Dynamical Casimir-Polder Interaction between a Moving Chiral Molecule and a Surface

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ABSTRACT— This paper aims to evaluate the time-dependent Casimir-Polder force of a moving chiral molecule and a magnetodielectric chiral body at finite temperature. The chiral body can be an ensemble of molecules in a biological environment. The temporal evolution of the Casimir-Polder force is considered. The dynamical Casimir-Polder is arising from the movement of the chiral molecule and self-dressing effect is calculated and specific dependence of the force on the velocity, distance and material properties are found. To give an example, the Casimir force of a dimethyl disulfide, which moves above a perfect mirror with positive chirality, is studied. It was observed that the self-dressing part of the Casimir-Polder force was larger than the velocity-dependent part.

KEYWORDS: Biological environment, Chiral molecule, Casimir polder, Dimethyl disulfide.

I. INTRODUCTION

One of the most surprising predictions of quantum field theory is the existence of vacuum energy and zero-point quantum field fluctuations. Casimir and Casimir-Polder forces are originating from field fluctuations [1, 2].

The Casimir force is due to the electromagnetic interactions between globally neutral macroscopic objects while the CP force arises due to the electromagnetic interactions between a globally neutral macroscopic object and a microscopic body. All of these forces have their roots in the quantum fluctuations of the electromagnetic field [3, 4]. These forces are categorized in the context of dispersion forces. It is shown that the interaction force between a ground state molecule and a perfect electric mirror is always attraction while between an excited molecule and a perfect electric mirror interaction force is proportional to the distance [5, 6]

Recently, the generalization of the CP force on chiral objects, i.e. chiral molecules, attracted the attention of many researchers. Chirality is a fundamental property of nature which can be found anywhere from elementary particle physics to macroscopic biological chiral molecules. Chiral molecules play an important role in our daily life. One of the most important problem in biology is the origin of biomolecular homochirality [7]. A chiral molecule is referred to a molecule which is not identical to its mirror image. The mirror image of a chiral molecule is called enantiomer. Enantiomers have the same chemical compositions and molecular structures, which means that their spectroscopic properties are identical. This issue makes their separation difficult task for distinguishing between them. To discriminate between chiral molecules, the characteristic feature is considered as the manner of their interactions with other chiral objects. The difference between the chiral properties of enantiomers appears in different phenomena. For example, the refractive indices of left- and right-handed circularly polarized light are not the same when passing through the chiral medium. This issue culminates to different propagation speeds for the two polarizations that is an important point

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in biological phenomena [8, 9]. The Casimir polder forces between chiral molecules have shown discriminatory effects, which is regarded as a suitable method to separate enantiomers.

Since the 1980's, because of the lack of consistent theory of the fluctuation electromagnetic interaction, material properties, thermal state and retardation effects of interacting bodies, a straight forward calculation of the dynamical Casimir-Polder was absent. The first calculation of the dynamical Casimir-polder force between a moving atom and a solid surface has been done by Ferrell and Ritchie [10].

So far, studies on the dynamical CP force have been restricted to static cases. For instance, the dynamical CP force was calculated for an electric ground state atom near an electric perfect conductor[11], or a partially dressed atomic state [12]. Another version of dynamical CP force was studied between a neutral atom and a real surface [13]. Recently, the dynamical CP force has been calculated for a static chiral molecule by including the self-dressing effect which arises due to the atomic transition frequency [14]. In this paper we investigate the dynamical CP force on the moving molecule. The necessity of study this problem is shown in various phenomena like the interactions between moving biomolecules and their surrounding in the present of the electromagnetic field. Here, the dynamical feature of CP force can be attributed to both of the velocity of the molecule and the self-dressing effect. To include the velocity, we utilize the developed Heisenberg equation of motion by using the small velocity approximation ($v/c<<1$). This dynamical CP force is calculated for an initial ground state chiral molecule and a perfect chiral mirror at finite temperature.

The paper is organized as follows: In section II, we introduce the studied system and solve the Heisenberg equation of motion for the molecule and body-assisted field. Section III will provide a detailed calculation of dynamical CP force by including velocity and self-dressing effects at finite temperature. In section IV, we investigate Casimir-Polder forces of the moving molecule with arbitrary speed near to a perfect mirror in retarded and non-retarded regimes. Finally, the conclusion section will sum up the results of the research.

II. DYNAMICS OF MOVING MOLECULE AND TOTAL HAMILTONIAN

Let us consider a chiral moving molecule which is modeled as a two-level system, interacting with the electromagnetic field in the presence of an arbitrary shape medium. The body-field system is prepared at uniform temperature and the molecule lies in an arbitrary incoherent superposition of internal energy eigenstates. In order to describe the dynamics of the molecule, we have to define a time-dependent flip operator $A_{mn}=|m\rangle\langle n|$, in which $|m\rangle$ is an energy eigenstate.

To obtain the dynamical Casimir-polder between the chiral molecule and the body, it is necessary to know the total Hamiltonian which enables us to describe the dynamics of the system. The total Hamiltonian of the system constitutes of three parts as follows

$$\hat{H}=\hat{H}_m+\hat{H}_f+\hat{H}_{fm}, \tag{1}$$

where $\hat{H}_m$ is Hamiltonian of the molecule, $\hat{H}_f$ is Hamiltonian of the medium-assisted electromagnetic field and $\hat{H}_{fm}$ describes the interaction Hamiltonian between the molecule and the medium-assisted electromagnetic field. The exact form of these Hamiltonians are written as below:

$$\begin{aligned}
&\hat{H}_m=\sum_n E_n\hat{A}_{nn,}\\
&\hat{H}_f=\sum_{\lambda=e,m}\int d^3r\int_0^\infty d\omega\,\hbar\omega\,\hat{f}_\lambda^\dagger(r,\omega).\hat{f}_\lambda(r,\omega)\\
&\hat{H}_{fm}=-\hat{d}.\hat{E}(r_A)-\hat{m}.\hat{B}(r_A)\\
&+\sum_{\alpha\in A}\frac{q_\alpha^2}{8m_\alpha}[\hat{r}_\alpha\times B(r_A)]^2-\frac{5}{8m_\alpha}\left[\hat{d}\times\hat{B}(r_A)\right]^2+\hat{\dot{r}}_A.\left(\hat{d}\times\hat{B}(r_A)\right)
\end{aligned} \tag{2}$$

