial coupling and comes from diffraction in a cavity possessing a large Fresnel number. A large Fresnel number provides a large number of transverse spatial modes whose existence and interaction play a key role in the formation and control of spatiotemporal structures. Cavity Solitons (CSs) are one of such structures which appear as diffraction-free bright localized spots anchoring on dark backgrounds. They have been shown to enhance resonance frequency and modulation bandwidth [18], provide ultra-low energy switches [19,20], generate excitable dynamics [21], act as core elements in delay line memories [22-24], high bit-rate demodulators [25], micromotors [26], and frequency mixers and switches [27,28] among many others [29]. Here we show that CSs in a broad-area semiconductor laser with a saturable absorber subject to continuous wave (cw) injection can perform high-frequency period-one (P1) oscillations feasible in microwave photonics.

Various optical techniques for generation of microwave signals have already been demonstrated including, for example, direct and external modulation of semiconductor lasers, optical heterodyne with an optical phase-locked loop (OPLL), dual-mode and mode-locked lasers, optoelectronic oscillators (OEOs), and

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periodic oscillations of semiconductor lasers [30]. In particular, period-one oscillation state induced by cw external injection can directly generate microwave signals with frequency exceeding the intrinsic relaxation resonance frequency of the laser with no need for external modulation or microwave input. Here we show that these features can be enhanced by CSs. CSs in the P1 regime are advantageous for microwave applications since they have optimal shapes anchoring on an almost zero background, have large amplitude due to the amplification properties of CSs, and benefit from a wide tunability range via cavity detuning as the control parameter. However, the effect of linewidth enhancement factor on the P1 dynamics of CSs remains to be studied.

In semiconductor lasers, variations in the carrier density result in changes of both the optical gain and the refractive index. The coupling between the optical gain and the refractive index is commonly characterized by the linewidth enhancement factor. This implies that variations in the carrier density due to temporal changes in the optical intensity will lead to fluctuations of the optical phase and thus to frequency variations. We particularly show that the value of the factor can be favorable or unfavorable for the P1 dynamics depending on the cavity detuning value.

The remaining of paper is organized as follows, Section II introduces the dynamical equations and the various parameters; Section III discusses the results, and conclusions are expressed in Section IV.

II. MODEL EQUATIONS

The effective model appropriate for an injected broad-area semiconductor laser with an intracavity saturable absorber is basically the one used in [31] for investigation of CSs but with additional terms representing the cw external injection [32],

$$\begin{cases} \dfrac{\partial F}{\partial t} = F_{inj} + \left[(1-i\alpha)D + (1-i\beta)d - (1-i\theta) + i\nabla_{\perp}^{2}\right]F, \\ \dfrac{\partial D}{\partial t} = b[\mu - (1+|F|^{2})D - BD^{2}, \\ \dfrac{\partial D}{\partial t} = b[\mu - (1+|F|^{2})D - BD^{2}, \end{cases} \quad (1)$$

where F is the slowly varying amplitude of the electric field, and D, d are the population variables defined as

$$D = \eta_1\left(\frac{N_1}{N_{1,0}} - 1\right); d = \eta_2\left(\frac{N_2}{N_{2,0}} - 1\right) \quad (2)$$

In Eq. 2, N_1 and N_2 are the carrier densities in the active and passive materials, respectively; $N_{1,0}$ and $N_{2,0}$ are their transparency values, and η_1, η_2 are dimensionless coefficients related to gain and absorption, respectively. The parameters $\alpha(\beta)$ and b are the linewidth enhancement factor in the active (passive) material and the ratio of the photon lifetime to the carrier lifetime in the active material. Here we keep α fixed and use the linewidth enhancement factor in the passive material β as a variable. μ is the pump current parameter of the active material, γ is the absorption parameter of the passive material, s is the saturation parameter, and B is the coefficient of radiative recombination. F_{inj} is the strength of the optical injection and θ is the frequency detuning of the injected monochromatic light with respect to the solitary laser frequency at the threshold. The diffraction of intracavity light F is described by the Laplace operator acting on the transverse plane (x,y). Time is scaled to the photon lifetime, and space to the diffraction length. Typically, a time unit is a few ps and a space unit is ~4 μm.

