Linear and Nonlinear Dust Ion Acoustic Waves in a Dense Quantum Magneto Plasma

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ABSTRACT— In this article, linear and nonlinear dust ion acoustic (DIA) waves are studied in a magnetized quantum dusty plasma which consists of inertialess electrons and positrons, cold ions and negatively charged dust grains. For this purpose, quantum Hydrodynamic model (QHD) and reductive perturbation method are employed. To investigate linear and nonlinear waves, dispersion relation and a quantum Zakharov-Kuznetsov (ZK) equation are derived respectively. A stationary solution of the ZK equation is obtained to investigate the effects of plasma parameters on the amplitude of the solitons.

KEYWORDS: Dust ion acoustic waves, quantum Hydrodynamic model, quantum magnetoplasma.

I. INTRODUCTION

Ordinary plasma is composed of freely moving charged particles which are particularly electrons and ions. This could be implied that few electron volts (10^5K), break neutral atoms and molecules to free electrons and ions. At high temperatures and low densities, thermal energy dominates, electrons and ions interact under classical dynamical laws subject to Newtonian mechanics and Maxwellian statistical physics. Quantum plasmas are known to exist under high-density conditions where the plasma electrons behave fully quantum mechanically such as plasmas which are created in many modern experiments with short laser pulses, ion beams or combination of both [1, 2]. Quantum effects also are expected to occur in many dense astrophysical plasmas [3], for example in the core of the giant planets such as Jupiter, Saturn, and brown or white dwarf stars [4], Neptune [5-8], neutron stars [9]. Furthermore the exotic state of the Universe after the Big Bang is believed to have been a quantum plasma. In all these cases, a description based on classical mechanics and thermodynamics, or classical kinetic theory is unsuccessful. A theoretical description based on the Schrodinger equation and quantum many-body theory involving methods such as quantum statistics, quantum kinetic theory, Hartree–Fock and nonequilibrium Green's functions (NEGF) use to carry out such investigations. Quantum hydrodynamic (QHD) model is a popular model which widely is used for description of hydrodynamic and statistical behaviors of plasma particles in quantum ranges[10, 11]. This model is a generalization of the classical fluid model of plasmas where QHD transport equations are expressed in terms of conservation laws for particle momentum and energy. Zakharov-Kuznetsov (ZK) has been extensively used to study the IA waves in magnetized plasmas[12, 13]. ZK equation governs the behavior of weakly nonlinear waves in a plasma consisting of cold ions and hot isothermal electrons in the presence of a uniform magnetic field and it has been almost employed for other conditions. Moslem et al. derived the ZK equation for electron-ion quantum plasma[14]. The authors have also examined the stability of the wave and discussed the importance of their results. Khan and et al. studied DIA solitary waves in a magnetized quantum dusty plasma by using the quantum hydrodynamic model and derived

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ZK equation [15]. In the present paper, we study the propagation of linear and nonlinear acoustic DIA waves in a dense magnetized quantum plasma consisting of inertialess electrons and positrons, cold ions and negatively charged dust grains by employing the QHD model. We derive dispersion relation and ZK equation taking into account the effects of quantum pressure and Fermi-Dirac statistics for the lighter species electrons and positrons. The manuscript is organized as follows. The governing equations are given in Sec II. Dispersion relation is derived for linear waves in section III and The ZK equation is derived for nonlinear waves in Sec IV. Section V is devoted to the solitary wave solution, results and discussions and The conclusion is given in section VI.

II. GOVERNING EQUATIONS

We study the linear and nonlinear propagation of DIA waves in a dense quantum magneto plasma composed of inertialess electrons and positrons, cold ions and negatively charged dust grains immersed in a uniform external magnetic field acting along the z-direction $\left(\mathbf{B_0}=\hat{\mathbf{k}}B_0\right)$. At equilibrium the charge neutrality requires $1+p=e+d$, where $p=\frac{n_{p0}}{n_{i0}}$, $e=\frac{n_{e0}}{n_{i0}}$, and $d=\frac{Z_{d0}n_{d0}}{n_{i0}}$, n_{p0}, n_{e0}, n_{i0}, and n_{d0} are respectively the equilibrium number density of positron, electron, ion, and dust. Z_{d0} is the dust charging state. The nonlinear dynamics of DIA wave in this plasma model, is governed by the following QHD equations

