The Simultaneous Effect of the Temperature and Density Gradient on the Relativistic Self-Focusing of the Gaussian Laser Beam in an Under-Dens Plasma

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ABSTRACT— In this paper, the interaction of Gaussian laser beam with under-dense plasma by taking the weakly relativistic ponderomotive nonlinearity has been investigated. For this purpose, the effect of the linear plasma electron temperature and upward exponential electron density profile on the laser propagation has been studied. The nonlinear second-order differential equation of the dimensionless beam-width parameter, *f*, on the distance of propagation, η, is derived by following WKB and paraxial approximation and solved numerically for the several initial electron temperatures. It is found that, the electron temperature ramp combined with upward ramp density profile would be caused stronger self-focusing where the beam width oscillates with less amplitude and smaller spot size. This could lead to further penetration of the laser beam inside the plasma by reducing the effects of diffraction.

KEYWORDS: Gaussian laser beam, Self- focusing, Relativistic, Ponderomotive

I. INTRODUCTION

One of the most important research topics on the interaction of a laser beam with plasma is the self-focusing phenomenon due to its technological application such as plasma-based accelerators, high harmonic generation, x-ray lasers, laser fusion, etc. [1-10]. Initial research on this phenomenon was discovered by Askaryan [11] and eventually verified by some other researchers [12-13]. The necessity for the self-focusing is that the laser beam is sufficiently powerful and intense to propagate considerable distance without divergence. In most cases, laser self-focusing of the laser beam inside the plasma is as ponderomotive self-focusing (PSF) and a relativistic self-focusing (RSF) [14-17]. Ponderomotive self-focusing (PSF) is caused by the ponderomotive force, which pushes electrons away from the region where the laser beam is more intense, therefore increasing the refractive index and inducing a focusing effect. The effect of this force on ions can be neglected due to their high mass. Relativistic self-focusing (RSF) can be appeared as a result of the relativistic mass increase of plasma electrons travelling at speed approaching speed of light, which modifies the plasma refractive index. Density profile and plasma electron temperature has a vital role for self-focusing of laser beam. So far, many studies have been done on the role of plasma electron temperature and electron density profile in the self-focusing of laser beams. For examples: plasma density ramp for relativistic self-focusing of an intense laser is studied by D. N. Gupta *et al*. [18]. Self-focusing of a laser pulse in plasma with periodic density ripple is studied by S. Kaur *et al*. [19]. Ponderomotive self-focusing of a short laser pulse under a plasma density ramp is studied by N Kant *et al*. [20]. Improving the relativistic self-focusing of intense laser beam in plasma using density transition is investigated by Sadighi-Bonabi *et al*. [21]. Non-stationary self-