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In this relation $f_\lambda^\dagger(r,\omega)$ and $\hat{f}_\lambda(r,\omega)$ are generalized bosonic matter-field operators for elementary electric $(\lambda = e)$ and $(\lambda = m)$ magnetic excitations of the system that obey the bosonic commutation relation as follows

$$[\hat{f}_\lambda(r,\omega), \hat{f}_{\lambda'}^\dagger(r',\omega')] = \delta_{\lambda\lambda'}\delta(\omega-\omega')\delta(r-r')$$

where $\hat{\mathbf{d}}$ and $\hat{m}$ are the molecule's electric and magnetic dipole moment operators respectively. r_A is the position of the molecule. Moreover q_α and r_α are the charge and relative position of α particle respectively. Furthermore, m_A and m_α represent the mass of molecule and particle respectively and $\hat{\dot{r}}_A$ is the velocity of the molecule. The Fourier component of electric and magnetic field reads

$$\hat{E}(r) = \int_0^\infty d\omega \hat{E}(r,\omega),$$
$$\hat{B}(r) = \int_0^\infty d\omega \hat{B}(r,\omega). \quad (3)$$

Also the commutator of electric and magnetic field may be written as [14]

$$\left[\hat{E}(r,\omega), \hat{E}^\dagger(r',\omega')\right] = \frac{\hbar\mu_0}{\pi} ImG(r,r',\omega)\omega^2\delta(\omega-\omega'),$$
$$\left[\hat{E}(r,\omega), \hat{B}^\dagger(r',\omega')\right] = \frac{-i\hbar\mu_0}{\pi} ImG(r,r',\omega)\times\vec{\nabla}'\omega\delta(\omega-\omega'),$$
$$\left[\hat{B}(r,\omega), \hat{E}^\dagger(r',\omega)\right] = \frac{-i\hbar\mu_0}{\pi} \nabla\times ImG(r,r',\omega)\omega\delta(\omega-\omega'),$$
$$\left[\hat{B}(r,\omega), \hat{B}^\dagger(r',\omega')\right] = \frac{-i\hbar\mu_0}{\pi} \nabla\times ImG(r,r',\omega)\times\vec{\nabla}'\omega\delta(\omega-\omega'), \quad (4)$$

Here, G is the classical Green tensor of the electromagnetic field where defined by the inhomogeneous Helmholtz equation

$$\left[\nabla\times\frac{1}{\mu(r,\omega)}\nabla\times - \frac{\omega^2}{c^2}\varepsilon(r,\omega)\right]$$
$$G(r,r',\omega) = \delta(r-r') \quad (5)$$

That $\varepsilon(r,\omega)$ and $\mu(r,\omega)$ are permittivity and permeability of the system respectively.

Since we are going to keep only the linear terms of velocity v and the electromagnetic fields are intrinsically fluctuating, so the interaction Hamiltonian may be given as follows [9]

$$\hat{H}_{fm} = -\hat{d}.\hat{E}(r_A) - \hat{m}.\hat{B}(r_A) + V.\left(\hat{d}\times\hat{B}(r_A)\right) \quad (6)$$

As shown, this Hamiltonian explicitly depends on the atomic velocity via the Rontgen term.

Hereafter, we have used V instead of $\dot{r}_A$ and for the sake of simplicity, we assume that molecule moves with constant velocity $v = (v_x, v_y, v_z)$

Schematic view of chiral molecule dynamics above the perfect chiral mirror is shown in Fig. 1.

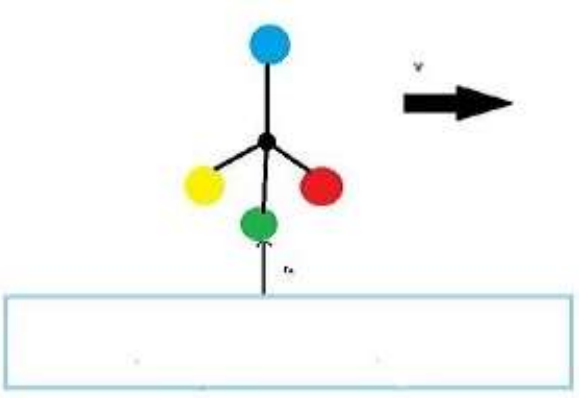

Fig. 1. Dynamical chiral molecule next to a surface.