The integration of the dynamical equations is performed by a split-step method separating the time and space derivatives on a 128×128 spatial grids with space step 0.5 implying the physical distance of 2 μm between two consecutive grid points. In all simulations, we use periodic boundary conditions and the following parameter values throughout the paper: s=1, B=0.1, α=2, b=0.01, γ=2, μ=5, F_{inj}=0.55, and r=1. θ and β are used as the control parameters [31, 33-34].

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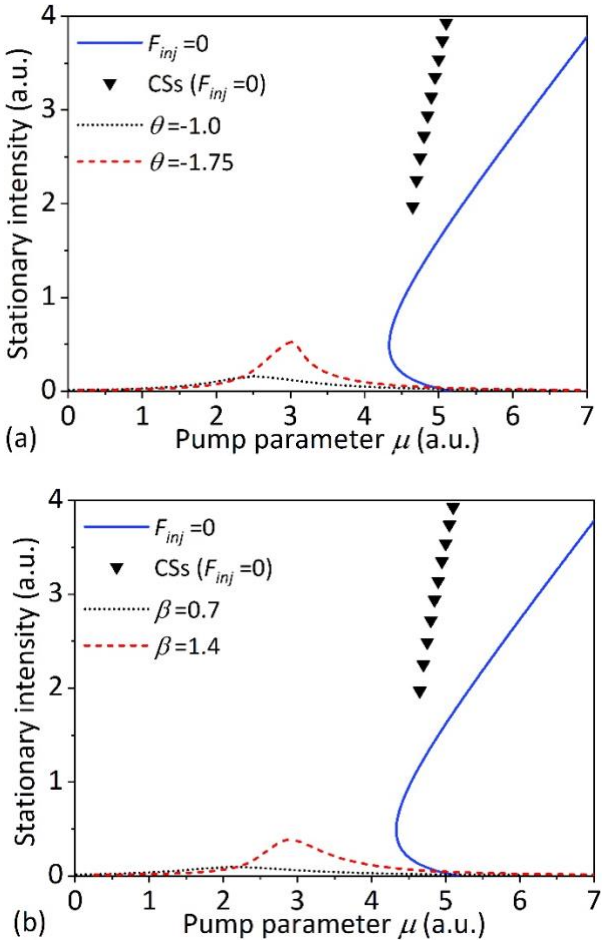

Fig. 1. Stationary intensity vs. the pump parameter. The case of zero injection is shown with solid blue line along with the corresponding CS branch with black triangles. Red-dashed and black-dotted curves belong to F_{inj}=0.55 and (a) varied θ with fixed β=1, (b) varied β and fixed θ=-0.5.

Plots of stationary intensity as a function of the pump parameter are shown in Fig.1 (a) and (b) for the case of zero injection (solid blue line) along with the corresponding CS branch with black triangles. Red-dashed and black-dotted curves belong to F_{inj} =0.55 and varied θ and β respectively in Fig.1 (a) and (b). It is seen that by adding injection a low intensity branch appears and is affected by both θ and β in a more or less the same way. The external optical field reduces the gain necessary for the injected laser and the carrier density. This results in an increase in the refractive index of the gain medium through the linewidth enhancement factor. Therefore, the laser cavity length experienced by the circulating optical field is increased, leading to a reduction of the cavity resonance frequency. Larger detuning value weakens the influence of the injection and smaller resonance frequency shifts are experienced by the system. The same happens by the larger β values, see Fig. 1 (b).