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focusing of intense laser beam in plasma using ramp density profile is presented by Habibi and Ghamari. [22]. The periodic self-focusing/defocusing of cosh-Gaussian beam due to relativistic–ponderomotive non-linearity in a ripple density plasma is observed by Aggarwal *et al*. [23]. Strong self-focusing of ChG laser beam in collisionless magneto plasma of ramped density profile is studied by Nanda and Kant [24]. Strong relativistic self-focusing of X-ray laser beam in warm quantum plasmas with upward density ramp is studied by Habibi and Ghamari. [25]. Enhanced relativistic self-focusing of Hermite-cosh-Gaussian laser beam in plasma under density transition is studied by Nanda and Kant [26]. Propagation characteristics of Hermite-cosh-Gaussian laser beam in rippled density plasmas is investigated by S. Kaur *et al*. [27]. Self-focusing of an elliptic-Gaussian laser beam in relativistic ponderomotive plasma using a ramp density profile is studied by H. Kumar *et al*. [28]. In all of the examples mentioned, the main focus for the study of laser beam self-focusing has been on the electron density profile and the role of temperature in the study of results is not considered. The effect of plasma temperature on propagation characters of the Gaussian laser beam in cold collisional plasma, considering the plasma temperature variation and ponderomotive nonlinearity is analyzed by Wang *et al*. [29]. Their study is given regardless of electron temperature variation. The constant temperature effect on self-focusing and defocusing of Gaussian laser beam propagation through the plasma in weakly relativistic regime is investigated by Milani *et al*. [30]. Self-focusing and defocusing of the Gaussian laser beam in plasmas under consideration of linear electron temperature ramp as an individual variable is studied by Z. Zhou *et al*. [31]. Their results are presented for uniform electron density without relativistic considerations. Self-focusing and defocusing of cosh Gaussian laser beam in the presence of nonlinearity of ponderomotive force and exponential electron temperature gradient regardless of relativistic effects reported by H. Rezapour *et al*. [32]. The present paper examines the interaction between Gaussian laser beam with underdense plasma in weakly relativistic regime and simultaneous effect of linear temperature gradient and upward exponential density profile on improved self-focusing. Due to the extensive application of self-focusing of the laser beam in plasma in the field of laser-driven fusion and plasma-based accelerators, we have studied the self-focusing of Gaussian laser beam in plasma in relativistic-ponderomotive nonlinearity regime and considering the role of temperature and gradient density in changing the behavior of the laser beam. Plasma electron temperature is an effective parameter in the study of microwave propagation, thermal conduction and collisional nonlinearity. [33-34]. The plasma electron temperature indicates a distributing inhomogeneous inside the plasma and varies complexly for different plasma and indicates a distributing inhomogeneous inside the plasma. Considering the importance of plasma electron temperature in laser plasma interaction and the results of the research, which indicates the quasi-linear temperature variations for the plasma electron temperature along the laser beam propagation, in this paper we regard linear temperature variations for the plasma electron temperature. In this work, we study the evolution of laser beam-width parameter during the propagation in under dense plasma for several initial electron temperatures under relativistic poderomotive nonlinearity. We find that, the simultaneous effect of both gradients in relativistic ponderomotive regime reduces the laser spot size, which implies a better penetration of the laser beam inside the plasma.

This paper organized as follows: In section II, using the ponderomotive force and the relativistic factor, the dielectric permittivity for under-dense plasma has been derived, also, we have obtained the nonlinear second-order differential equation of the dimensionless beam-width parameter, $\boldsymbol{f}$, on the distance of propagation, η In Section III the numerical results and discussions are presented. Finally, the conclusion is given in section IV.

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II. BASIC EQUATIONS

We consider the Gaussian laser beam in an under-dense plasma in a weak relativistic regime. The relativistic ponderomotive force exerted on the electrons changes the density of the plasma. This force is expressed by the following equation. [35-36].

$$\vec{F}_{pe} = m_{e0}c^2\nabla(\gamma - 1) \tag{1}$$

m_0 is the electron mass, $\gamma = \left(1 + \alpha EE^*\right)^{\frac{1}{2}}$ is the relativistic factor which arises from the quiver motion of the electron in laser field with $\alpha = \frac{e^2}{m_0^2\omega^2c^2}$ .By ignoring the space-charg force, the momentum transfer equation in the stationary steady is presented as:

$$-m_{e0}n_ec^2\nabla(\gamma - 1) = T_e\nabla n_e \tag{2}$$

T_e, n_e are electron temperature and electron density, respectively, the answer of Eq. (2) can be expressed in the following form

$$n_e = n_{e0}\exp\left[\frac{-m_{e0}c^2}{T_e}(\gamma - 1)\right] \tag{3}$$

To solve the equation, we assume that the amplitude of electrical field and electron density of the plasma are only a function of z. n_{e0} is the maximum electron density corresponding to the point at which the laser electric field is zero, in which the electric field of the laser is zero. We assume that the plasma is cold, that is, the average kinetic energy of electrons is much greater than their thermal energy [37], hence the dielectric permittivity is defined as follows:

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\gamma\omega^2} \tag{4}$$

Using Eq. (3) in Eq. (4), the dielectric permittivity for a cold plasma presents as:

$$\varepsilon = 1 - \frac{\omega_{pe0}^2}{\gamma\omega^2}\exp\left[\frac{-m_{e0}c^2}{T_e}(\gamma - 1)\right] \tag{5}$$

$\omega_{pe0} = \left(\frac{4\pi n_{e0}e^2}{m_{e0}}\right)^{\frac{1}{2}}$ is the plasma frequency. The wave equation governing the electric field E of the Gaussian laser beam in the direction of the z axis by considering the effective dielectric constant is given as follows:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{\partial}{r\partial r} + \frac{\partial^2}{\partial z^2}\right)E(r,z,t) + \frac{\omega^2}{c^2}\varepsilon(r,z)E = 0 \tag{6}$$

where ω, c, $\varepsilon(r,z)$ are incident wave frequency, speed of light in vacuum and effective dielectric constant of plasma, respectively. Using WKB approximation the solution of this equation has the form [38-39].

$$E(r,z) = A(r,z)\exp(i\omega t - i\int_0^z k(z)dz) \tag{7}$$

$k(z) = \frac{w}{c}(\varepsilon_0)^{\frac{1}{2}}$, where ε_o is the linear part of the dielectric constant. The complex amplitude $A(r,z)$ can be written as:

$$A(r,z) = A_0(r,z)\exp[-iK(z)S(r,z)] \tag{8}$$

eikonal function $S(r,z)$ in the evolution equation of laser beam extends as follows:

$$S(r,z) = S_0(z) + \frac{r^2}{2f(z)}\frac{df}{dz} \tag{9}$$

where $S_0(z)$ is the axial phase. Suppose that the laser beam has Gaussian profile

$$EE^* = A_0^2(r,z) = \frac{A_{00}^2}{f^2}\exp\left(-\frac{r^2}{f^2r_0^2}\right) \tag{10}$$

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In the paraxial approximation, the dielectric constant of the non-absorbing plasmas can be separated as a series of r^2 [40].

$$\varepsilon(r,z)=\varepsilon_0-\varepsilon_2(z)r^2 \tag{11}$$

Substituting the expression for $E(r,z)$ from Eqs. (7,8) and dielectric constant from Eq. (11) into the wave equation, Eq. (6), assuming the field to be slowly varying along the propagation direction, neglecting the term $\frac{\partial^2 A}{\partial z^2}$ in the resultant equation and separating real and imaginary parts of the resulting equation, the real part can be explained as follows:

$$\begin{aligned}&2\left(\frac{\partial S}{\partial z}\right)+\left(\frac{\partial S}{\partial r}\right)^2+\frac{2S}{K(z)}\frac{dK(z)}{dz}\\&-\frac{1}{K^2A_0}\left(\frac{\partial^2 A_0}{\partial r^2}+\frac{1}{r}\frac{\partial A_0}{\partial r}\right)+r^2\frac{\varepsilon_2(z)}{\varepsilon_0(z)}=0\end{aligned} \tag{12}$$

Substituting Eq. (9) and (10) into Eq. (12), equating the coefficients of r^2 on both sides of the final result, the second order differential equation governing beam-width parameter, $f(z)$, achieved as:

$$\varepsilon_0\frac{d^2f}{dz^2}=\frac{1}{2}\frac{d\varepsilon_0}{dz}\frac{df}{dz}-\frac{c^2}{\omega^2r_0^4f^3}+f\varepsilon_2 \tag{13}$$

where ε_0 and $\varepsilon_2(z)$ are represented as follows:

$$\begin{aligned}\varepsilon_0&=1-\frac{\omega_{pe0}^2}{\omega^2}\left(1+\frac{\alpha A_{00}^2}{f^2}\right)^{\frac{-1}{2}}\times\\&\exp\left[-\frac{m_0c^2}{T_e}\left(1+\frac{\alpha A_{00}^2}{f^2}\right)^{\frac{1}{2}}-1\right]\\\varepsilon_2&=-\frac{\omega_{pe0}^2}{\omega^2}\left\{\frac{\alpha A_{00}^2}{r_0^2f^4}\left(1+\frac{\alpha A_{00}^2}{f^2}\right)^{\frac{-3}{2}}\times\right.\\&\left.\times\exp\left[-\frac{m_0c^2}{T_e}\left(1+\frac{\alpha A_{00}^2}{f^2}\right)^{\frac{1}{2}}-1\right]\times\right.\\&\left.\times\left[1+\frac{m_0c^2}{T_e}\left(1+\frac{\alpha A_{00}^2}{f^2}\right)^{\frac{1}{2}}\right]\right\}\end{aligned} \tag{a-14, b-14}$$

Introducing the dimensionless variable $\eta=\frac{z}{R_d}$ where R_d is known as the Rayleigh length, Eq. (13) reduces to:

$$\frac{d^2f}{d\eta^2}+\frac{1}{2}\frac{d\varepsilon_0}{d\eta}\frac{df}{d\eta}-\frac{1}{f^3}+R_d^2f\frac{\varepsilon_2}{\varepsilon_0}=0 \tag{15}$$

The first term on the right-hand side of Eq. (14) is due to the diffraction effect, the second term is due to the plasma inhomogeneities, and the third term is the nonlinear term that is responsible for self-focusing due to relativistic ponderomotive nonlinearity. It is assumed that the boundary conditions are

$$\frac{df}{d\eta}=0,\ f=1 \quad at \quad \eta=0 \tag{16}$$

III. NUMERICAL RESULTS AND DISCUSSION

Eq. (15) is a nonlinear second-order nonlinear differential equation, called as characteristic Gaussian laser beam propagation equation. This equation governs the behavior of beam-width parameter f as a function of propagation distance η. Using fourth order Rung-Kutta method, we obtain the numerical solution of Eq. (15). To this end, defining $y_1=f, y_2=\frac{df}{d\eta}$, we first transform Eq. (10) to the following equivalent first order system of differential equation $Y'=F(\eta,Y)$, where $Y=[y_1,y_2]^T$ is the vector of exact solution. Under given initial conditions and in the second we pick a step size $h > 0$ and define

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$$\begin{cases} \eta_{n+1} = \eta_n + h, \\ K_1 = F(\eta_n, Y_n), \\ K_2 = F\left(\eta_n + \frac{h}{2}, Y_n + \frac{h}{2}k_1\right), \\ K_3 = F\left(\eta_n + \frac{h}{2}, Y_n + \frac{h}{2}k_2\right), \\ K_4 = F\left(\eta_n + \frac{h}{2}, Y_n + hk_3\right), \\ Y_{n+1} = Y_n + \frac{h}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right), \end{cases}$$

with $Y_n = [y_1(\eta_n, y_2(\eta_n))]^T$. All the numerical computations have been done using Matlab for an initially plane wave front $\left(\frac{df}{d\eta} = 0, f = 1 \quad at \quad \eta = 0\right)$. Numerical calculations of relativistic ponderomotive nonlinearity are done for the following set of typical parameters: $\alpha A_{00}^2 = 0.8eV, r_0 = 20\mu m$

$n_0 = 2.4\times10^{18} cm^{-3}, \omega = 6.28\times10^{14} s^{-1}$ [12,30]. First, we consider isothermal plasma without any density ramp ($T_e = T_{e0}, n_e = n_{e0}$). In Figs. (1,2), the effect of plasma electron temperature in determining laser beam propagation characteristic is given for six different initial electron temperatures. Fig. (1) describes the variation of Gaussian beam-width parameter f with the normalized propagation distance η for different initial electron temperatures as $T_{e01} = 5keV, T_{e02} = 10keV, T_{e03} = 20keV$, which named oscillatory divergence region. Increasing electron temperature leads to decrease in oscillatory amplitude and an increase in oscillatory frequency. Temperature variations from low to high, cause changes in laser beam behavior, so that, if the temperature is higher than $37keV$, the laser beam starts to self-focusing.