Using the total Hamiltonian the system, Heisenberg equation for the atomic flip operators is obtained as follows:

$$\dot{\hat{A}}_{mn} = \frac{1}{i\hbar}\left[\hat{A}_{mn}, \hat{H}\right] = i\omega_{mn}\hat{A}_{mn}$$
$$+\frac{i}{\hbar}\hat{K}_{mn}.\left(\hat{E}(r_A) + V\times\hat{B}(r_A)\right) + \frac{i}{\hbar}\hat{Q}_{mn}.\left(\hat{B}(r_A)\right) \quad (7)$$

where $\hat{K}_{mn}$ and $\hat{Q}_{mn}$ are equivalent to:

$$\hat{K}_{mn} = \left[\hat{A}_{mn}, \hat{d}\right], \hat{Q}_{mn} = \left[\hat{A}_{mn}, \hat{m}\right] \quad (8)$$

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The electric field at the position of the molecule consists of two term: the radiation reaction and the free field. As shown in the literature for a purely electric atom, the radiation reaction field gives rise to frequency shifts and spontaneous decay rate of the molecule [15, 16]. By renormalizing the field and splitting off the radiation reaction field, the quantum average of the dynamical flip operators can be written as follows:

$$\begin{aligned}\left\langle \dot{\hat{A}}_{mn}\right\rangle &= \left[i\tilde{\omega}_{mn} - \frac{\Gamma_n + \Gamma_m}{2}\right]\left\langle \hat{A}_{mn}\right\rangle \\ &+ \frac{i}{\hbar}\left\langle V \times \hat{B}^{(0)}\left(r_A\right)\right\rangle + \frac{i}{\hbar}\left\langle \hat{Q}_{mn}\cdot \hat{B}^{(0)}\left(r_A\right)\right\rangle + \frac{i}{\hbar}\left\langle \hat{K}_{mn}\cdot \hat{E}^{(0)}\left(r_A\right)\right\rangle \\ &+ \left[\delta\omega_n\left(V\right) - \delta\omega_m\left(V\right) - \frac{1}{2}\left(\Gamma_n\left(V\right) + \Gamma_m\left(V\right)\right)\right]\left\langle \hat{A}_{mn}\right\rangle \end{aligned} \tag{9}$$

Where the expectation value is taken over the field thermal state and the molecular integral state. $\tilde{\omega}_{mn}$ is the Lamb-shifted molecular frequencies and Γ_n the decay rates, which have electric and magnetic and chiral parts. $\delta\omega_n\left(V\right)$ and $\Gamma_n\left(V\right)$ are frequency shifts and transition rates that contain motion-induced corrections.

$p = n(t) = \left\langle \hat{A}_{nn}(t)\right\rangle$ is populations and the $\sigma_{nm}(t) = \left\langle \hat{A}_{nm}(t)\right\rangle$ is coherence and the expectation value is taken over the field thermal state and the molecular internal state.

Complete form of electric and magnetic field is clear when we know the dynamics of $f_\lambda(r,\omega)$ and $f_\lambda^\dagger(r,\omega)$. To this purpose, we use the Heisenberg equation for this operators:

$$\begin{aligned}\dot{\hat{f}}\left(r,\omega\right) &= \frac{1}{i\hbar}\left[\hat{f}_\lambda\left(r,\omega\right), \hat{H}\right] = -i\omega\cdot f_\lambda\left(r,\omega\right) \\ &+ \frac{i}{\hbar}\sum_{mn} G_\lambda^{*T}\left(r_A, r, \omega\right)\cdot d_{mn}\hat{A}_{mn} \\ &- \frac{1}{\hbar\omega}\sum_{m,n}\left[G(r, r_A, \omega)\times\vec{\nabla}'\right]\times d_{mn}\cdot V\,\hat{A}_{mn} \\ &+ \frac{1}{\hbar\omega}\sum_{m,n}[\left[G_\lambda^{*T}(r, r_A, \omega)\times\vec{\nabla}'\right]\cdot m_{mn}\hat{A}_{mn}. \end{aligned} \tag{10}$$

By solving this equation we reach to the following result

$$\begin{aligned}\hat{f}_\lambda\left(r,\omega,t\right) &= e^{-i\omega(t-t_0)}\,\hat{f}_\lambda\left(r,\omega\right) \\ &+ \frac{i}{\hbar}\sum_{mn}\int_{t_0}^{t} dt' e^{-i\omega(t-t')} G_\lambda^{*T}\left(r_A(t'), r, \omega\right)\cdot\hat{A}_{mn}(t')d_{mn} \\ &+ \frac{1}{\hbar\omega}\sum_{m,n}\int_{t_0}^{t} dt' e^{-i\omega(t-t')} S\left(r, r_A(t'), \omega\right)\times d_{mn}\cdot V\,\hat{A}_{mn}(t') \\ &+ \frac{1}{\hbar\omega}\sum_{m,n}\int_{t_0}^{t} dt' e^{-i\omega(t-t')} S(r, r_A(t'), \omega)\cdot\hat{A}_{mn}(t')m_{mn} \end{aligned} \tag{11}$$

in which $S\left(r, r_A(t'), \omega\right)$ is defined as

$$S\left(r, r_A(t'), \omega\right) \equiv G_\lambda^{*T}\left[r, r_A(t'), \omega\right]\times\vec{\nabla}' \tag{12}$$

Now we suppose that molecule has small and constant velocity in which (V<<c). Assuming so, we are able to expand the atom -field dynamics up to first order in V/c and neglect higher

$$r_A(t') = r_A\left(t\right) + (t-t')V. \tag{13}$$

Using the above assumption and keep only the linear terms in expansion V/c, annihilation operator $\hat{f}_\lambda(r,\omega,t)$ in Eq. (11) is written as follows:

$$\begin{aligned}\hat{f}_\lambda\left(r,\omega,t\right) &= e^{-i\omega(t-t_0)}\,\hat{f}_\lambda(r,\omega) \\ &+ \frac{i}{\hbar}\sum_{mn}\int_{t_0}^{t} dt' e^{-i\omega(t-t')}\mathcal{L}_{mn}\left(r_A, r, \omega\right)\hat{A}_{mn}(t') \end{aligned} \tag{14}$$

in which $\mathcal{L}_{mn}\left(r_A, r, \omega\right)$ has the following definition

$$\begin{aligned}\mathcal{L}_{mn}\left(r_A, r, \omega\right) &= G_\lambda^{*T}\left(r_A, r, \omega\right).d_{mn} \\ &-(t-t')G_\lambda^{*T}\left(r_A, r, \omega\right)\cdot d_{mn}(\vec{\nabla}'\cdot V) \\ &-\frac{i}{\omega}\left(S\left(r, r_A, \omega\right)\times d_{mn}V - \frac{i}{\omega}S\left(r, r_A, \omega\right)m_{mn}\right) \\ &+\frac{i}{\omega}(t-t')S\left(r, r_A, \omega\right)\cdot m_{mn}\left(\vec{\nabla}'\cdot V\right) \end{aligned} \tag{15}$$

