Modelling Dispersion Characteristics of Circular Optical Waveguide with Helical Winding–Comparison for Different Pitch Angles

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Abstract— In this Article dispersion characteristic of conventional optical waveguide with helical winding at core – cladding interface has been obtained. The model dispersion characteristics of optical waveguide with helical winding at core-cladding interface have been obtained for five different pitch angles. This paper includes dispersion characteristics of optical waveguide with helical winding, and compression of dispersion characteristics of optical waveguide with helical winding at core-cladding interface for five different pitch angles. Boundary conditions have been used to obtain the dispersion characteristics and these conditions have been utilized to get the model Eigen values equation. From these Eigen value equations dispersion curve are obtained and plotted for modified optical waveguide for particular values of the pitch angle of the winding and the result has been compared.

KEYWORDS: Optical fiber communication, fiber dispersion, helical winding, helix pitch angle, modal cutoff.

I. INTRODUCTION

Optical fibers with helical winding are known as complex optical waveguides. The use of helical winding in optical fibers makes the analysis much accurate [1]. Singh [13] have proposed an analytical study of dispersion characteristics of helically cladded step – index optical fiber with elliptical core. The sheath helix [12] is a cylindrical surface with high conductivity in a preferential direction which winds helically at constant angle around the core–cladding boundary surfaces. We assume that the waveguide have real constant refractive index of core and cladding is n_1 and n_2 respectively ($n_1 > n_2$).

II. THEORY AND CALCULATIONS

The optical waveguide is the fundamental element that interconnects the various devices of an optical integrated circuit, just as a metallic strip does in an electrical integrated circuit. The mode theory [10] is used to describe the properties of light that ray theory is unable to explain. A set of guided electromagnetic waves is called the modes [13, 16] of the fiber. For a given mode, a change in wavelength can prevent the mode from propagating along the fiber. The mode is no longer bound to the fiber. The mode is said to be cut off [13]. The wavelength at which a mode ceases to be bound is called the cutoff wavelength [11] for that mode. we consider the following boundary conditions [8].

$$E_{z1} \sin\psi + E_{\phi 1} \cos\psi = 0 \tag{1}$$

$$E_{z2} \sin\psi + E_{\phi 2} \cos\psi = 0 \tag{2}$$

$$\left(E_{z1} - E_{z2}\right)\cos\psi - \left(E_{\phi 1} - E_{\phi 2}\right)\sin\psi = 0 \tag{3}$$

$$\left(H_{z1} - H_{z2}\right)\sin\psi + \left(H_{\phi 1} - H_{\phi 2}\right)\cos\psi = 0 \tag{4}$$

The expressions for E_z and H_z inside the core are, when (r<a)

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$$E_{z1} = AJ_1(Ua)F(\phi)e^{j(\omega t-\beta z)} \quad (5)$$

$$H_{z1} = BJ_1(Ua)F(\phi)e^{j(\omega t-\beta z)} \quad (6)$$

The expressions for E_z and H_z outside the core are, when (r>a),

$$E_{z2} = CK_1(Wa)F(\phi)e^{j(\omega t-\beta z)} \quad (7)$$

$$H_{z2} = DK_1(Wa)F(\phi)e^{j(\omega t-\beta z)} \quad (8)$$

where,

$$\mathrm{U}^2 = k_1^2 - \beta^2 = \omega^2\mu\varepsilon_1 - \beta^2 \quad (9)$$

$$\mathrm{W}^2 = \beta^2 - k_2^2 = \beta^2 - \omega^2\mu\varepsilon_2 \quad (10)$$

The transverse field components can be obtained by using Maxwell's standard relations. So the electric and magnetic field components E_φ and H_φ can be written as,

Transverse components of the electric and magnetic fields $E_{\varphi 1}$ and $H_{\varphi 1}$ for core region can be written as:

$$E_{\phi 1} = -\left(j/U^2\right)F(\phi)e^{j(\omega t-\beta z)} \left[j(\beta/a)AJ_1(Ua) - \mu\omega UBJ_1'(Ua)\right] \quad (11)$$

$$H_{\phi 1} = -\left(j/U^2\right)F(\phi)e^{j(\omega t-\beta z)} \left[j(\beta/a)BJ_1(Ua) + \omega\varepsilon_1 UAJ_1'(Ua)\right] \quad (12)$$

The axial field components of the electric field and magnetic field $E_{\varphi 2}$ and $H_{\varphi 2}$ for clad region can be can written as:

$$E_{\phi 2} = -\left(j/W^2\right)F(\phi)e^{j(\omega t-\beta z)} \left[j(\beta/a)CK_1(Wa) - \mu\omega WDK_1'(Wa)\right] \quad (13)$$

$$H_{\phi 2} = -\left(j/U^2\right)F(\phi)e^{j(\omega t-\beta z)} \left[j(\beta/a)DK_1(Wa) + \omega\varepsilon_2 WCK_1'(Wa)\right] \quad (14)$$

We get four equations which involves four unknown constants A, B, C and D.