Figure (2) illustrates the variation of dimensionless beam-width parameter f with normalized propagation distance η for different values of electron temperatures as: $T_{e04} = 50keV$, $T_{e05} = 100keV$, $T_{e06} = 175keV$, which called self-focusing region.

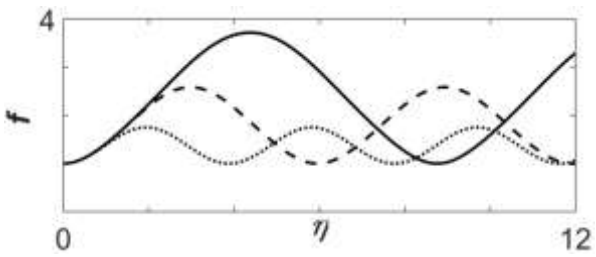

Fig. **1.** Variation of beam width parameter, *f*, respect to propagation distance, *η*, at three different electron temperatures in oscillatory divergence region: $T_{e01} = 5keV$ (solid curve), $T_{e02} = 10keV$ (dashed curve), $T_{e03} = 20k$ (dotted curve) for $\alpha A_{00}^2 = 0.8eV$, $n_0 = 2.4\times10^{18} cm^{-3}$, $r_0 = 20\mu m$, $\omega = 6.28\times10^{14} s^{-1}$

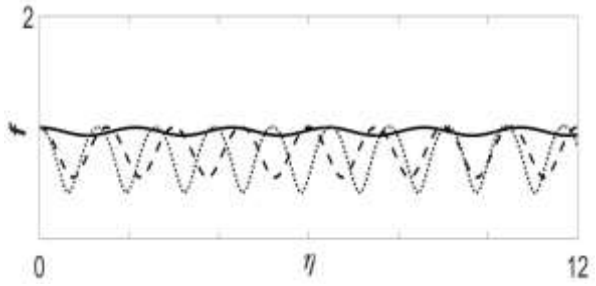

Fig. **2**. Variation of beam width parameter, *f*, respect to propagation distance, *η*, at three different electron temperatures in self-focusing region: $T_{e04} = 50keV$ (solid curve), $T_{e05} = 100keV$ (dashed curve), $T_{e06} = 175keV$ (dotted curve) for $r_0 = 20\mu m$, $\alpha A_{00}^2 = 0.8eV$, $n_0 = 2.4\times10^{18} cm^{-3}$, $\omega = 6.28\times10^{14} s^{-1}$

It is apparent that sharp self-focusing is observed at $T_{e06} = 175keV$. For an increasing value of the electron temperature, the oscillatory amplitude increase and earlier and stronger self-focusing occurs. This is due to the dominance relativistic ponderomotive self-focusing over the defocusing caused by diffraction. If the electron temperature increases again, we will witness the overcome of spatial diffraction at large distance of propagation η and consequently restart the laser beam divergence. As is shown in Figs. (1,2) Plasma electron temperature plays a key role in determining Gaussian laser beam

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behavior .The effect increasing linear gradient electron temperature and exponential density gradient on Gaussian laser beam propagation regime have reported in Figs. (3-8). we consider the linear temperature gradient along the propagation as follows: [31].

$$T_e = T_{e0}(1+\sigma\eta) \tag{17}$$

T_{e0}, and σ are initial electron temperature and electron temperature gradient variation, respectively. Here we assume $\sigma = 0.1$. For inhomogeneous plasma, the charge density is not uniform in space. In axial inhomogeneous plasma, the electron density profile changes along axial direction then the electron density profile presented as [41].