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Using the relations (14) and (15) one may write the electric and magnetic fields as follows:

$$\begin{aligned}&\hat{E}(r,\omega,t)=e^{-i\omega(t-t_0)}\hat{E}(r,\omega)\\&+\frac{i\mu_0}{\pi}\sum_{mn}\int_{t_0}^{t}dt'e^{-i\omega(t-t')}\varepsilon_{mn}(r,r_A,\omega)\hat{A}_{mn}(t')\\&\hat{B}(r,\omega,t)=e^{-i\omega(t-t_0)}\hat{B}(r,\omega)\\&+\frac{i\mu_0}{\pi}\sum_{mn}\int_{t_0}^{t}dt'e^{-i\omega(t-t')}\mathcal{B}_{mn}(r,r_A,\omega)\hat{A}_{mn}(t')\end{aligned}\tag{16}$$

In this relation $\varepsilon_{mn}(r,r_A,\omega)$ and $\mathcal{B}_{mn}(r,r_A,\omega)$ have following definition

$$\begin{aligned}&\mathcal{B}_{mn}(r,r_A,\omega)=-i\omega\nabla\times ImG(r,r_A,\omega)\cdot d_{mn}\\&+i\omega(t-t')(\nabla\times ImG(r,r_A,\omega)\cdot d_{mn}(\overleftarrow{\nabla}'\cdot V)\\&-\left\{\left(\nabla\times ImG(r,r_A,\omega)\right)\times\overleftarrow{\nabla}'\right\}\cdot n\\&+(t-t')\left\{\left(\nabla\times ImG(r,r_A,\omega)\right)\times\overleftarrow{\nabla}'\right\}\cdot m_{mn}(\overleftarrow{\nabla}'\cdot V)\\&-\left\{\left(\nabla\times ImG(r,r_A,\omega)\right)\times\overleftarrow{\nabla}'\right\}\times d_{mn}\cdot V\end{aligned}\tag{17}$$

One can check that in the limit of $v\longrightarrow 0$, we restore the results of [14]. The flip operator and fields are sum of the free terms, as they would be in absence of coupling and induced terms.

III. DYNAMICAL CASIMIR-POLDER FORCE

Utilizing the electric and magnetic field representation shown in (16) and (17), we can obtain the dynamics of atom-field. We will calculate CP force, which is generated by the interaction between a constant-velocity molecule with electric, magnetic and chiral properties and the field of a macroscopic object. The dynamical CP force for a moving molecule is obtained by

$$\begin{aligned}F=&\nabla\left\langle\hat{d}\cdot(\hat{E}(r_A)+V\times\hat{B}(r_A))\right\rangle\\&+\nabla\left\langle\hat{m}\cdot\hat{B}(r_A)\right\rangle\Big|_{r=r_A}\end{aligned}\tag{18}$$

To obtain CP force from above relation, we need all operators that are calculated by solving the Heisenberg equation and the expectation value is taken over the thermal field state and the internal molecular state [9]. We can express the electric field in terms of its free part and the source field due to the molecule Eq. (16)

$$\begin{aligned}&F(t)=\int_0^\infty d\omega\sum_{m,n}\nabla\left\langle\hat{A}_{mn}(t)d_{mn}.\hat{E}^{(0)}(r,\omega,t)\right\rangle/_{r=r_A}\\&+\int_0^\infty d\omega\sum_{m,n}\nabla\left\langle V\times B^{(0)}(r,\omega,t)\right\rangle/_{r=r_A}\\&+\int_0^\infty d\omega\sum_{m,n}\nabla\left\langle\hat{A}_{m,n}(t)m_{m,n}.\hat{B}^{(0)}(r,\omega,t)\right\rangle/_{r=r_A}\\&+\frac{i\mu_0}{\pi}\sum_{m,n}\sum_{p,q}\int_0^\infty d\omega\int_0^t dt'e^{-i\omega(t-t')}\left\langle A_{mn}(t)A_{pq}(t')\right\rangle\nabla\mathcal{I}_{mnpq}(\mathrm{r},\mathrm{r}_A,\omega)\end{aligned}\tag{19}$$

Here $\mathcal{I}_{mnpq}(\mathrm{r},\mathrm{r}_A,\omega)$ has following definition

$$\begin{aligned}\mathcal{I}_{mnpq}(\mathrm{r},\mathrm{r}_A,\omega)=&\mathcal{I}^{(0)}_{mnpq}(\mathrm{r},\mathrm{r}_A,\omega)+\omega\mathcal{I}^{(1)}_{mnpq}\\&(\mathrm{r},\mathrm{r}_A,\omega)+\omega^2\mathcal{I}^{(2)}_{mnpq}(\mathrm{r},\mathrm{r}_A,\omega)\end{aligned}\tag{20}$$