$$\frac{\partial n_d}{\partial t}+\vec{\nabla}\cdot\left(n_d\mathbf{u_d}\right)=0 \tag{1}$$

$$\frac{\partial n_i}{\partial t}+\vec{\nabla}\cdot\left(n_i\mathbf{u_i}\right)=0 \tag{2}$$

$$\nabla^2\varphi=\mu_d\left(1-d\right)n_e+dn_d-n_i+\left(1-\mu_d\right)\left(1-d\right)n_p \tag{3}$$

$$\frac{\partial\mathbf{u_d}}{\partial t}+\left(\mathbf{u_d}.\vec{\nabla}\right)\mathbf{u_d}=\mu\vec{\nabla}\phi-\mu\Omega_{ci}\left(\mathbf{u_d}\times\hat{\mathbf{z}}\right) \tag{4}$$

$$\frac{\partial\mathbf{u_i}}{\partial t}+\left(\mathbf{u_i}\cdot\vec{\nabla}\right)\mathbf{u_i}=-\vec{\nabla}\phi+\Omega_{ci}\left(\mathbf{u_i}\times\hat{\mathbf{z}}\right)-\sigma n_i\vec{\nabla}n_i \tag{5}$$

$$\vec{\nabla}\phi=\Omega_{ci}\left(\mathbf{u_e}\times\hat{\mathbf{z}}\right)+n_e\vec{\nabla}n_e-\frac{H^2}{2}\vec{\nabla}\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right) \tag{6}$$

$$\vec{\nabla}\phi=\Omega_{ci}\left(\mathbf{u_p}\times\hat{\mathbf{z}}\right)-n_p\vec{\nabla}n_p-\frac{H^2}{2}\vec{\nabla}\left(\frac{\nabla^2\sqrt{n_p}}{\sqrt{n_p}}\right) \tag{7}$$

The number density n_j, is normalized by its unperturbed density n_{j0}, where *j=e* for electron, *i* for ion, *p* for positron and *d* for dust particle. The electrostatic wave potential φ is normalized by $\frac{2k_BT_{Fe}}{e}$. The fluid velocity u_j (*j=e, i, p, d*) is normalized by the quantum ion-acoustic speed C_{si}. The space and time coordinates *x* and *t* are normalized respectively, by the quantum Debye length λ_D and the ion plasma frequency ω_{pi}. Further, $\lambda_D=\sqrt{\frac{2k_BT_{Fe}}{4\pi e^2n_{i0}}}$, $\omega_{pi}=\sqrt{\frac{4\pi e^2n_{i0}}{m_i}}$, $C_{si}=\sqrt{\frac{2k_BT_{Fe}}{m_i}}$, $\mu=\frac{Z_{d0}m_i}{m_d}$, $\mu_d=1+\frac{P}{1-d}$, $\gamma=\frac{T_{Fi}}{T_{Fp}}$, $\sigma=\frac{T_{Fi}}{T_{Fe}}$, $\Omega_{ci}=\frac{\omega_{ci}}{\omega_{pi}}$, where T_{Fj} (j=*e*, *i*, *p*) is Fermi temperature and ω_{ci} is plasma cyclotron frequency. The non-dimensional quantum diffraction parameter *H* is defined as $H=\sqrt{\frac{\hbar^2\omega_{pi}^2}{m_{e,p}m_iC_{si}^4}}$.

III. THE LINEAR DAWS-DISPERSION RELATION

To study the properties of linear DIA waves, Fourier transforming of the first order perturbed quantities of the Eqs. 1-7, we obtain the dispersion relation for DAWs in form of

$$\omega^4-A\omega^2+B=0 \tag{8}$$

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where ω and k $\left(k^2 = k_x^2 + k_y^2\right)$ are the normalized frequency and wave number, respectively, and

$$A = \Omega^2(1-\mu^2) + k^2\sigma + \frac{k^2(1-\mu\beta d)}{S}$$

$$B = \mu^2\Omega^2\left(\Omega^2 + k^2\sigma\right) + \frac{\mu\beta d}{S}\left(\Omega^2 + k^4\sigma\right)$$

$$S = \frac{k^2 + \mu_d(1-\beta d) + (1-\mu_d)(1-\beta d)}{\left(1+\frac{H^2}{4}k^2\right)}$$

Note that, we assume the Fermi temperatures of electrons and positrons to be equal ($\sigma = \gamma$). Fig.1 shows the plot of dispersion relation for DIA waves.

