Quantum Squeezed Light Propagation in an Optical Parity-Time ($\mathcal{PT}$)-Symmetric Structure

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ABSTRACT— We investigate the medium effect of a parity-time ($\mathcal{PT}$)-symmetric bilayer on the quantum optical properties of an incident squeezed light at zero temperature (T=0 K). To do so, we use the canonical quantization approach and describe the amplification and dissipation properties of the constituent layers of the bilayer structure by Lorentz model to analyze the quadrature squeezing of the outgoing state from the bilayer structure. Our results show that despite the apparent compensation of the losses within the bilayer in the symmetry phase, the outgoing light is no longer squeezed. The results also show that the quantum optical effective medium theory correctly predicts the quantum features of the light outgoing from the $\mathcal{PT}$-symmetric bilayer structure.

KEYWORDS: Parity-time ($\mathcal{PT}$)-symmetry, Quadrature squeezing, Electromagnetic field quantization, Quantum optical effective-medium theory.

I. INTRODUCTION

Physical systems exhibiting parity-time ($\mathcal{PT}$) symmetry was first suggested by Bender and Bottcher [1-2]. They have shown that the Hamiltonian hermiticity is not a required condition for its eigenvalues to be real. In other words, it is possible for a non-hermitian Hamiltonian to exhibit real eigenvalues, provided the $\mathcal{PT}$-symmetry conditions are satisfied. A necessary condition for a Hamiltonian with a complex potential to be $\mathcal{PT}$-symmetric is $V(\mathbf{r}) = V^*(-\mathbf{r})$, where $\mathbf{r}$ is the three-dimensional position vector and V is the potential energy with a complex conjugate V^*. Another intriguing property of these systems is the possibility of a phase transition from a real to a complex spectrum because of the $\mathcal{PT}$-symmetry spontaneous breakdown.

An optical analogy of a $\mathcal{PT}$-symmetric potential is a complex refractive index, n, satisfying the condition $n(\mathbf{r}) = n^*(-\mathbf{r})$. Thus, a typical $\mathcal{PT}$-symmetric optical structure can be implemented by coupling a pair of gain and loss slabs characterized by the refractive indices both satisfying $\mathrm{Re}[n(\mathbf{r})] = \mathrm{Re}[n(-\mathbf{r})]$ and $\mathrm{Im}[n(\mathbf{r})] = -\mathrm{Im}[n(-\mathbf{r})]$. This suggests optics as a fertile ground for experimental investigations on $\mathcal{PT}$-symmetric systems. Moreover, $\mathcal{PT}$-symmetry leads to a series of intriguing optical phenomena such as coherent perfect absorbers, anisotropic transmission resonances, and unidirectional invisibility [3-4].

The dielectric slabs can have many applications in classical and quantum optics. Particularly, quantum light transmission through dielectric slabs is one of the important problems in the quantum optics because the quantum light is an important prerequisite for

different tasks of quantum information processing, such as quantum cryptography, quantum computing, and quantum voting. A dielectric slab is macroscopically described by an electric permittivity $\varepsilon(\omega)$ that is a complex function of frequency, whose imaginary part determines whether the slab is a gain/loss medium. According to the dissipation-fluctuation theory, there is a quantum noise due to the dissipative nature of these media. Thus, the absorption in a dielectric medium adds noise to a beam of light, having detrimental effects on the nonclassical features of the quantum light. Given the fact that the effect of loss in a $\mathcal{PT}$-symmetric structure can be compensated at some exceptional points, it would be of great interest to study the effects of propagation across these optical structures on the nonclassical properties of an incident quantum light. Note that at these points the absolute values of the real and imaginary parts of the refractive index become identical. In order to describe the aforementioned effects, one can treat an electromagnetic wave in the framework of full quantum theory as a stream of photons interacting with the media.