where

$$\begin{aligned}&\mathcal{I}^{(0)}_{mnpq}(\mathrm{r},\mathrm{r}_A,\omega)=-m_{mn}\cdot\left\{\left(\nabla\times ImG(r,r_A,\omega)\right)\times\overleftarrow{\nabla}'\right\}\cdot m_{pq}\\&+V\left\{-i\omega d_{mn}\cdot\left(\nabla\times ImG(r,r_A,\omega)\right)\cdot d_{pq}\right.\\&+imd_{mn}(t-t')\left(\nabla\times ImG(r,r_A,\omega)\right).d_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)\\&-d_{mn}\left\{\left(\nabla\times ImG(r,r_A,\omega)\right)\times\overleftarrow{\nabla}'\right\}\cdot m_{pq}\\&+d_{mn}(t-t')\left\{\left(\nabla\times ImG(r,r_A,\omega)\right)\times\overleftarrow{\nabla}'\right\}\cdot m_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)\\&+m_{mn}(t-t')\left[\nabla\times ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right]\cdot m_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)\\&-m_{mn}\left[\nabla\times ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right]\times d_{pq}V+C.C.-\\&\left.-d_{mn}\left(ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right)\times d_{pq}\cdot V\right\}\\&+m_{mn}(t-t')\left[\nabla\times ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right]\cdot m_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)\\&-m_{mn}\left[\nabla\times ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right]\times d_{pq}V+C.C.\\&\omega\mathcal{I}^{(1)}_{mnpq}(r,r_A,\omega)=-id\cdot\left\{ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right\}\cdot m_{pq}\\&-imd_{mn}\left\{\nabla\times ImG(r,r_A,\omega)\right\}\cdot d_{pq}\\&+id_{mn}(t-t')\left(ImG(r,r_A,\omega)\times\overleftarrow{\nabla}'\right)\cdot m_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)-\end{aligned}$$

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$$
\begin{aligned}
&-d_{mn}\left(ImG\left(r,r_A,\omega\right)\times\overleftarrow{\nabla}'\right)\times d_{pq}\cdot V\\
&+imd_{mn}\left(t-t'\right)\left(\nabla\times ImG\left(r,r_A,\omega\right)\right)\cdot d_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)+C.C.\\
&\omega^2\mathcal{I}^{(2)}_{mnpq}\left(\mathrm{r},\mathrm{r}_A,\omega\right)=d_{mn}.ImG\left(r,r_A,\omega\right)\cdot d_{pq}\\
&-d_{mn}\left(t-t'\right)ImG\left(r,r_A,\omega\right)d_{pq}\left(\overleftarrow{\nabla}'\cdot V\right)
\end{aligned}
\tag{21}
$$

In this step, we use the known formula for the field fluctuation [9].

$$
\begin{aligned}
&\left\langle E^{(0)\dagger}\left(r,\omega\right)E^{(0)}(r',\omega')\right\rangle=\\
&\qquad\frac{\hbar\mu_0}{\pi}ImG\left(r,r',\omega\right)\omega^2\delta\left(\omega-\omega'\right)n\left(\omega\right).\\
&\left\langle E^{(0)\dagger}\left(r,\omega\right)B^{(0)}\left(r',\omega'\right)\right\rangle=\\
&\qquad\frac{i\hbar\mu_0}{\pi}ImG\left(r,r',\omega\right)\times\overleftarrow{\nabla}'\omega\delta\left(\omega-\omega'\right)n(\omega)\\
&\left\langle B^{(0)\dagger}\left(r,\omega\right)E^{(0)}\left(r',\omega'\right)\right\rangle=\\
&\qquad\frac{i\hbar\mu_0}{\pi}\vec{\nabla}\times ImG\left(r,r',\omega\right)\omega\delta\left(\omega-\omega'\right)n\left(\omega\right)\\
&\left\langle B^{(0)\dagger}\left(r,\omega\right)B^{(0)}\left(r',\omega'\right)\right\rangle=\\
&-\frac{\hbar\mu_0}{\pi}\vec{\nabla}\times ImG\left(r,r',\omega\right)\times\overleftarrow{\nabla}'\omega\delta\left(\omega-\omega'\right)n\left(\omega\right)
\end{aligned}
\tag{22}
$$

Here, $n(\omega)$ is the Bose-Einstein distribution function

$$
n\left(\omega\right)=\frac{1}{e^{\hbar\omega/K_BT}-1}. \tag{23}
$$

In Eq. (19) we need to calculate the two-time correlation function of the flip operator where we use Lax regression theorem to simplify as follows

$$
\begin{aligned}
&\int_0^{r_2}F(r,\varphi)\,dr\,d\varphi=\left[\sigma r_2/(2\mu_0)\right]\times\\
&\int_0^{\infty}e^{-\lambda\left|z_j-z_i\right|}\lambda^{-1}\,J_1\left(\lambda r_2\right)J_0\left(\lambda r_i\right)d\lambda\,.
\end{aligned}
\tag{24}
$$

The small velocity approximation will enable us to write $f_{mn}\left(t-t'\right)$ as

$$
\begin{aligned}
f_{mn}\left(t-t'\right)=e^{\left(i\tilde{\omega}_{mn}-\frac{\Gamma_n+\Gamma_m}{2}\right)\left(t-t'\right)}\Big[&1+\delta\omega_n(V)\\
&-\delta\omega_m(V)-\frac{i}{2}\left(\Gamma_n(V)+\Gamma_m\left(V\right)\right)\Big]
\end{aligned}
\tag{25}
$$