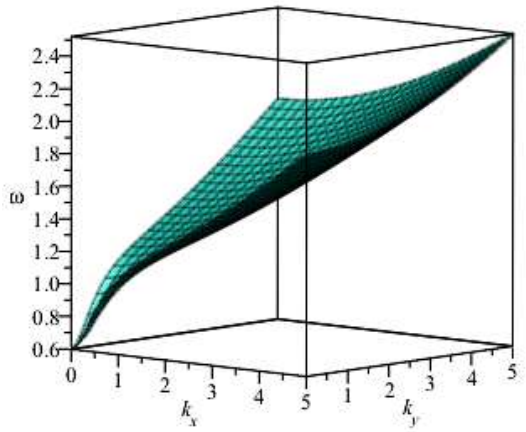

Fig. **1.** Plot of dispersion relation of linear DIAWs with fixed values of $\Omega = 0.6$, $\gamma = 1$, $\alpha = 20^\circ$, $\mu = 3\times10^{-3}$, $u_0 = 1$, $\mu_d = 0.5$ $H = 2.11\times10^{-3}$,

IV. The Nonlinear DAWs- Limit and Small Amplitude

We employ the reductive perturbation method to Eqs. 1-7 to obtain the nonlinear Zakharov-Kuznetsov (ZK) equation for DIA waves. For This purpose, plasma parameters can be expanded in powers of ε as

$$u_{jz} = \varepsilon u_{j1z} + \varepsilon^2 u_{j2z} + \ldots$$

$$u_{jy} = \varepsilon^{\frac{3}{2}} u_{j1y} + \varepsilon^2 u_{j2y} + \ldots$$

$$u_{jx} = \varepsilon^{\frac{3}{2}} u_{j1x} + \varepsilon^2 u_{j2x} + \ldots$$

$$\varphi = \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \ldots \qquad (9)$$

where ε is a small parameter measuring the weakness of the dispersion and V_0 is the wave phase speed normalized by the ion acoustic speed (C_{si}). We also introduce the following independent variables scaled as $X = \varepsilon^{\frac{1}{2}}x$, $Y = \varepsilon^{\frac{1}{2}}y$, $Z = \varepsilon^{\frac{1}{2}}(z - V_0 t)$ and $\tau = \varepsilon^{\frac{3}{2}}t$. Substituting Eq. 8 into Eqs. 1-7 and collecting the various order terms of ε, we obtain a ZK equation

$$\frac{\partial\varphi_1}{\partial\tau} + A_1\varphi_1\frac{\partial\varphi_1}{\partial Z} + A_2\frac{\partial^3\varphi_1}{\partial Z^3} + A_3\frac{\partial}{\partial Z}\left(\frac{\partial^2\varphi_1}{\partial X^2} + \frac{\partial^2\varphi_1}{\partial Y^2}\right) = 0 \qquad (10)$$

with the coefficients

$$A_1 = \frac{1}{D}\left[-\mu_d(1-d) - \frac{(1-\mu_d)(1-d)}{\gamma^2} + \frac{2\mu^2 d}{V_0^4} - \frac{2V_0^2+\sigma}{(V_0^2-\sigma)^3}\right]$$

$$A_2 = \frac{1}{D}\left(\frac{\mu_d(1-d)H^2}{4} - \frac{(1-\mu_d)(1-d)H^2}{4\gamma^2} - 1\right)$$

$$A_3 = \frac{1}{D}\left[\frac{\mu_d(1-d)H^2}{4} - \frac{(1-\mu_d)(1-d)H^2}{4\gamma^2} - \frac{d}{\mu\Omega^2} - \frac{V_0^2}{(V_0^2-\sigma)}\left(\frac{1}{\Omega^2} + \frac{\sigma}{\Omega^2(V_0^2-\sigma)}\right) - 1\right]$$

