Investigation and Optimization of Sub-Doppler DAVLL Error Signal for ECDL Stabilization

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Abstract— Sub-Doppler dichroic atomic vapor laser lock (DAVLL) is a modulation-free laser stabilization method that combines DAVLL and saturated absorption spectroscopy (SAS). The performance of this highly sensitive stabilization technique strongly depends on the characteristics of the generated error signal. The slope of the error signal determines the lock sensitivity or how fast the frequency compensation could be made in the feedback loop, and the amplitude of the error signal determines the lock stability or how much noise the feedback loop can tolerate before laser unlocking. We have analytically modeled the error signal of the sub-Doppler DAVLL considering all possible transitions between Zeeman sublevels and compared it with the experimental results. Using the analytical and experimental results, it is shown that the values of the required magnetic fields for maximizing the slope and amplitude of the error signal are close to each other. Selecting the mentioned values of the magnetic field for optimization of the sub-Doppler DAVLL error signal is highly useful for sensitive and stable laser locking.

KEYWORDS: Sub-Doppler DAVLL, laser stabilization, saturated absorption spectroscopy, cesium D1 line.

I. INTRODUCTION

Laser frequency stabilization is essential in different atomic physics experiments. Laser cooling, atom interferometry, atomic clocks, and many other applications require laser locking to an atomic transition. There are different techniques for locking a tunable laser to an atomic transition, such as saturated absorption spectroscopy [1], polarization spectroscopy [2], dichroic atomic vapor laser lock (DAVLL) [3-8], and other methods [9-11]. The importance of using stabilized lasers for atomic applications has led to different works on the optimization of the locking method [12-14]. The sub-Doppler DAVLL is a combination of saturated adsorption spectroscopy and DAVLL. This technique has the benefits of modulation-free of DAVLL and Doppler-free narrow line width of the saturation absorption spectroscopy [15, 16]. In both DAVLL and sub-Doppler DAVLL, the error signal for servo laser stabilization is generated by subtracting the absorption profiles for right and left circularly polarized input lights [3, 17]. The separation of the absorption profiles in the presence of a magnetic field could be simply explained by considering a two-level atomic system [18]. As shown in Fig. 1, for an atom with the ground state $F = 0$, and the excited state $F' = 1$, in the absence of external magnetic field, the Zeeman sublevels $m_{F'} = -1,0,1$ of $F' = 1$ are degenerate. Applying a weak uniform magnetic field B_0 in the direction of the light propagation will split the Zeeman sublevels which change the absorption frequency for the left and right circularly polarized light.

It should be noted that for real atoms (Cesium), a more sophisticated model for sub-Doppler DAVLL is presented in the following. Although the mechanism for generation of the error signal for DAVLL and sub-Doppler DAVLL is the subtraction of σ^+ and σ^-

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absorption profiles, however, sub-Doppler DAVLL the mentioned profiles are related to the saturated absorption spectroscopy. Because of that, in contrast to DAVLL, the sub-Doppler DAVLL can be used for locking on hyperfine transition lines very close to each other [19].

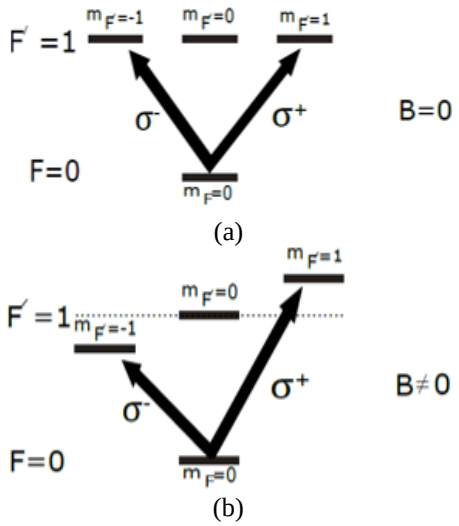

Fig. 1. Level diagram and frequency of σ^+ and σ^- transitions in a two-level atom, (a) in the absence of magnetic field, (b) in the presence of magnetic field.

The sub-Doppler DAVLL has been studied for different alkali atoms [17, 20-23]. The error signal in sub-Doppler DAVLL has a significantly smaller width relative to DAVLL which leads to the much smaller required magnetic field for generating this error signal. The small width of the error signal indicates that more attention should be paid to its characteristics for obtaining fast and stable laser locking. In this work, we have investigated the sub-Doppler DAVLL error signal for efficient locking of the external cavity diode laser (ECDL) on atomic transition. In this regard, a theoretical model is developed and the effect of the applied magnetic field on the characteristics of the error signal is investigated and compared with the experimental results. A very good agreement between the experimental and analytical results is obtained which is used for the comprehensive investigation of the error signal. It is shown that by applying an appropriate magnetic field, the slope and amplitude of the error signal could be simultaneously maximized which results in fast and stable laser locking.