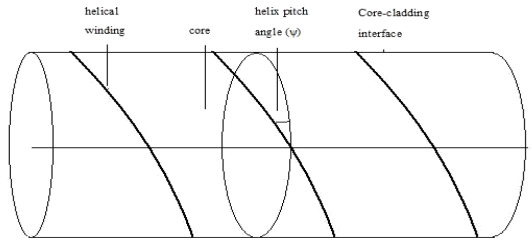

Fig. 1 Fiber with conducting helical winding at core cladding interface

$$AJ_\nu(ua)\left[\sin\psi + \frac{\beta\nu}{u^2a}\cos\psi\right] + BJ_\nu'(ua)\left[\frac{j\mu\omega}{u}\right]\cos\psi = 0 \quad (15)$$

$$CK_\nu(wa)\left[\sin\psi + \frac{\beta\nu}{w^2a}\cos\psi\right] + DK_\nu'(wa)\left[\frac{j\mu\omega}{w}\right]\cos\psi = 0 \quad (16)$$

$$AJ_\nu(ua)\left[\cos\psi - \frac{\beta\nu}{u^2a}\sin\psi\right] - BJ_\nu'(ua)\left[\frac{j\mu\omega}{u}\right]\sin\psi - CK_\nu(wa)\left[\cos\psi - \frac{\beta\nu}{w^2a}\sin\psi\right] + DK_\nu'(wa)\left[\frac{j\mu\omega}{w}\right]\sin\psi = 0 \quad (17)$$

$$-AJ_\nu'(ua)\left[\frac{j\omega\varepsilon_1}{u}\right]\cos\psi + BJ_\nu(ua)\left[\sin\psi + \frac{\beta\nu}{u^2a}\cos\psi\right] + CK_\nu'(wa)\left[\frac{j\omega\varepsilon_2}{w}\right]\cos\psi - DK_\nu(wa)\left[\sin\psi + \frac{\beta\nu}{w^2a}\cos\psi\right] = 0 \quad (18)$$

Eq. (15), Eq. (16), Eq. (17) and Eq. (18) will yield a non–trivial solution if the determinant whose elements are the coefficient of these unknown constants is set equal to zero. Thus we have:

$$\Delta = \begin{vmatrix} A1 & A2 & A3 & A4 \\ B1 & B2 & B3 & B4 \\ C1 & C2 & C3 & C4 \\ D1 & D2 & D3 & D4 \end{vmatrix} \quad (19)$$

where,

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$$A1 = J_\nu(ua)\left[\sin\psi + \frac{\beta\nu}{u^2 a}\cos\psi\right]$$
$$A2 = J_\nu{}'(ua)\left[\frac{j\mu\omega}{u}\right]\cos\psi \tag{20}$$

$$A3 = A4 = B1 = B2 = 0$$

$$B3 = K_\nu(wa)\left[\sin\psi + \frac{\beta\nu}{w^2 a}\cos\psi\right]$$
$$B4 = K_\nu{}'(wa)\left[\frac{j\mu\omega}{w}\right]\cos\psi \tag{21}$$

$$C1 = J_\nu(ua)\left[\cos\psi - \frac{\beta\nu}{u^2 a}\sin\psi\right]$$
$$C2 = -J_\nu{}'(ua)\left[\frac{j\mu\omega}{u}\right]\sin\psi \tag{22}$$
$$C3 = -K_\nu(wa)\left[\cos\psi - \frac{\beta\nu}{w^2 a}\sin\psi\right]$$
$$C4 = K_\nu{}'(wa)\left[\frac{j\mu\omega}{w}\right]\sin\psi$$

$$D1 = -J_\nu{}'(ua)\left[\frac{j\omega\varepsilon_1}{u}\right]\cos\psi$$
$$D2 = J_\nu(ua)\left[\sin\psi + \frac{\beta\nu}{u^2 a}\cos\psi\right] \tag{23}$$
$$D3 = K_\nu{}'(wa)\left[\frac{j\omega\varepsilon_2}{w}\right]\cos\psi$$
$$D4 = -K_\nu(wa)\left[\sin\psi + \frac{\beta\nu}{w^2 a}\cos\psi\right]$$