$$n_e(\eta) = n_{e0} D(\eta) \tag{18}$$

where n_{e0} is the plasma density at $z = 0$ and the density ramp function $D(\eta)$ is defined as $D(\eta) = \exp(\mu\eta)$ while μ is a constant parameter which determines the slop and adjustable and in this paper, we consider $\mu = 0.1$. Variation of beam-width parameter f on the dimensionless distance of propagation η in case of relativistic ponderomotive nonlinearity correspond to without any electron density and electron temperature gradient (solid curve), only with electron density gradient (dotted curve), only electron temperature gradient (dashed curve) and both electron density and electron temperature gradient (dashed-dotted curve) has shown in Figs. (3-8). From Figs. (3-5) for oscillatory divergence regions, the simultaneous effect of the temperature and density gradient in comparison with the uniform plasma distribution and isothermal plasma, leads to the avoidance of further divergence of laser beam, also the oscillatory amplitude decreases and oscillatory frequency increases. As is shown in Fig. (3) when electron temperature is $T_{e01} = 5keV$ after some distance of propagation (more than $\eta = 6$) the beam-width parameter decreases under the initial value

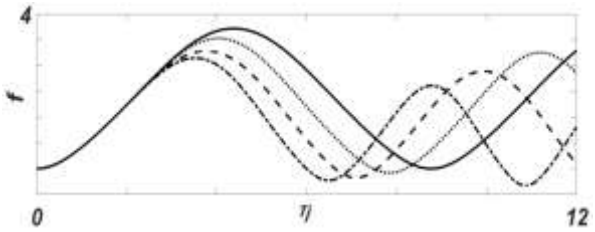

Fig. **3**. Variation of beam width parameter, f, respect to propagation distance, η, at $T_{e01} = 5keV$ for $\alpha A_{00}^2 = 0.8eV$, $n_0 = 2.4\times10^{18}cm^{-3}$, $r_0 = 20\mu m$, and $\omega = 6.28\times10^{14}s^{-1}$.

Although the effect of both gradients of temperature and density avoids the greater divergence of the laser beam, the temperature gradient's effect is clearer than the density gradient. The simultaneous effect of both gradients leads to a maximum reduction in the laser beam divergence.

Figure 4 presents the propagation mode of Gaussian laser beam with the initial electron temperature as $T_{e02} = 10keV$. The dependence of Gaussian laser beam propagation behavior on the electron temperature ramp is similar to that of Fig. 3, with the difference that the reduction of the laser beam width parameter below the initial value occurs in the range of $4 < \eta < 6$. Along with decreasing the laser beam divergence due to the effect of the gradient of density and temperature, the oscillation amplitude decreases and the oscillation frequency increases.

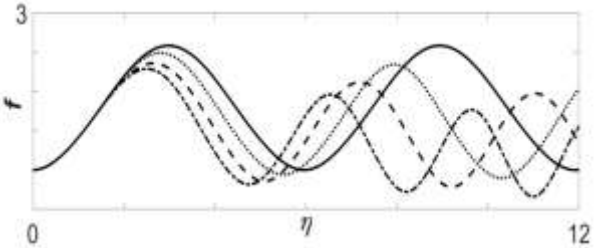

Fig. **4.** Variation of beam width parameter, f, respect to propagation distance, η, at $T_{e02} = 10keV$ for $\alpha A_{00}^2 = 0.8eV$, $r_0 = 20\mu m$, $n_0 = 2.4\times10^{18}cm^{-3}$, $\omega = 6.28\times10^{14}s^{-1}$.

Figure 5 reports the variation of dimensionless beam width-parameter, f, with respect to

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dimensionless propagation distance, η, when plasma temperature is $T_{e03} = 20kev$, which belongs to oscillatory divergence region. Under the influence of temperature gradient with $\sigma = 0.1$ and density gradient with $\mu = 0.1$, divergence of the laser beam is prevented, It reduces the laser beam width parameter below the initial value in the range of $\eta < 4$. The electron temperature and plasma electron density variations along the laser beam propagation direction, play a significant role in propagation mods. Applying temperature gradient and density gradient leads to a decrease in amplitude and an increase in oscillation frequency.