II. Theoretical Model of Sub-Doppler DAVLL

As we have experimentally investigated the sub-Doppler DAVLL for Cesium D1-line, the same atom is used for the description of the analytical model. The Cesium D1 line transition contains hyperfine levels of F=3 and F=4 in the ground state and F'=3 and F'=4 in the excited state. As shown in Fig. 2, considering the transition between F=4 to F'=3, the ground state has nine Zeeman sublevels and the excited state has seven Zeeman sublevels. Furthermore, the Landé g-factor for the two states are different and so the energy splitting of the ground state and excited state in the weak field is approximately given by $\Delta E_g = g_F \mu_B B_0 m_F$ and $\Delta E_e = g_{F'} \mu_B B_0 m_{F'}$, respectively.

Applying circularly polarized light, there are fourteen possible sub-transitions $\sigma^{1+},...,\sigma^{7+}$ and $\sigma^{1-},...,\sigma^{7-}$ which should be all considered for describing the dichroic separation [17, 20-23]. The developed method is based on considering Gaussian profiles for each sub-transition. So, for right and left circularly polarized light, the absorption profile could be written as Eq. 1

$$A_{\sigma^\pm} = A_{\pm1}\exp\left[-\frac{(E-E_{\pm1})^2}{2g^2}\right] + A_{\pm2}\exp\left[-\frac{(E-E_{\pm2})^2}{2g^2}\right] + A_{\pm3}\exp\left[-\frac{(E-E_{\pm3})^2}{2g^2}\right] + A_{\pm4}\exp\left[-\frac{(E-E_{\pm4})^2}{2g^2}\right] + A_{\pm5}\exp\left[-\frac{(E-E_{\pm5})^2}{2g^2}\right] + A_{\pm6}\exp\left[-\frac{(E-E_{\pm6})^2}{2g^2}\right] + A_{\pm7}\exp\left[-\frac{(E-E_{\pm7})^2}{2g^2}\right] \quad (1)$$

where $A_{\pm i}$ is the transition probability rate which is different for each mentioned sub-transition. This transition rate between the ground state (F, M) and excited state (f, m) could be obtained according to Eq. 2 [24]. The parameter q in Eq. 2 determines the light polarization and has the value of -1, 0, and +1 for circularly left polarization, linearly polarization, and right circularly polarization, respectively.

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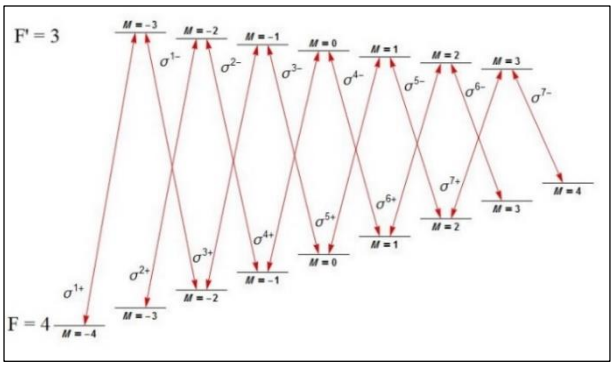

Fig. 2. Zeeman sublevels and possible σ^+ and σ^- transitions for F=4 to F'=3 transition in Cesium D1 line.

$$R_{F,M,f,m} = (2F+1)(2f+1)\begin{pmatrix} f & q & F \\ -m & m-M & M \end{pmatrix}^2 \begin{Bmatrix} j & f & I \\ F & J & 1 \end{Bmatrix}^2 \quad (2)$$

In Eq. 1, $E_{\pm i}$ is the frequency shift related to each transition in the presence of a magnetic field compared to the case of zero magnetic field, which is determined by $m_{F,F'}$ and $g_{F,F'}$. The positive and negative signs are for the right and left light circularly polarizations, respectively. For example, the value of $E_{\pm 1}$ is obtained according to:

$$E_{+1} = 4g_F\mu_B B_0 + 3g_{F'}\mu_B B_0 =$$
$$= (40.35 + 3(-0.12))\mu_B B_0 = 1.04\ \mu_B B_0$$

Also, in Eq. 1, g is proportional to the FWHM of the sub-Doppler peak at the B=0 field and its value depends on the experimental parameters and the corresponding atomic transition characteristics. In general, the value of g is about the natural width of the transition. In our experiment, its value for the transition between F=4 to F'=3 is considered equal to 20MHz (according to Fig. 5). This value is measured by comparing the frequency difference between F=4 to F'=3 and F=4 to F'=4 transitions (1167.68 MHz). Figure 3(a) shows the calculated saturation absorption spectroscopy line shape for the cesium D1 line (F=4 to F'=3) at room temperature. The obtained analytical results of the two separated profiles and their difference (error signal) are shown in Fig. 3(b).