After simplifying the determinant, we get a simplified equation for lowest order modes.

$$U\frac{J_1(Ua)}{J_1'(Ua)}\left(\sin\psi + \frac{\beta}{U^2 a}\cos\psi\right)^2 - \frac{k_1^2}{U}\frac{J_1'(Ua)}{J_1(Ua)}\cos^2\psi$$
$$-W\frac{K_1(Wa)}{K_1'(Wa)}\left(\sin\psi + \frac{\beta}{W^2 a}\cos\psi\right)^2 + \frac{k_2^2}{W}\frac{K_1'(Wa)}{K_1(Wa)}\cos^2\psi = 0 \tag{24}$$

III. Results

It is now possible to interpret the characteristic equation (Eq. 24) in numerical terms. This will give us an insight into model properties of our waveguide. For this we can use following relations,

$$U\frac{J_1(Ua)}{J_1'(Ua)}\left(\sin\psi + \frac{\beta}{U^2 a}\cos\psi\right)^2$$
$$-\frac{k_1^2}{U}\frac{J_1'(Ua)}{J_1(Ua)}\cos^2\psi$$
$$-W\frac{K_1(Wa)}{K_1'(Wa)}\left(\sin\psi + \frac{\beta}{W^2 a}\cos\psi\right)^2 \tag{25}$$
$$+\frac{k_2^2}{W}\frac{K_1'(Wa)}{K_1(Wa)}\cos^2\psi = 0$$

$$b = \left(\frac{aw}{V}\right)^2 = \left(\frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}\right) \tag{26}$$

$$V^2 = (u^2 + w^2)a^2 = \left(\frac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2) \tag{27}$$

where b and V are known as normalization propagation constant and normalized frequency parameter respectively. We make some simple calculations based. We choose n_1=1.50, n_2=1.46 and λ=1.55μm.

A. *Dispersion Curve*

1. Dispersion Curve for Pitch Angle ψ=0^0

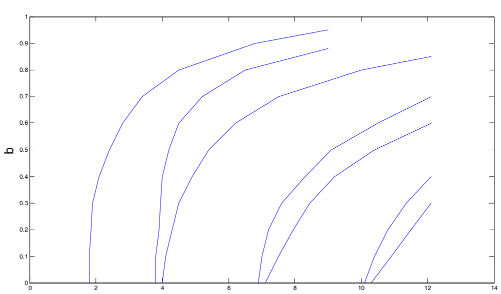

Fig. 2 Dispersion curve for pitch angle ψ=0^0

2. Dispersion Curve for Pitch angle ψ=30^0

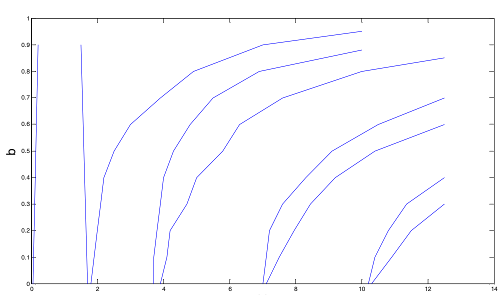

Fig. 3 Dispersion curve for pitch angle ψ=30^0

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3. Dispersion Curve for Pitch Angle $\psi=45^0$