In this paper, we investigate the medium effect of a $\mathcal{PT}$-symmetric bilayer on the quantum optical properties of an incident squeezed light at zero temperature 0 K. To do so, we calculate the quadrature squeezing of the outgoing states and show how the quantum features of the incident state are degraded when transmitted through the $\mathcal{PT}$-symmetric bilayer structure.

II. QUADRATURE SQUEEZING

We consider a $\mathcal{PT}$-symmetric bilayer structure composed of two slabs of identical thicknesses *l*, and dielectric permittivities $\varepsilon_g(\omega,\mathbf{r})$ and $\varepsilon_l(\omega,\mathbf{r})$, satisfying the $\mathcal{PT}$-symmetry condition $\varepsilon_l(\omega,\mathbf{r})=\varepsilon_g^*(\omega,-\mathbf{r})$. In other words, the absolute magnitudes of the gain and loss parameters are equal. A schematic of the proposed bilayer structure, surrounded by vacuum, is illustrated in Fig. 1. Consider an optical beam of light normally incident (i.e., along the *x*-direction) upon the bilayer.

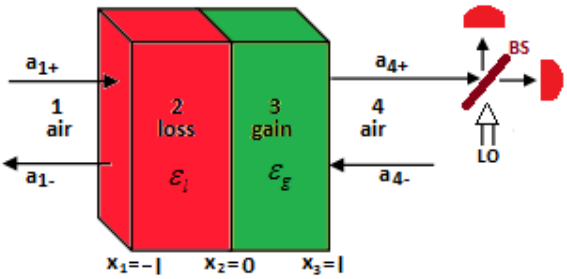

Fig. 1. A schematic of a $\mathcal{PT}$-symmetric bilayer composed of two slabs of identical thicknesses, l, immersed in the vacuum and illuminated by a quantum squeezed state from the left side. $\varepsilon_{l(g)}$ represents the loss (gain) medium permittivity. The arrows, →(←), together with the operators $a_{j+(-)}$ with j=1 and 4 indicate the incoming (outgoing) fields in the regions 1 and 4. The homodyne detector at the right side of the bilayer is used to measure the output quadrature variance (see Eq.(12)).

According to the canonical quantization of the electromagnetic field in the presence of a medium, the positive frequency component of the vector potential operator is [5,6]:

$$\hat{A}(x,t)=\int_0^\infty d\omega\sqrt{\frac{\hbar}{4\pi\varepsilon_0 c\,\omega\sigma}}\left(\hat{a}_{\Omega+}(\omega)e^{-\frac{i\omega x}{c}}+\hat{a}_{\Omega-}(\omega)e^{\frac{i\omega x}{c}}\right)e^{-i\omega t} \tag{1}$$

where Ω=1 or 4, indicating the light is incident from the left or right, σ is the area of quantization in the *y*-*z* plane, and the indices + and − refer to the right going and left going propagating modes. The negative frequency component of the vector potential operator is obtained by taking the Hermitian conjugate of Eq. (1). Using the quantum input-output relations, the annihilation operators of the output modes, $\hat{a}_{1-}(x_1,\omega)$ and $\hat{a}_{4+}(x_3,\omega)$, can be expressed in terms of the annihilation operators of the input modes, $\hat{a}_{1+}(x_1,\omega)$ and $\hat{a}_{4-}(x_3,\omega)$, and the noise operators, $\hat{F}_-(\omega)$ and $\hat{F}_+(\omega)$ [5-7]:

$$\begin{pmatrix} \hat{a}_{1-}(x_1,\omega) \\ \hat{a}_{4+}(x_3,\omega) \end{pmatrix} = \mathcal{A}\begin{pmatrix} \hat{a}_{1+}(x_1,\omega) \\ \hat{a}_{4-}(x_3,\omega) \end{pmatrix} + \begin{pmatrix} \hat{F}_-(\omega) \\ \hat{F}_+(\omega) \end{pmatrix}. \quad (2)$$