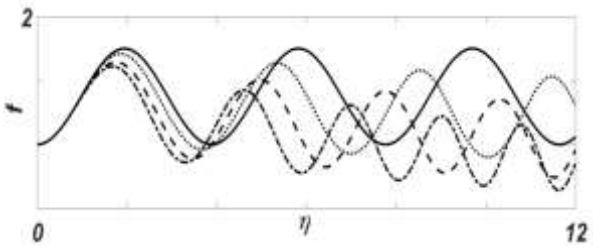

Fig. **5.** Variation of beam width parameter, *f*, respect to propagation distance, η, at $T_{e03} = 20kev$ for $\alpha A_{00}^2 = 0.8eV$, $r_0 = 20\mu m$, $n_0 = 2.4\times10^{18} cm^{-3}$, $\omega = 6.28\times10^{14} s^{-1}$.

Figure 6 denotes comparison for the Gaussian laser beam self-focusing under exponential electron density and linear electron temperature variation for initial electron temperature as $T_{e04} = 50keV$. This figure shows that the combined effect of electron temperature ramp and plasma density gradient enhances the laser focusing. In the absence of temperature gradient (where there is only a density gradient effect) the beam width parameter decrease with distance of propagation to $f = 0.8562$ at $\eta = 12$. Conversely, if we only consider the effect of the temperature gradient in the absence of a density gradient, the laser beam width parameter decreases to a value of $f = 0.0.7496$ at $\eta = 12$. .In conclusion, the combined role of the exponential plasma density ramp and the linear electron temperature makes it possible to achieve the minimum value for the laser beam width parameter, $f_{\min} = 0.5925$, which implies a reduction of about 41% initial value, which implies the overcoming of self-focusing term on diffraction, as much as possible.

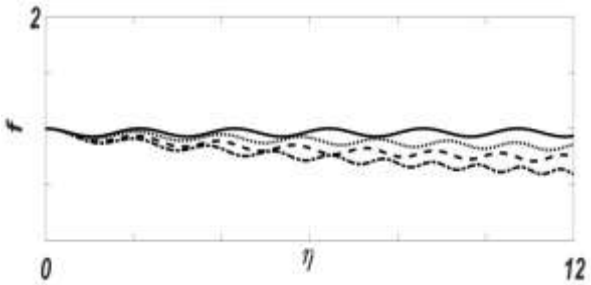

Fig. **6.** Variation of beam width parameter, *f*, respect to propagation distance, η, at $T_{e04} = 50keV$ for $\alpha A_{00}^2 = 0.8eV, r_0 = 20\mu m$, $n_0 = 2.4\times10^{18} cm^{-3}, \omega = 6.28\times10^{14} s^{-1}$.

From Fig. 7, we observe the variation of beam-width parameter, f again normalized propagation distance, η for electron temperature $T_{e05} = 100keV$. It is clear that for $\eta < 6$ the effect of electron density gradient on the self-focusing is more effective than the electron temperature gradient and vice versa at $6 < \eta < 12$ the temperature gradient is dominant influence. The simultaneous effect of the gradient of temperature and density shrink the laser spot size as the laser beam penetrate into the plasma, so that the spot size of the laser beam is reduced to 35% of its initial value.