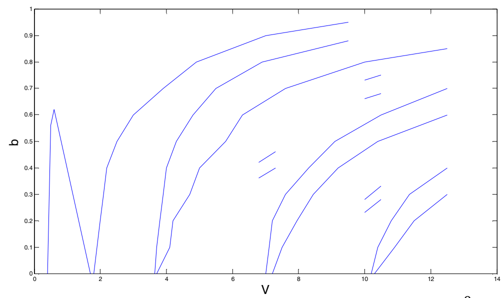

Fig. 4 Dispersion curve for pitch angle $\psi=45^0$

4. Dispersion Curve for Pitch Angle $\psi=60^0$

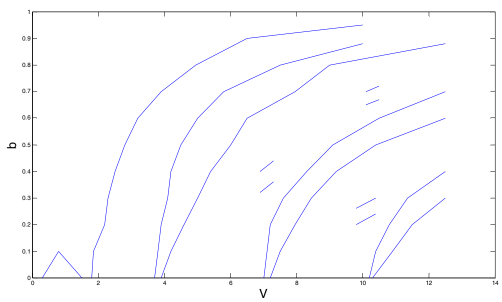

Fig. 5 Dispersion curve for pitch angle $\psi=60^0$

5. Dispersion Curve for Pitch Angle $\psi=90^0$

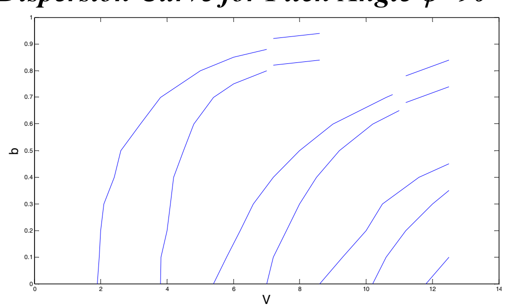

Fig. 6 Dispersion curve for pitch angle $\psi=90^0$

We observed that, they all have standard expected shape. This effect is undesirable for the possible use of. This means that one effect of conducting helical winding is to these waveguide for long distance communication in pairs that is cutoff values for two adjacent modes converge.

We found that some curves have band gaps of discontinuities between some value of V. These represent the band gaps or forbidden bands of the structure. These are induced by the periodicity of the helical windings.

B. *Dependence of Cutoff Values on Pitch Angle*

From Table I, we note particularly that the dependence of the cutoff V–value (V_c) on ψ is such that as ψ is increased, there is a drastic fall in V_c at $\psi=30^0$, and then a small increase as ψ goes from 30^0 to 60^0; then there is a quick rise as ψ changes from 60^0 to 90^0 (Fig. 7).

Thus the two most sensitive regions in respect of the influence of helical pitch angle ψ on the cutoff values and the model properties of waveguides are ranges from $\psi=0^0$ to $\psi=30^0$ and $\psi=60^0$ to $\psi=90^0$ and these ranges of pitch angle expected to have potential applications with ψ as a means for controlling the model properties split the modes and remove a degeneracy which is hidden in conventional waveguide without windings.

TABLE I CUTOFF V_C VALUES FOR SOME LOWER ORDER MODES

Pitch Angle Ψ (^{0})	**0**	**30**	**45**	**60**	**90**
Cutoff Normalized Frequency V_c	1.80	0.05	0.40	0.30	1.90
	3.80	1.70	1.70	1.50	3.80
	4.00	1.80	1.80	1.80	5.40
	6.90	3.70	3.65	3.70	7.00
	7.10	3.90	3.70	3.90	8.60
	10.10	7.00	7.00	7.00	10.20
	10.30	7.10	7.20	7.20	11.80
	-	10.20	10.20	10.20	-
	-	10.30	10.30	10.30	-

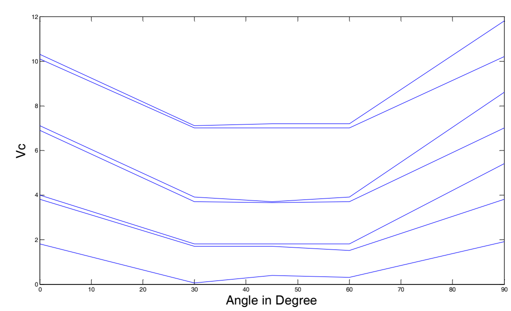

Fig. 7 Dependence of cutoff values V_c on the pitch angle ψ

IV. CONCLUSION

We observed that, they all have standard expected shape, but except for lower order modes they comes in pairs, that is cutoff values for two adjacent mode converge. This

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means that one effect of conducting helical winding is to split the modes and remove a degeneracy which is hidden in conventional waveguide without windings.

From the above discussions we can conclude that the modal cutoff for helical pitch angle $\psi=30^0$, 45^0 and 60^0 are higher than the modal cutoff for helical pitch angle $\psi=0^0$ and 90^0. This means, for some specific range of cutoff values V_c, one can have greater number of modes for helical pitch angle $\psi=30^0$, 45^0 and 60^0 than for helical pitch angle $\psi=0^0$ and 90^0. So the helical pitch angle $\psi=30^0$, 45^0 and 60^0 are better than helical pitch angle $\psi=0^0$ and 90^0.

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