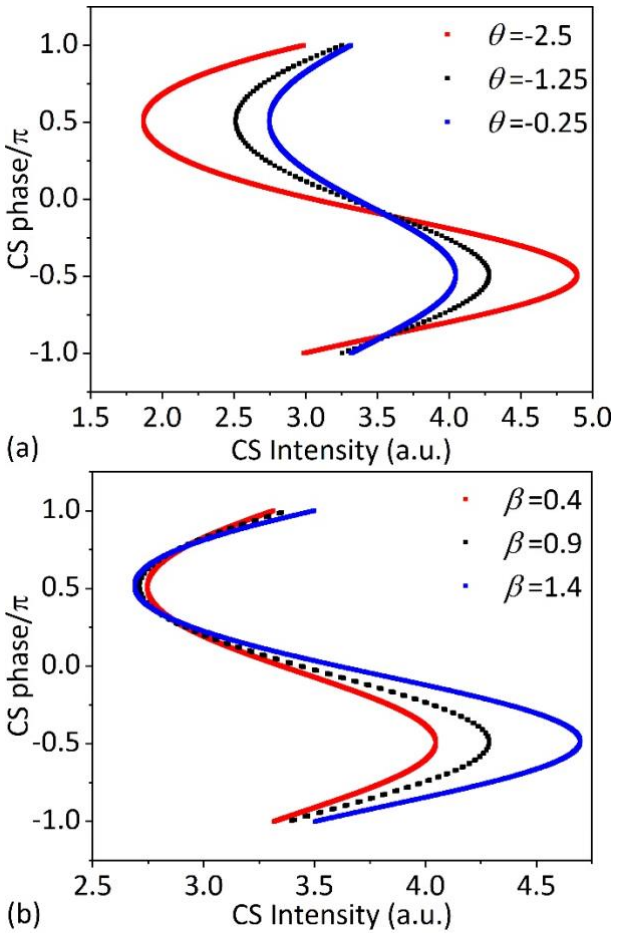

Fig. 2. CS trajectories in the intensity-phase subspace under different values of (a) cavity detuning θ with fixed β=1 and (b) linewidth enhancement factor β and fixed cavity detuning θ=-0.25.

III. Results and Discussion

We first study P1 oscillations of CSs by their trajectories in the intensity-phase subspace. As the phase is unlocked and varies between (π,-π), intensity oscillations adopt larger amplitudes by increasing the values of detuning θ and the linewidth enhancement factor of the passive material β, see Fig. 2 (a) and (b) respectively. An important point to notice is the asymmetric oscillation amplitudes by increased values of the cavity detuning and the linewidth enhancement factor. The asymmetry is more substantial for in the linewidth enhancement factor β in Fig. 2 (b) since β nonlinearly couples the amplitude of the intracavity optical field to its phase. The optical and power spectra of the

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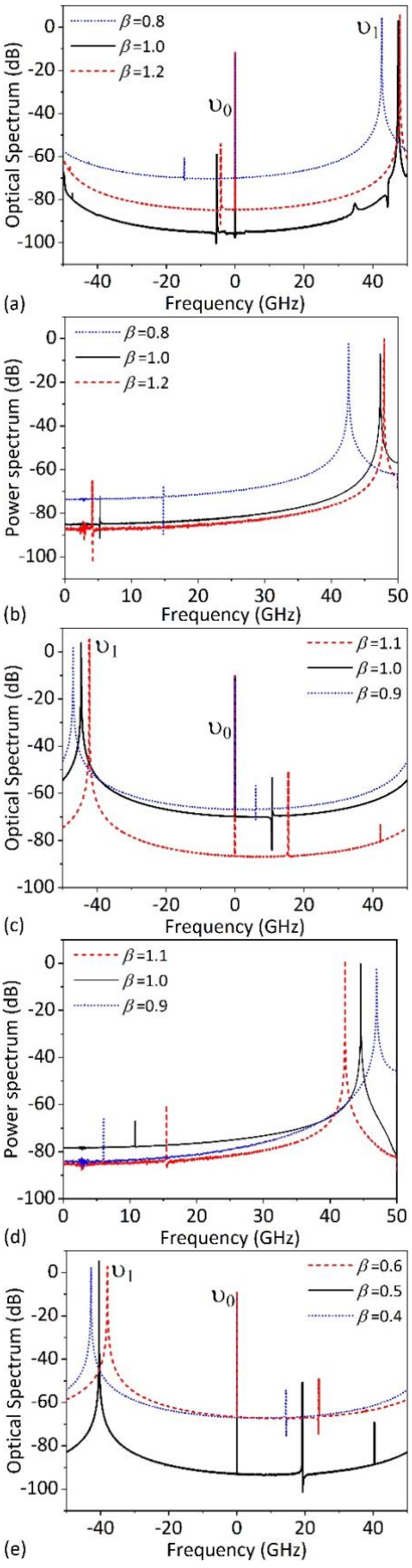
emitted light by the period-one CSs have also been studied by varying the values of θ and β.