The quantum noise originating from the loss and the gain layers is given by:

$$\begin{pmatrix} \hat{F}_-(\omega) \\ \hat{F}_+(\omega) \end{pmatrix} = \mathcal{B}^{(2)}\begin{pmatrix} c_{2+}(\omega) \\ c_{2-}(\omega) \end{pmatrix} + \mathcal{B}^{(3)}\begin{pmatrix} c_{3+}(\omega) \\ c_{3-}(\omega) \end{pmatrix}. \quad (3)$$

Here, the coefficient matrices $\mathcal{A}$ and $\mathcal{B}^{(j)}$ $(j=2,3)$ are written as:

$$\mathcal{A} = A_{22}^{(-1)}\begin{pmatrix} -A_{21} & 1 \\ A_{11}A_{22} - A_{12}A & A_{12} \end{pmatrix} \quad (4a)$$

$$\mathcal{B}^{(j)} = A_2^{(-1)} \times \begin{pmatrix} -B_{21}^{(j)} & -B_{22}^{(j)} \\ B_{11}^{(j)}B_{22} - B_{12}B_{21}^{(j)} & B_{12}^{(j)}B_{22} - B_{12}B_{22}^{(j)} \end{pmatrix}, \quad (4b)$$

where the matrices B and A satisfy the relations $B^{(2)} = B^{(3)} \cdot R^{(3)} \cdot S^{(2)}$, $B^{(3)} = S^{(3)}$ and $A = B^{(2)} \cdot R^{(2)} \cdot S^{(1)}$. Also, $R^{(j)}$ are diagonal 2×2 matrices with $R_{11}^{(j)} = 1/R_{22}^{(j)} = e^{-\kappa_j l\omega/c}$, in which $\kappa_2 = \kappa_l$ and $\kappa_3 = \kappa_g$ are, respectively, the imaginary parts of the refractive indices of the loss and gain layers. Furthermore, the elements of the scattering matrices $S^{(j)}$ are given by:

$$S_{11}^{(j)} = \sqrt{\frac{\eta_j}{\eta_{j+1}}}\frac{n_{j+1}+n_j}{2n_j} e^{-\frac{i(\eta_{j+1}-\eta_j)\omega x_j}{c}}, \quad (5a)$$

$$S_{12}^{(j)} = \sqrt{\frac{\eta_j}{\eta_{j+1}}}\frac{n_{j+1}-n_j}{2n_j} e^{-\frac{i(\eta_{j+1}+\eta_j)\omega x_j}{c}}, \quad (5b)$$

$$S_{21}^{(j)} = \sqrt{\frac{\eta_j}{\eta_{j+1}}}\frac{n_{j+1}-n_j}{2n_j} e^{-\frac{i(\eta_{j+1}+\eta_j)\omega x_j}{c}}, \quad (5c)$$

$$S_{22}^{(j)} = \sqrt{\frac{\eta_j}{\eta_{j+1}}}\frac{n_{j+1}+n_j}{2n_j} e^{-\frac{i(\eta_{j+1}-\eta_j)\omega x_j}{c}}. \quad (5d)$$

Here, $n_2 = \sqrt{\varepsilon_l}$, $n_3 = \sqrt{\varepsilon_g}$ are, respectively, the refractive indices of the loss and gain layers, and $\eta_2 = \eta_l$ and $\eta_3 = \eta_g$ denote the real parts of the corresponding complex parameters.