Using the above formulas and performing time integration, we can write the total CP force as being summed over internal states

$$
F\left(t\right)=\sum_n p_n\left(t\right)F_n\left(t\right). \tag{26}
$$

In which $p_n\left(t\right)$ is the population of the energy-state $\left|n\right\rangle$. We can decompose the CP force $F_n(t)$ into the electric, magnetic and chiral part as follows

$$
F(t)=F_n^e(t)+F_n^m(t)+F_n^c(t), \tag{27}
$$

The electric part of the CP force is written as

$$
\begin{aligned}
&F_n^e\left(t\right)=\frac{\mu_0}{2\pi}\int_0^{\infty}d\omega\omega^2\sum_k\psi'_{kn}\left(\omega,t\right)\nabla_A tr\left(d_{nk}ImG\left(r,r_A,\omega\right)d_{kn}\right)\\
&+\frac{\mu_0}{2\pi}\sum_K\left[\delta\omega_n\left(V\right)-\delta\omega_k\left(V\right)-\frac{i}{2}\left(\Gamma_n\left(V\right)+\Gamma_k\left(V\right)\right)\right]\\
&\times\int_0^{\infty}d\omega\omega^2\psi'(\omega,t)\nabla_A tr\left(d_{nk}ImG\left(r,r_A,\omega\right)d_{kn}\right)\\
&+\frac{i\mu_0}{2\pi}\sum_k\int_0^{\infty}d\omega\omega\frac{1-e^{-it\left(\omega+\omega_{kn}^{(-)}\right)}}{\left(\omega+\omega_{kn}^{(-)}\right)^2}d_{nk}\cdot d_{kn}\\
&\times\Big\{\omega\nabla_A tr\left(ImG\left(r,r_A,\omega\right)\cdot V\cdot\overleftarrow{\nabla}'\right)\\
&+\left(\omega+\omega_{kn}^{(-)}\right)\nabla_A tr\left\{V\times\nabla\times ImG\left(r,r_A,\omega\right)\right\}\\
&+\left(\omega+\omega_{kn}^{-}\right)\times\nabla_A tr\left\{V\times\nabla\times ImG\left(r,r_A,\omega\right)\right\}\\
&-i\nabla_A tr\left\{V\times\nabla\times ImG\left(r,r_A,\omega d\omega\right)V\cdot\overleftarrow{\nabla}'\right\}\Big\}
\end{aligned}
\tag{28}
$$

The magnetic part is also as below:

$$
\begin{aligned}
&F_n^m\left(t\right)=-\frac{\mu_0}{3\pi}\int_0^{\infty}d\omega\sum_k m_{nk}.m_{kn}\psi'_{kn}\left(\omega,t\right)\\
&\qquad\times\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\times\overleftarrow{\nabla}'\right\}\\
&-\frac{\mu_0}{3\pi}\sum_K\left[\delta\omega_n\left(V\right)-\delta\omega_k\left(V\right)-\frac{i}{2}\left(\Gamma_n\left(V\right)+\Gamma_k\left(V\right)\right)\right]\\
&\times\int_0^{\infty}d\omega\,m_{nk}\cdot m_{kn}\psi'_{kn}\left(\omega,t\right)\times\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\times\overleftarrow{\nabla}'\right\}
\end{aligned}
$$

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$$-\frac{i\mu_0}{3\pi}\sum_k\int_0^\infty d\omega\omega\frac{1-e^{-it\left(\omega+\omega_{kn}^{(-)}\right)}}{\left(\omega+\omega_{kn}^{(-)}\right)^2}$$
$$\times\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\times\overleftarrow{\nabla}'\left(V\cdot\overleftarrow{\nabla}'\right)\right\} \quad (29)$$

And the chiral force which, is the main purpose of our work, is given as follows:

$$F_n^c(t)=\frac{2i\mu_0}{3\pi}\int_0^\infty d\omega\omega\sum_k\psi'_{kn}(\omega,t)$$
$$\times d_{nk}\cdot m_{kn}\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\right\}$$
$$+\frac{2i\mu_0}{3\pi}\sum_K\left[\delta\omega_n(V)-\delta\omega_k(V)-\frac{i}{2}(\Gamma_n(V)+\Gamma_k(V))\right]$$
$$\times\int_0^\infty d\omega\omega\psi'_{kn}(\omega,t)\,d_{nk}.m_{kn}\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\right\}$$
$$-\frac{i\mu_0}{3\pi}\sum_k\int_0^\infty d\omega d_{nk}\cdot m_{kn}\frac{1-e^{-it\left(\omega+\omega_{kn}^{(-)}\right)}}{\left(\omega+\omega_{kn}^{(-)}\right)^2}$$
$$\times\left[2n(\omega)(\omega+\omega_{kn}^{(-)})\nabla_A tr\left\{\nabla\times\left(\nabla\times ImG\left(r,r_A,\omega\right)\right)\times\overleftarrow{\nabla}'\right\}\right.$$
$$+2\omega\left(V\cdot\overleftarrow{\nabla}'\right)\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\right\}$$
$$\left.+i\nabla_A tr\left\{\nabla\times\nabla\times ImG\left(r,r_A,\omega\right)\times\overleftarrow{\nabla}'\left(V\cdot\overleftarrow{\nabla}'\right)\right\}\right] \quad (30)$$

where

$$\psi'_{kn}(\omega,t)=\frac{1+n(\omega)}{(\omega_{kn}+\omega)}(1-\cos\cos[(\omega_{kn}+\omega)t])$$
$$+\frac{n(\omega)}{(\omega_{kn}-\omega)}\left(1-\cos\left[(\omega_{kn}+\omega)t\right]\right) \quad (31)$$

and $\omega_{kn}^{(\pm)}=\tilde{\omega}_{kn}\pm i(\Gamma_n+\Gamma_k)/2$.

Under reflection, the electric dipole moment changes sign, while the magnetic dipole moment does not. Hence, the electric and magnetic parts of the dynamical interaction do not change sign if the molecule is substituted with its enantiomer (mirror image), while the chiral part of the CP force changes sign. This shows the discriminatory effect for the chiral part of the dynamical interaction [14]. The CP force on a moving atom thus depends crucially on the atomic level shifts and widths.