V. Solitary Waves Solution of the ZK Equation

To study the properties of the solitary waves propagating in a direction making an angle δ with the Z−axis, i.e., with the external magnetic field and lying in the (Z−X) plane, we first rotate the coordinate axes (X, Z) through an angle δ, keeping the Y − axis fixed. Thus, we transform our independent variables to

$$\zeta = X\cos\alpha - Z\sin\alpha$$
$$\xi = X\sin\alpha + Z\cos\alpha \qquad (11)$$
$$\eta = Y$$

Using the transformation we get,

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$$\frac{\partial}{\partial X}=\cos\alpha\frac{\partial}{\partial\zeta}+\sin\alpha\frac{\partial}{\partial\xi}$$
$$\frac{\partial}{\partial Y}=\frac{\partial}{\partial\eta} \quad (12)$$
$$\frac{\partial}{\partial Z}=\cos\alpha\frac{\partial}{\partial\xi}-\sin\alpha\frac{\partial}{\partial\zeta}$$

This transformation of these independent variables allows us to write the ZK equation in the form

$$\frac{\partial\varphi_1}{\partial\tau}+\alpha_1\varphi_1\frac{\partial\varphi_1}{\partial\xi}+\alpha_2\frac{\partial^3\varphi_1}{\partial\xi^3}+$$
$$\alpha_3\varphi_1\frac{\partial\varphi_1}{\partial\zeta}+\alpha_4\frac{\partial^3\varphi_1}{\partial\zeta^3}+\alpha_5\frac{\partial^3\varphi_1}{\partial\xi^2\partial\zeta}+\alpha_6\frac{\partial^3\varphi_1}{\partial\xi\partial\zeta^2} \quad (13)$$
$$+\alpha_7\frac{\partial^3\varphi_1}{\partial\xi\partial\eta^2}+\alpha_8\frac{\partial^3\varphi_1}{\partial\zeta\partial\eta^2}=0$$

We now look for a steady state solution of this ZK equation in the form $\varphi_1=\varphi_0(\eta)$ where $\eta=\xi-u_0\tau$ and u_0 is a constant speed normalized by the IA speed(C_{si}).Using this transformation, Eq. 12 can be written as

$$\frac{d\varphi_0}{d\tau}-u_0\frac{d\varphi_0}{d\eta}+\alpha_1\varphi_0\frac{d\varphi_0}{d\eta}+\alpha_2\frac{d^3\varphi_0}{d\eta^3}=0 \quad (14)$$

where $\alpha_1=A_1\cos\alpha$ and $\alpha_2=A_2\cos^3\alpha+A_3\sin^2\alpha\cos\alpha$. At stationary state $\frac{d\phi_0}{d\tau}\to 0$, so we can write the ZK equation in steady state form as

$$-u_0\frac{d\varphi_0}{d\eta}+\alpha_1\varphi_0\frac{d\varphi_0}{d\eta}+\alpha_2\frac{d^3\varphi_0}{d\eta^3}=0 \quad (15)$$

Now, using the appropriate boundary conditions, viz., $\frac{d^2\varphi_1}{d\eta^2},\frac{d\varphi_1}{d\eta},\varphi_1\to 0$ at $|\eta|\to\infty$ one can write

$$\frac{d\varphi_0}{d\eta}=(\sqrt{\frac{u_0}{\alpha_2}}\varphi_0\sqrt{1-\frac{\alpha_1}{3u_0}\varphi_0} \quad (16)$$

Therefore, the solitary wave solution is given by

$$\varphi_0(\eta)=\varphi_m\sec h^2(\kappa\eta) \quad (17)$$

where $\varphi_m=\frac{3u_0}{\alpha_1}$ is the amplitude and $\kappa=\sqrt{\frac{u_0}{4\alpha_2}}$ is the inverse of the width (Δ) of the solitary waves. Figures 2-4 show the variation of the electrostatic potential φ_1 as a function of η. Figure 2 shows the effect of ion to electron Fermi temperature ratio σ and μ_d on the amplitude of electrostatic potential φ_0.