Since the input optical fields before arriving at the $\mathcal{PT}$-symmetric bilayer cannot sense the presence of the slabs in free space, the optical input operators satisfy the bosonic commutation relation [7],

$$\left[\hat{a}_{1+}(\omega), \hat{a}_{1+}^{\dagger}(\omega')\right] = \left[\hat{a}_{4-}(\omega), \hat{a}_{4-}^{\dagger}(\omega')\right] = \delta(\omega-\omega'). \quad (6)$$

Substituting Eq. (5) into (2) results in a similar bosonic commutation relation for the outgoing operators,

$$\left[\hat{a}_{4+}(\omega), \hat{a}_{4+}^{\dagger}(\omega')\right] = \left[\hat{a}_{1-}(\omega), a_{1-}^{\dagger}(\omega')\right] = \delta(\omega-\omega'). \quad (7)$$

Using Eqs. (2)-(6) together with the properties of the incoming fields and the noise operators, one can calculate the quantum properties of the output fields at any position outside the $\mathcal{PT}$-symmetric bilayer. Hence, having a squeezed quantum state incident from the left and a vacuum state incident from the right, one can demonstrate the quantum optics effect of the $\mathcal{PT}$-symmetric structure.

Nowadays, the most widely used methods for generating a squeezed state of light rely on the parametric down-conversion, a process in which one photon is converted into two phase-correlated photons of lower frequencies. Mathematically, this squeezed incident quantum state of light can be written as $|L\rangle = \hat{S}(\{\rho(\omega), \phi(\omega)\})|0\rangle$ with the following squeeze operator [6]:

$$\hat{S}(\{\rho(\omega), \phi(\omega)\}) = \exp\int_0^{\Delta\omega} d\omega\left(\rho^*(\omega) \times e^{-i\phi_\rho(\omega)}\hat{a}_{1+}^{\dagger}(\omega)\hat{a}_{1+}^{\dagger}(2\Omega-\omega) - \text{h.c.}\right) \quad (8)$$

where $\phi_\rho(\omega)$ and $\rho(\omega)$ are the phase and amplitude of the squeezed state at a given frequency (ω) that controls the strength of the light squeezing. The squeezing strength can be measured by a balanced homodyne detector (see Fig. 1), where the fields from an optical signal and a local oscillator are superimposed on a beam splitter. The difference in the photocurrents generated by the two outputs of the beam splitter can be represented by an operator [6]:

$$\hat{O} = i \int_{t_0}^{t_0+T_0} dt \left\{ \hat{a}_{4+}^{\dagger}(t)\hat{a}_{LO}(t) - \text{h.c.} \right\} \tag{9}$$

The detector is considered to be on during the time interval $t_0 \le t \le t_0 + T_0$. The operator $\hat{a}_{LO}$, in Eq. (8), represents the local-oscillator field,

$$\hat{\alpha}_{LO}(t) = \hat{F}_{LO}^{1/2} \exp\{i\phi_{LO} - i\omega_{LO}t\}, \tag{10}$$

which is assumed to be in a coherent state $|\{\alpha_{LO}\}\rangle$. Here, $\hat{F}_{LO}$ represents the local-oscillator mean photon flux and ϕ_{LO} and ω_{LO} denote the corresponding phase and frequency. Assuming the local oscillator field to be stronger than that of the input signal, the measurement operator (8) can be written in terms of a dimensionless homodyne electric field operator as,

$$\hat{O} = \left(\hat{F}_{LO}T_0\right)^{\frac{1}{2}} \hat{E}(\phi_{LO}, \omega_{LO}) \tag{11}$$

From the above definitions, the variance of the transmitted light through the $\mathcal{PT}$-symmetric bilayer for a sufficiently long time interval (i.e., a narrow detector bandwidth), is given by:

$$\begin{aligned} &\left\langle \left[\Delta\hat{E}(\phi_{LO}, \omega_{LO})\right]^2 \right\rangle^{\text{out}} = \\ &1 + 2\left\langle \hat{a}_{4+}^{\dagger}(\phi_{LO}, \omega_{LO}), \hat{a}_{4+}(\phi_{LO}, \omega_{LO}) \right\rangle + \\ &2\text{Re}\left[\left\langle \hat{a}_{4+}^{\dagger}(\phi_{LO}, \omega_{LO}), \hat{a}_{4+}^{\dagger}(\phi_{LO}, \omega_{LO}) \right\rangle e^{2i\phi_{LO}} \right] \end{aligned} \tag{12}$$