IV. EXAMPLE

In this section, we consider the interaction between a ground-state chiral molecule and a perfectly reflecting chiral plate at zero temperature. We assume chiral molecule moving parallel to the plate in the z direction. In particular, the explicit dependence of the force on the Doppler shifts $\delta\omega_n(v),\delta\omega_k(v)$ and widths $\Gamma_n(v),\Gamma_k(v)$ make calculations difficult. For simplicity, we assume that an atom satisfies two conditions. Firstly, Due to translational invariance of the system the Doppler term does not contribute in this case, so $\delta\omega_n(v),\ \delta\omega_k(v),\ \Gamma_n(v)$, and $\Gamma_k(v)=0$. Secondly, the atom must be sufficiently far from anybody, So that its level widths are well approximated by the values of them in free space values, $\Gamma_{k''}=\Gamma_{k'}$ whenever $k',k''\in\bar{k}$ [17]. Eventually, by considering these assumptions the chiral part of the CP force simplifies to

$$F_n^c(t)=\frac{2i\mu_0}{3\pi}\int_0^\infty d\omega\,\omega\sum_k\psi'_{kn}(\omega,t)$$
$$\times d_{nk}\cdot m_{kn}\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\right\}$$
$$-\frac{2\mu_0}{3\pi}\sum_k\int_0^\infty d\omega\,\omega\,d_{nk}\cdot m_{kn}\frac{1-e^{-it\left(\omega+\omega_{kn}^{-}\right)}}{\left(\omega+\omega_{kn}^{-}\right)^2} \quad (32)$$
$$\times\nabla_A tr\left\{\nabla\times ImG\left(r,r_A,\omega\right)\left(V\cdot\overleftarrow{\nabla}'\right)\right\}$$

The first term is obtained from self-dressing and the second part is a stem from the molecule speed effect.

For ground-state atoms, the velocity-dependence force is very small and always decelerating, hence commonly known as quantum friction (force antiparallel to its velocity). The quantum friction is proportional to the atomic damping parameters Γ_k which in turn are proportional to the metallic damping parameter γ. As expected for a dissipative force, quantum friction vanishes in the absence of damping. The dependence on the γ indicates that the kinetic energy lost by the moving atom leads to Ohmic heating of the plate.

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The Green function of this system is obtained from Eq. (5), as follows

$$\frac{\partial}{\partial d}\left\{\omega\nabla\times \mathrm{Im}\,G(r_A,r_A,\omega)\right\}=$$
$$\pm\frac{3c}{8\pi d^4}\lim_{m\to 1}\left[1-\frac{\partial}{\partial m}+\frac{1}{3}\frac{\partial^2}{\partial m^2}\right]\cos(mx)\Big|_{x=2d\omega/c} \quad (33)$$

By calculating the frequency integration in the above relation, the explicit form of the force is found as follows

$$F_0^c(v)=\pm\frac{\mu_0 v_z}{d^4\pi^2}\lim_{m\to 1}\left[1-\frac{\partial}{\partial m}+\frac{5}{12}\frac{\partial^2}{\partial m^2}-\frac{1}{12}\frac{\partial^3}{\partial m^3}\right]$$
$$\times\sum_k R_{ok}\left[-\frac{\cos(ax_k)}{2}\left\{\cos\left[x(a+m)\right]+\cos\left[x(a-m)\right]\right\}\right.$$
$$\left.+\frac{\sin(ax_k)}{2}\left\{\sin\left[x(a+m)\right]+\sin\left[x(a-m)\right]\right\}\right] \quad (34)$$

In this relation, $R_{ok}=\mathrm{Im}\left(d_{ok}.m_{k0}\right)$ is the rotatory strength of molecule and $x=\frac{2d\omega}{c}$, $x_k=\frac{2d\omega_k}{c}$, and $a=\frac{ct}{2d}$. Moreover, $(\pm)$ signs refer to the chirality of mirror in which (+) sign refers to the positive chirality and (-) sign refers to the negative chirality of mirror. As an example of a chiral molecule, consider dimethyl disulphide (CH3)2S2. We have chosen only the first transition when the orientation between the two CH3-S-S planes is 90^0. Transition frequency between the excited state and the ground state is $\omega_{10}=9.17\times10^{15}Hz$, the square of the dipole moment is $\left|d_{01}\right|^2=8.264\times10^{-60}Cm^2$ and the rotatory strength is $R_{10}=3.328\times10^{-64}C^2m^3s^{-1}$ [18]. In Fig. 2, chiral dynamical Casimir-Polder, the interaction between a ground-state dimethyl disulfide and a perfect mirror of positive chirality, is shown. The chiral part of CP force is sketched in the case of positive chirality of mirror, the molecule dimethyl disulfide is in its ground state and time being in the order of $T=10^{-15}s$. As it is shown in Fig. 2 and from relation (33), the chiral force is in direct proportional with molecule's velocity and decreased with increasing distance. Since the Casimir-Polder is due to the exchange of one virtual photon between the molecule and the mirror. Therefore by increasing distance, the photon needs more time to be reflected and absorbed by the molecule and by increasing time, daming parameter is increased therefore Casimir-polder force is decreased. From the mathematical point, the trace of the Green tensor scales differently for small and large distances leading to different dependences of the force on the distance.