As shown in Fig. 3(b), we have considered the difference between the local extremums of the error signal as its amplitude and the derivative at υ_0, as its slope. For experimental reasons both the amplitude and slope of the error signal should be maximum for efficient laser locking.

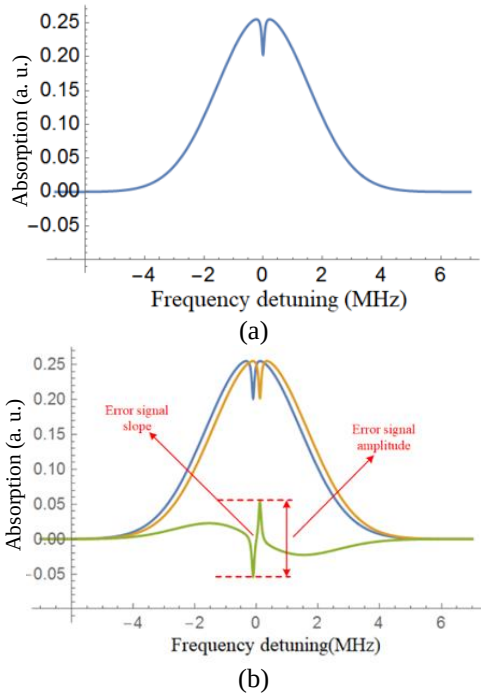

Fig. 3. Calculated sub-Doppler DAVLL error signal for cesium D1 line (F=4 to F'=3) (a) saturation absorption spectroscopy line shape (b) separation of saturated absorption signal by applying magnetic field. The blue and red lines show absorption of the left and right circular polarized light, and the green line shows the error signal (without any gain).

III. Experimental Setup

The schematic of the implemented sub-Doppler DAVLL is shown in Fig. 4. The highlighted section corresponds to the saturation absorption spectroscopy setup added to the DAVLL setup. The details of the experimental setup are explained in [19, 25].

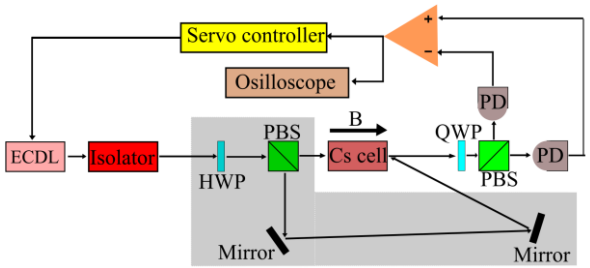

Fig. 4. Schematic of the sub-Doppler DAVLL setup. HWP (half-Waveplate), PBS (polarizing beam splitter), QWP (quarter-Waveplate), PD (photodiode).

Using this setup, an experimental result for absorption profiles of σ^+ and σ^- transitions of

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the Cesium D1 line (F=4 to F'=3) and their difference (sub-Doppler error signal), is shown in Fig. 5. In this experiment result, the applied magnetic field is about 20G.

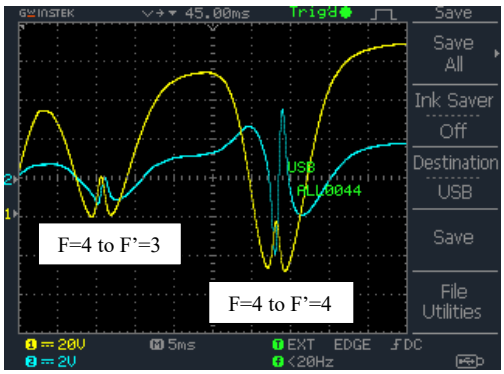

Fig. 5. Experimental results of the σ⁻ absorption lines (yellow traces), and the sub-Doppler error signal (blue trace).