By making use of the input-output relation (1), after some manipulations, the measured field variance reduces to,

$$\begin{aligned} \left\langle \left[\Delta\hat{E}(\phi_{LO}, \omega_{LO})\right]^2 \right\rangle^{\text{out}} &= 1 + \left\langle \hat{F}_+^{\dagger}(\omega)\hat{F}_+(\omega) \right\rangle + \\ &|\mathcal{A}_{21}|^2 \left\{ 2\sinh^2\rho - \right. \\ &\left. \text{Re}\left[e^{i\left(2\phi_{LO} - 2\arg(\mathcal{A}_{22}) - \phi_\rho\right)} \sinh 2\rho \right] \right\} \end{aligned} \tag{13}$$

Hereafter, we assume that the gain and loss slabs are both maintained at zero temperature (T=0 K). Hence, from Eq. (3), the average flux of the noise photons due to the spontaneous emission processes reduces to:

$$\begin{aligned} \left\langle \hat{F}_+^{\dagger}(\omega)\hat{F}_+(\omega) \right\rangle = -2\Big\{ &\sinh(\kappa_g l)\Big(\left|\mathcal{B}_{21}^{(3)}\right|^2 e^{-\kappa_g l} + \\ &\left|\mathcal{B}_{22}^{(3)}\right|^2 e^{\kappa_g l} \Big) - \frac{\sin(\eta_g l)}{\eta_g l}\Big(\mathcal{B}_{21}^{(3)*}\mathcal{B}_{22}^{(3)} e^{i\eta_g l} + \\ &\mathcal{B}_{21}^{(3)}\mathcal{B}_{22}^{(3)*} e^{-i\eta_g l} \Big) \Big\}. \end{aligned} \tag{14}$$

III. THE EFFECT OF THE $\mathcal{PT}$-SYMMETRIC BILAYER ON AN INCIDENT QUANTUM LIGHT

Due to the complexity of Eq. (12), it is difficult to obtain any results analytically. Hence, we numerically calculate the field variance $\left\langle \left[\Delta\hat{E}\right]^2 \right\rangle^{\text{out}}$ for a single-resonance $\mathcal{PT}$-symmetric bilayer of Lorentz type, as an example. Hence, the complex permittivity of the gain/loss (g/l) slab can be written as [8],

$$\varepsilon_{g/l}(\omega) = \varepsilon_0 - \frac{\alpha_{g/l}\omega_{0g/l}\gamma_{g/l}}{\omega^2 - \omega_{0g/l}^2 + i\omega\gamma_{g/l}}. \tag{15}$$

where ε_0 represents the medium background permittivity, ω_0 is the emission frequency, $\gamma_{g/l}$ represents the gain/absorption linewidth, and $\alpha_{g/l}$ is the gain/absorption coefficient. The $\mathcal{PT}$-symmetry conditions for this bilayer structure can be written as, $\text{Re}\left[\varepsilon_g(\omega)\right] = \text{Re}\left[\varepsilon_l(\omega)\right]$ and $\text{Im}[\varepsilon_g(\omega)] = -\text{Im}[\varepsilon_l(\omega)]$. Moreover, the loss slab parameters satisfy $\alpha_l > 0$ and $\gamma_l > 0$ while

those of the gain slab satisfy α_g <0 and γ_g >0. We have used the values ω_{0l}=1 PHz, ω_{0g}=1.2 PHz, α_l =20.86, α_g =−2, γ_l =0.067 PHz, γ_g =0.14 PHz, same as those used in [9] that satisfy the $\mathcal{PT}$-symmetry conditions and the symmetry phase.