In addition, a delay between the emission and reabsorption of the photon by the atom arises because of the velocity in Casimir-polder effect. So it is expected the Casimir-polder force increase with increasing speed. Fig. 3, the chiral force is shown for the same molecule by moving with speed $v=100m/s$ and distance between molecule and mirror is set at $d=0.1\mu m$ and the time is less than $t<\frac{2d}{c}=4\ fs$. This time proves that we sketch it for the non-retarded regime. According to the Fig. 3 the force exhibiting an oscillatory behavior in time around the stationary value. Depending on the time, the force can be attractive and repulsive for a given distance, contrary to the static case. It should be noted that in the figures, the amount of stationary state Casimir-polder force, is set to zero for simplicity. Therefore, the fluctuations in the figures are centered around zero. Figure 4, is sketched for the same situation as in Fig. 3 except that it is in the retarded regime, the time is needed for light emitted by the atom to be reflected by the mirror and return to the molecule, which means that $t>\frac{2d}{c}=4\ fs$. As it is evident, by passing the time the force strength is reduced to the stationary value of Casimir-polder force. As expected, the retarded interaction decreases more rapidly due to finite velocity of the light: During the time in which the virtual photon has been exchanged, the molecule will evolve. This associated loss of correlation leads to a more

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rapidly decreasing force. According to the Fig. 4 the velocity has increased the force strength. Fig. 5 shows the chiral force acted on the molecule, which locates at distance 1nm from the mirror and moving with speed 100m/s in the retarded regime, i.e. $t > \frac{2d}{c}$. Strength of force is increased with respect to Fig. 4 due to distance is decreased.

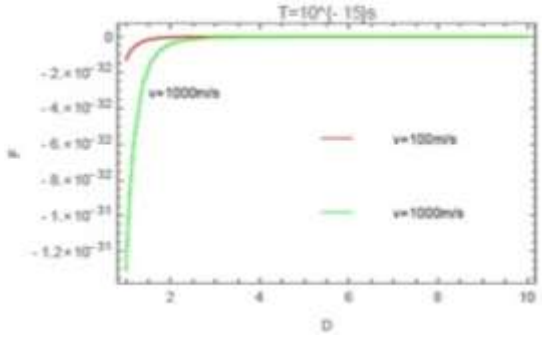

Fig. 2. Chiral dynamical Casimir-Polder interaction between a ground-state dimethyl disulfide and a positive perfect chiral medium. Casimir polder force in term of distance for $T = 10^{-15} s$.

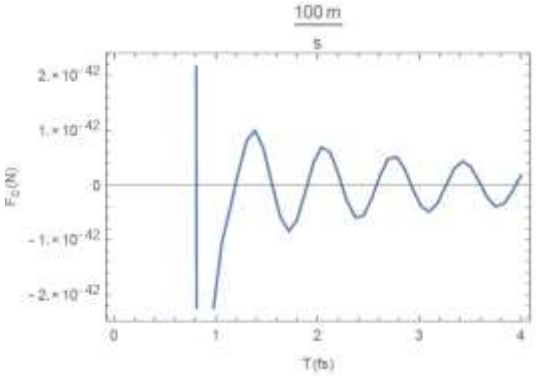

Fig. 3. Chiral dynamical Casimir-Polder interaction between a ground- state dimethyl disulfide and a positive perfect chiral medium for $d = 0.1\mu m$ and $t < 2d / c = 4\, fs$

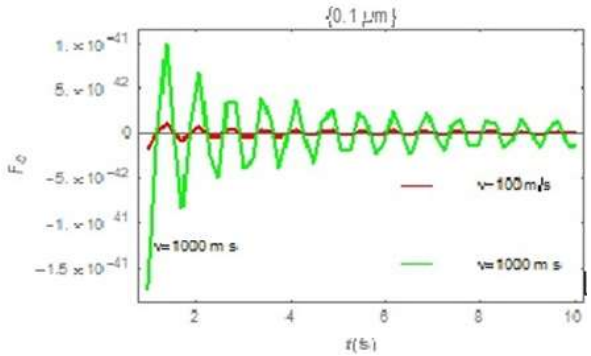

Fig. 4. Chiral dynamical Casimir-Polder interaction between a ground- state dimethyl disulfide and a positive perfect chiral medium for $d = 0.1\mu m$ and $t > 2d / c$.

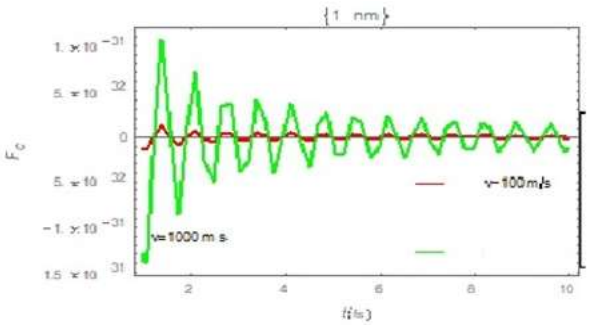

Fig. 5. Chiral dynamical Casimir-Polder interaction between a ground- state dimethyl disulfide and a positive perfect chiral medium for d=1nm and $t > 2d / c$.

V. CONCLUSION

In this paper, we studied the dynamical chiral CP force of a moving molecule with an arbitrary internal state. at finite temperature. We began with considering developed Heisenberg equations of motion which include the velocity of the molecule. We solved these equations for the atomic flip operator, annihilation operator and electromagnetic fields in the small velocity approximation ($V / c << 1$). These equations will enable us to find the CP force by using the Lorentz expression. As an example the chiral part of Casimir-polder force for a ground state molecule near to a perfect mirror was calculated at zero temperature $n(\omega) = 0$. We conclude that, in general, the velocity independent term in relation (32) has more effect on CP force than the velocity-dependent term (the second term). Velocity has a direct effect on CP force, since it appears as an individual factor in front of force. The relative strength of these two forces is also a good indicator of the velocity effect on chiral forces. As it is depicted in above-mentioned figures, at large distances ($d > 10^{-6} m$), the velocity-dependent force does not play a significant role, rather it has a minor role in comparison to the velocity independent force. However by lowering the distance and making the velocity to be large, the importance of velocity-dependent force manifest itself.

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