IV. Results and Discussion

In laser locking using the sub-Doppler DAVLL method, the amplitude and slope of the error signal are the main parameters that determine the performance of the locking stability. Greater amplitude of the error signal leads to greater stability of the laser against more intense noises. While the greater slope of the error signal quickly compensates for small wavelength changes and increases the sensitivity of the laser lock. Therefore, using the mentioned analytical model and experimental setup, we have investigated the dependence of the amplitude and the slope of the sub-Doppler error signal on the applied magnetic field. The results of the sub-Doppler error signal for different magnetic fields are shown in Fig. 6. In this figure, the analytical results are shown for the same values of a magnetic field that are used in the experimental tests. The experimental results are obtained with pumping laser power of 0.83 mW, probe beam power 0.018mW and the cell temperature is adjusted at 27 °C.

It can be inferred from Fig. 6 that by increasing the applied magnetic field from low values, at first both the amplitude and slope of the error signal increase, and then by increasing the magnetic field they both decrease. So, it is obvious that the slope and amplitude of the error signal would be maximized for particular values of the magnetic field.

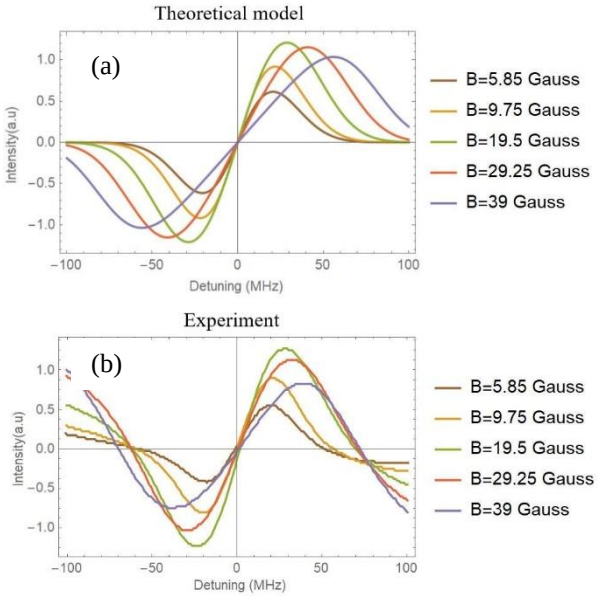

Fig. 6. The sub-Doppler error signal for different magnetic fields, (a) theoretical result, (b) experimental result.

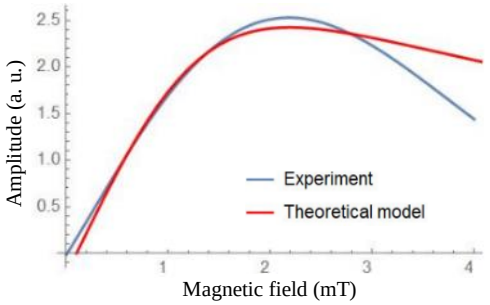

Fig. 7. The theoretical and experimental results for the amplitude of the error signal in terms of applied magnetic field.

For investigation of this issue, Figs. 7 and 8 show the amplitude and slope of the sub-Doppler error signal in terms of the applied magnetic field. These analytical and experimental results which have a good agreement with each other, clearly show that there are optimal magnetic fields to obtain the maximum slope and amplitude of the error signal. Although the required magnetic field for maximizing the slope is different from the field for maximizing the amplitude, these optimum field values are close to each other. In selecting the magnetic field for sub-Doppler DAVLL, we can usually optimize a magnetic field with a mean value between these two optimum values. This is while according to the priority of

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stability or sensitivity of the laser lock loop, a compromise between the mentioned values could be made.

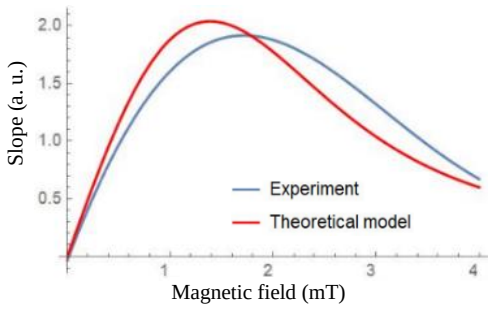

Fig. 8. The theoretical and experimental results for the slope of the error signal in terms of applied magnetic field.

V. CONCLUSION

In this work, we have analytically modeled the error signal of the sub-Doppler DAVLL for the cesium D1 line and compared it with experimental results. The model describes the amplitude and slope of the error signal which are the two important factors that affect the laser locking performance. A very good agreement between analytical and experimental results has been obtained. Investigation of the error signal characteristics versus the applied magnetic field shows that the required magnetic fields for achieving the error signal with maximum amplitude or maximum slope are very close to each other. Using this optimum magnetic field value is considerably important for sensitive and stable sub-Doppler DAVLL laser locking.

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