Figures 2(a) and 2(b), respectively, show the variance of the transmitted squeezed light for the cases that the homodyne detection is placed at the right and the left hand-sides of the bilayer structure, showing the transmitted and reflected lights. Quadrature squeezing occurs when the variance of the quantum fluctuations in one of the quadrature components of the electromagnetic fields drops below the vacuum state (i.e. <1). We observe that although the loss effect is compensated within the $\mathcal{PT}$-symmetric bilayer, the quadrature squeezing is greater than unity (i.e. >1). This means that the outgoing light is no longer squeezed. Furthermore, this deviation from unity becomes maximum when the frequency of the input light approaches that of the bilayer resonance. This is because of the quantum noise flux (Eq. (13)) reaches its maximum value near the bilayer resonance, where nearly complete population inversion is achieved and the excited-state population can decay spontaneously. Far from the resonance, $|\gamma_g|=\gamma_l \approx 0$ and the quantum noise flux vanishes. Therefore, the bilayer structure is seen like a lossless/gainless slab by the incident quantum light — i.e., the transmitted field is prepared in a state close to the squeezed vacuum state.

IV. Quantum Optical Effective Medium Theory for The Layered $\mathcal{PT}$-Symmetry

In this section, we present the quantum optical effective medium theory (QOEMT) in a layered metamaterial with a unit cell much smaller than the incident wavelength to describe the $\mathcal{PT}$-symmetric bilayer entirely in terms of its effective dielectric function [6, 7]. Due to the existence of the quantum noise in the system, the QOEMT differs from the usual effective index theories in classical optics [6, 7]. In this approach, the effective parameters

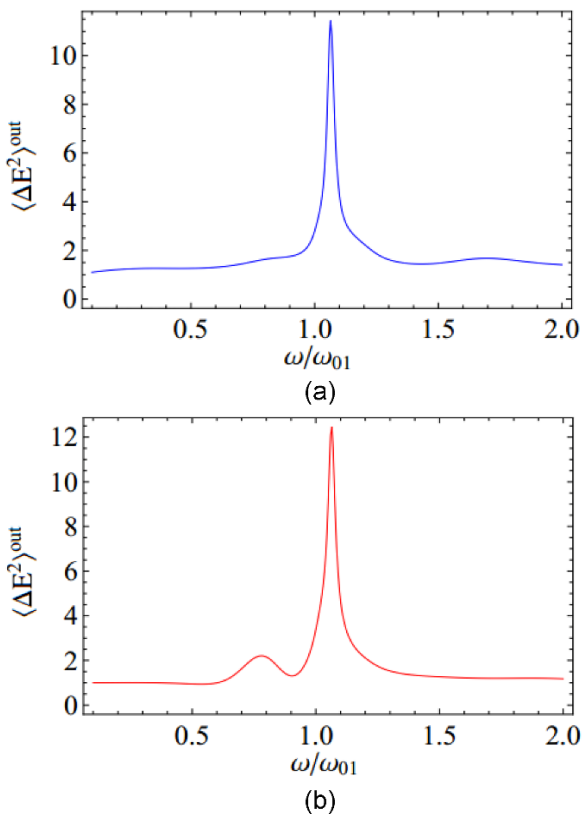

Fig. 2. Quadrature variance of Eq. (10) for an incident squeezed light with the squeezing strength $\rho = 0.2$ and the phase $\phi_\rho = 2\phi_{LO} - 2$. Here, the thickness of the loss and gain slabs is l=500nm. The Homodyne detection is placed at (a) the right and (b) the left hand-sides of the $\mathcal{PT}$-symmetric bilayer structure.