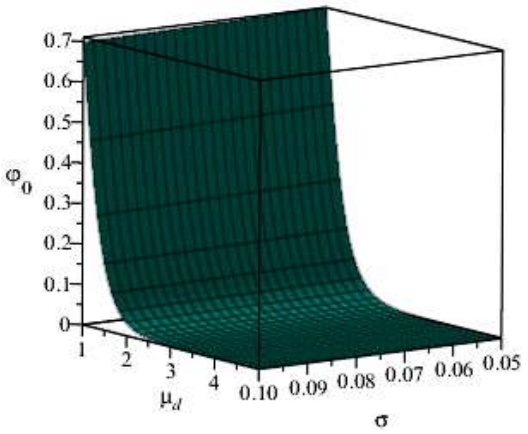

Fig. **2.** Plot of electrostatic potential in terms of η for different value of σ and μ_d with fixed values $\gamma=1$, $\alpha=20^\circ$ $\mu=3\times10^{-3}$, , $d=0.5$, $H=2.11\times10^{-3}$, $u_0=1$

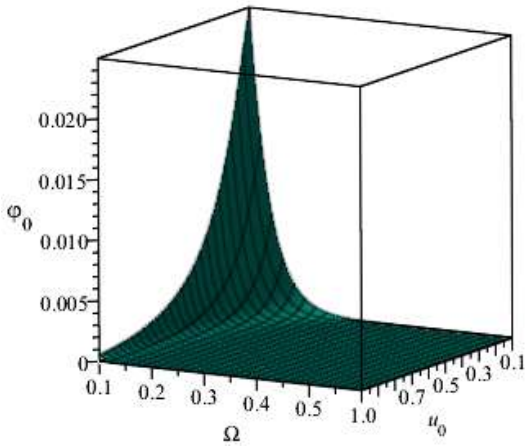

Fig. **3** Plot of electrostatic potential in terms of η for different value of Ω and u_0 with fixed values $\sigma=0.1$, $\gamma=1$ $\alpha=20^\circ$, $\Omega=0.6$, $\mu=3\times10^{-3}$, $\mu_d=5$, $d=0.5$, $H=2.11\times10^{-3}$

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It is found that by increasing the density of dust grains $\left(\mu_d = 1 + \frac{p}{1-d} = \frac{e}{1-d}\right)$ and decreasing the fermi temperature of electrons $\left(\sigma = \frac{T_{Fi}}{T_{Fe}}\right)$, the amplitude of electrostatic potential decreases. Fig.3 indicates that increasing IA speed and decreasing plasma cyclotron frequency ω_{ci} $\left(\Omega = \frac{\omega_{ci}}{\omega_{pi}}\right)$, lead to decreasing the amplitude of "electrostatic potential φ_0. In fig.4 we can observe DIA solitary wave and the effect of μ_d.

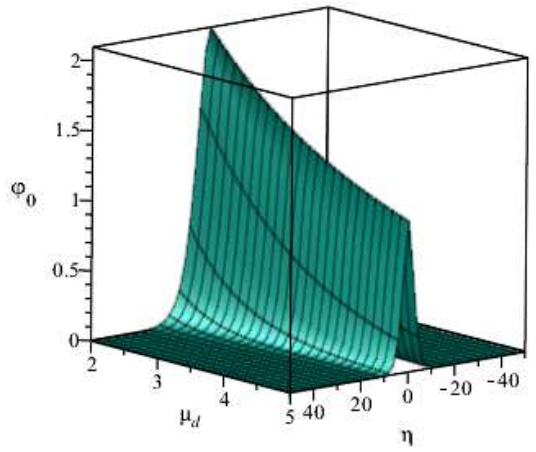

Fig. **4** Plot of electrostatic potential in terms of η for different value of μ_d with fixed values $\Omega = 0.6$ $\gamma = 1$, , $\alpha = 20^\circ$ $\mu = 3\times10^{-3}$, , $u_0 = 1$, $\mu_d = 5$, $H = 2.11\times10^{-3}$, $d = 0.5$

VI. CONCLUSION

In this paper by using the quantum hydrodynamics model (QHD), the properties of nonlinear DIAWs are investigated in a magnetized quantum dusty plasma consists of inertialess electrons and positrons, cold ions and negatively charged dust grains. To study the small amplitude DIAWs, reductive perturbation method is applied and a quantum Zakharov-Kuznetsov(ZK) equation is derived. Solitary Waves Solution of the ZK Equation is presented. From numerical analysis, it is found that the amplitude of electrostatic potential decreases as σ, μ_d and u_0 increase. Additionally, the effect of Ω on this amplitude is shown. This work is aimed to study some basic features of DIAWs in dense magnetized quantum plasmas, which are ubiquitous in massive astrophysical objects.

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