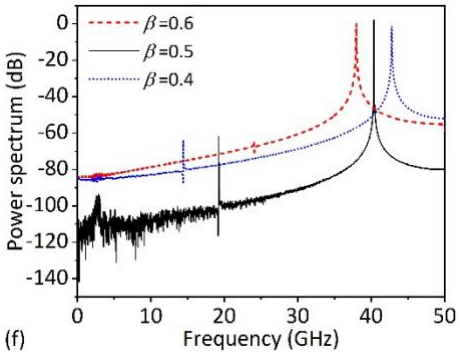


Fig. 3. Optical and power spectra of a P1 CS. (a,b) θ=-0.5, (c,d) -1, and (e,f)-2.

We observe from Fig. 3 (a, c, e) that as a result of injection the frequency of the emitted light shifts to a large value around $\nu_1 \approx 45$ GHz which appears in a larger peak than the injection frequency depicted as $\nu_0=0$. A side band also appears in the optical spectrum with a much smaller intensity which belongs to the background emission and is well-separated in frequency from that of the CS. These are also pronounced in the power spectrum as shown in Fig. 3 (b, d, f). It is seen that a very narrow peak with high intensity appears far from that of the background emission. Moreover, it is evident from both the optical and power spectra that linewidth enhancement factor can alter the frequency of emission and intensity oscillation for the CSs. Depending on the value of the cavity detuning θ, larger β values shift the frequencies to larger or smaller values. It also affects the frequency separation of the two peaks belonging to the P1 CSs and the background emission such that when the frequency associated with the CSs shift toward smaller values, their separation from the one of the backgrounds shrink as well, and vice versa. In Fig. 4 we sketch the P1 oscillation frequencies of the CSs in terms of cavity detuning θ and linewidth enhancement factor β. As explained earlier, it is seen that these two parameters can fortify or counteract each other in increasing or decreasing the P1 oscillation frequency of the CSs since they both deal with the resonance frequencies. We should note that these two factors determine the CS stability range too, which suggests that it is even possible to achieve wider frequency tunability range for P1 CSs with respect to θ and β by changing other control parameter such as μ and α.

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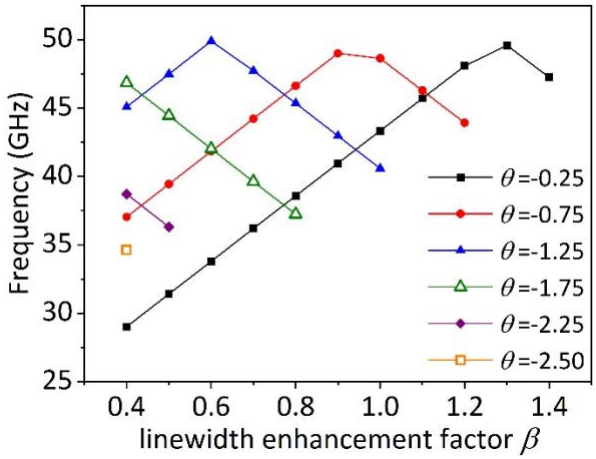

Fig. 4. Frequency of intensity oscilations for P1 CSs in terms of cavity detuning θ and linewidth enhancement factor β.

IV. Conclusion

By numerical simulation of an injected broad-area semiconductor laser with saturable absorber we showed that the linewidth enhancement factor plays an important role in the P1 dynamics of CSs and shifts the oscillation frequencies to smaller or larger values in cooperation with the cavity detuning value. It is particularly discussed that P1 dynamics of CSs in injected semiconductor lasers with respect to the linewidth enhancement factor is affected by dynamic competition between the injection-imposed oscillation frequency and the preferred cavity resonance frequency since they both deal with the resonance frequency. Choosing different values of the linewidth enhancement factor is possible practically and can be used to improve the performance of period-one CSs in microwave applications. Finally, it should be noted that the stability and accuracy of the results are checked via long simulations in the order of tens of nanoseconds much longer than the time steps used in the integration. Different box sizes (128×128 and 256×256) in the transverse section are also used to verify the convergence of the algorithm. In all simulations, CSs are asymptotically stable and preserve their transverse profiles.

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