can be obtained from the small-(ω, k) Taylor expansion of the known dispersion relation for the multilayer structure. In this manner, the effective parameters of the $\mathcal{PT}$-symmetric bilayer structure for a s-polarized light can be obtained from the Bloch dispersion relation,

$$\cos(\beta_{\text{eff}} d) = \cos(\beta_g l)\cos(\beta_l l) - \frac{1}{2}\left(\frac{\beta_g}{\beta_l} + \frac{\beta_l}{\beta_g}\right)\sin(\beta_g l)\sin(\beta_l l) \tag{16}$$

where $\beta_j = \sqrt{\omega^2 \varepsilon_j / c^2 - k^2}$ $(j = l, g)$ is the normal component of the wave vector in the jth layer, in which k is the in-plane wave

vector. In the long-wavelength limit, by taking the Taylor expansion, Eq. (14) reduces to:

$$\frac{\beta_{\text{eff}}^2}{\varepsilon_{\text{eff}\parallel}}+\frac{k^2}{\varepsilon_{\text{eff}\perp}}=\frac{\omega^2}{c^2}, \tag{17}$$

where $\varepsilon_{\text{eff}\parallel}$ $\varepsilon_{\text{eff},s\parallel}$ and $\varepsilon_{\text{eff}\perp}$ are the components of the standard effective dielectric tensor, corresponding to the components of the electric field parallel $(\parallel)$ and perpendicular $(\perp)$ to the layers, respectively. For the system under study with a normal incidence (i.e., k=0), we have:

$$\varepsilon_{\text{eff}\parallel}=\varepsilon_{\text{eff}\perp}=\left(\varepsilon_l+\varepsilon_g\right)/2 \tag{18}$$

In quantum optics, besides the effective index, another effective parameter,

$$N_{\text{eff}}(\omega)=-\frac{1}{2}+\frac{1}{2}\sum_{j=g,l}\xi_j\left[2N_{\text{th}}\left(\omega,\left|T_j\right|\right)+1\right], \tag{19}$$

known as the "effective noise photon distribution" is required [5-7]. Here, $N_{\text{th}}\left(\omega,T_j\right)=\left(\exp\left(\hbar\omega/k_BT_j\right)-1\right)^{-1}$ represents the mean number of the thermal photons, wherein the parameters $\hbar$ and k_B are the reduced Planck's constant and the Boltzmann's constant, respectively. Moreover,

$$\xi_j=p_j\left|\frac{\text{Im}[\varepsilon_j(\omega)]}{\text{Im}[\varepsilon_{\text{eff}}(\omega)]}\right|, \tag{20}$$

represents the dielectric parameter of the j-th layer in the unit cell, where p_j is the corresponding layer volume fraction that equals 0.5 in the present bilayer structure. The QOEMT predicts that the quantum noise contribution to the output variance of the $\mathcal{PT}$-symmetric bilayer is given by:

$$\left\langle\hat{F}_{\text{eff}}^{\dagger}(\omega)\hat{F}_{\text{eff}}(\omega')\right\rangle=\left\{N_{\text{eff}}\,\theta\left(\text{Im}[\varepsilon_{\text{eff}}]\right)-\left(N_{\text{eff}}+1\right)\theta\left(-\text{Im}[\varepsilon_{\text{eff}}]\right)\right\}\left(1-\left|r_{\text{eff}}\right|^2-\left|t_{\text{eff}}\right|^2\right)\times\delta\left(\omega-\omega'\right), \tag{21}$$

where the effective reflection coefficient, r_{eff}, and the effective transmission coefficient, t_{eff}, for the effective $\mathcal{PT}$-symmetric medium are given by [6,7]:

$$r_{\text{eff}}=\frac{\left(\beta_{\text{eff}}^2-\beta_0^2\right)\left(\exp\left[4i\beta_{\text{eff}}l\right]-1\right)}{\left(\beta_{\text{eff}}+\beta_0\right)^2-\left(\beta_{\text{eff}}-\beta_0\right)^2\exp\left[4i\beta_{\text{eff}}l\right]}, \tag{22a}$$

$$t_{\text{eff}}=\frac{4\beta_{\text{eff}}\beta_0\exp\left[2i(\beta_{\text{eff}}-\beta_0)l\right]}{\left(\beta_{\text{eff}}+\beta_0\right)^2-\left(\beta_{\text{eff}}-\beta_0\right)^2\exp\left[4i\beta_{\text{eff}}l\right]}. \tag{22b}$$