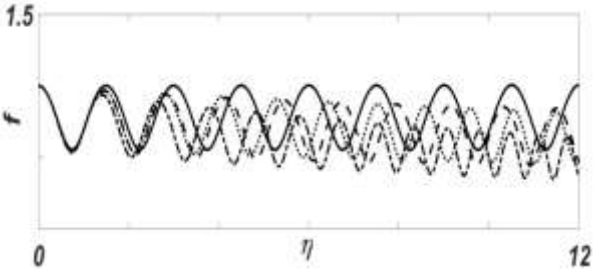

Fig. **7**. Variation of beam width parameter, *f*, respect to propagation distance, η, at $T_{e05} = 100keV$ for $\alpha A_{00}^2 = 0.8eV$, $r_0 = 20\mu m$, $n_0 = 2.4\times10^{18} cm^{-3}, \omega = 6.28\times10^{14} s^{-1}$

Figure 8 depicts the influence of linear electron temperature gradient and upward exponential

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ramp density profile on the self-focusing of the Gaussian laser beam in relativistic ponderomotive regime for electron temperature $T_{e06} = 175keV$. In the total distance of the laser beam propagation, the role of the electron density gradient in self-focusing improvement is undeniable. In contrast to the previous Figure, the effect of the density gradient on the temperature gradient prevails. It is observed that the simultaneous effect of both gradient leads to significant decrease in spot size laser beam. The minimum value obtained for the width parameter in such a situation is $f_{\min} = 0.2617$, which implies 74% reduction in initial spot size.

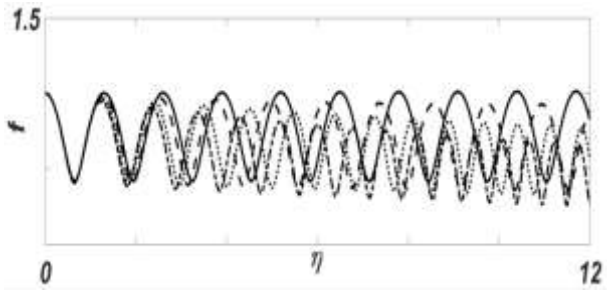

Fig. **8.** Variation of beam width parameter, *f*, respect to propagation distance, *η*, at $T_{e06} = 175keV$ for $\alpha A_{00}^2 = 0.8eV$, $r_0 = 20\mu m$, $n_0 = 2.4\times10^{18} cm^{-3}$, $\omega = 6.28\times10^{14} s^{-1}$.

Table **1.** Minimum beam-width parameter variation in self-focusing region.

Temperature (keV) / Method	**50**	**100**	**175**
No any gradient	0.9280	0.5604	0.4188
Only temperature gradient	0.7069	0.4517	0.3583
Only density gradient	0.8081	0.4707	0.3327
Both temperature and density gradient	0.5925	0.3554	0.2617

In Table 1, the effect of plasma electron density gradient and plasma temperature gradient on the reduction of the beam-width parameter for self-focusing region has been reported.

IV. CONCLUSION

In this paper, propagation of a Gaussian laser beam in plasma is analyzed. The impact of initial electron temperature on laser beam propagation regimes is studied. The dielectric permittivity for under-dense plasma is obtained. The Relativistic ponderomotive force is considered as the effective nonlinearity mechanism. The equation of beam-width parameter and computational figures are presented by numerical solving of nonlinear second-order differential equation. The key role of electron temperature in laser beam propagation characters is discussed. We observed that each temperature corresponds with distinguished propagation regime. The influence of linear temperature gradient and upward exponential density profile on laser beam propagation is investigated. The regarded temperature gradient dissociated the stationery oscillation modes. The results show that combined effect of plasma density ramp and temperature gradient enhances the self-focusing property of laser beam. Appropriate choice of both factors not only reduce the radius of laser beam efficiently but also maintain it longer propagation distances. The results indicate that starting self-focusing is at $37keV$ and stronger self-focusing occurs at $175keV$, so that, the least amount of laser beam-width parameter is observed at $T_{e06} = 175keV$. Using this design for study of self-focusing of Gaussian laser beam in plasma with combined role of an exponential plasma density ramp and linear temperature gradient cause to the laser beam not only focused inside the plasma, but it can extend over a long distance without divergence. The presented results analysis about could be applicable in the application plasma-based accelerators and laser-driven fusion.

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