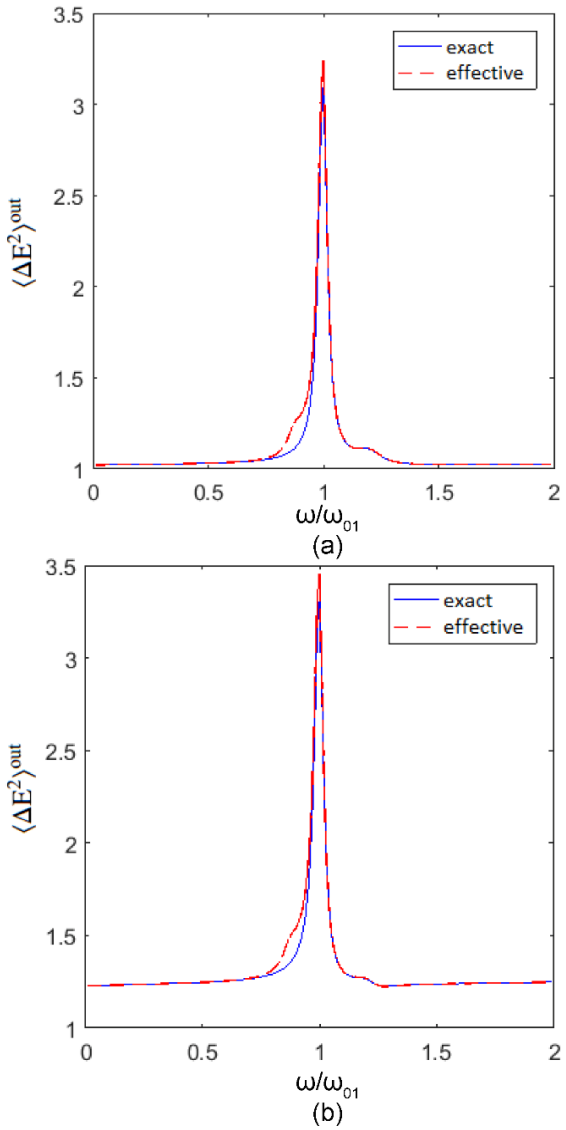

Fig. 3. Comparison of the quadrature variance for an incident squeezed light of strength $\rho=0.2$ $\rho=0.2$ and phase, $\phi_\rho=2\phi_{\text{LO}}-2$, on the effective $\mathcal{PT}$-symmetric structure with l=2.5 nm, obtained from the exact calculation (blue solid curve) and the QOEMT (red dashed curve), when the homodyne

detector is placed at the (a) right- and (b) left-hand sides of the structure.

By substituting Eqs. (15) and (17)-(21) into Eq. (12), the variance of the transmitted light is obtained in the QOEMT.

Figure 3(a) compares the quadrature variance, obtained from the exact calculations (blue solid curve) with those obtained from the QOEMT (red dashed curve) when the homodyne detector is placed at the right-hand side of the $\mathcal{PT}$-symmetric bilayer. Figure 3(b) depicts a similar comparison for the case in which the homodyne detector is placed at the left-hand side of the $\mathcal{PT}$-symmetric bilayer. We observe that the quantum optical effective medium theory correctly predicts the quadrature squeezing of the light outgoing from the $\mathcal{PT}$-symmetric bilayer structure.

V. CONCLUSION

In this paper, we have investigated the medium effect of the $\mathcal{PT}$-symmetric bilayer on the quantum optical properties of an incident squeezed light. By calculating the quadrature squeezing of the output state at zero temperature, we have found that even in the symmetry phase, the squeezing of the transmitted quantum light is severely degraded, so that the output state is not squeezed near the resonance frequency of the $\mathcal{PT}$-symmetric bilayer. The results also show that there are excellent agreements between the exact method and the QOEMT for a squeezed light passing through a $\mathcal{PT}$-symmetric bilayer structure.

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