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IJOP Editorial Office
Optics and Photonics Society of Iran (OPSI)
P O Box 14395-115
Tehran, 1439663741, Iran
Phone: (+98) 21-44255936
Fax: (+98) 21-44255937
Email: **info@ijop.ir**

Self-Fields Effects on Gain in a Helical Wiggler Free Electron Laser with Ion-Channel Guiding and Axial Magnetic Field

Hasan Ehsani Amri[a] and Taghi Mohsenpour[b*]

[a]Department of Physics, Islamic azad University,nour branch, nour, Iran
[b]Department of Physics, Faculty of Basic Sciences, University of Mazandaran, Babolsar, Iran

***Corresponding Author Email:** mohsenpour@umz.ac.ir

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ABSTRACT— **In this paper, we have investigated the effects of self-fields on gain in a helical wiggler free electron laser with axial magnetic field and ion-channel guiding. The self-electric and self-magnetic fields of a relativistic electron beam passing through a helical wiggler are analyzed. The electron trajectories and the small signal gain are derived. Numerical investigation is shown that for group I orbits, gain decrement is obtained relative to the absence of the self-fields, while for group II orbit gain enhancement is obtained.**

Keywords: Free-electron laser, self-fields, ion-channel, axial magnetic field, gain

I. INTRODUCTION

A Raman free-electron laser (FEL) produces coherent radiation by passage of a cold intense relativistic electron beam through a static magnetic (wiggler) field which is spatially periodic along the beam axis. In Raman regime, due to the high density and low energy of the electron beam, an axial magnetic field generated by current in a solenoid is usually employed to focus on the beam [1-4]. Also the ion-channel guiding is used to focus the electrons against the self-repulsive electrostatic force generated by the beam itself [5-9].

In Raman regime, equilibrium self-electric and self-magnetic fields, due to the charge and current densities of the beam, can have considerable effects on the equilibrium orbits. It has been shown that self-field can induce chaos in the single-particle trajectories [10]. The effects of self-fields on the stability of equilibrium trajectories and gain were studied in a FEL with a one-dimensional helical wiggler and axial magnetic field [11, 12] or ion-channel guiding [13-14].

In recent years, electron trajectories and gain in a planar and helical wiggler FEL with ion-channel guiding and axial magnetic field were studied, detailed analysis of the stability and negative mass regimes were considered [15-18]. Esmaeilzadeh and Willett have investigated the effects of self-fields on gain in a FEL with an idealized (one-dimensional) helical wiggler and axial magnetic field [19]. Also, Esmaeilzadeh *et al*. have investigated the electron orbits and gain in free electron laser with realizable helical wiggler in the present of ion channel guiding [20]. The purpose of this paper is to study the effects of the self-fields on gain in a FEL with ion-channel guiding and axial magnetic field. In Section II, steady-state trajectories are obtained in the absence of self-fields. In Section III, self-fields are calculated using Poisson's equation and Ampere's law. Equilibrium orbits are found under the influence of self-fields. In Section 2.3, the derivation of the gain equation is presented. In Section 3, the numerical results are given. In Section 4, concluding remarks are given.

II. THEORY DETAILS

A. *Basic assumptions*

The evolution of the motion of a single electron in a FEL is governed by the relativistic Lorentz equation

$$\frac{d(\gamma m\mathbf{v})}{dt}=-e\left[\mathbf{E}_i+\frac{1}{c}\mathbf{v}\times\left(\mathbf{B}_w+\mathbf{B}_0\hat{\mathbf{e}}_z\right)\right], \tag{1}$$

where $\mathbf{E}_i$ is the transverse electrostatic field of an ion-channel and it can be written as

$$\mathbf{E}_i=2\pi e n_i\left(x\hat{\mathbf{x}}+y\hat{\mathbf{y}}\right), \tag{2}$$

and $\mathbf{B}_w$ is the idealized helical wiggler magnetic field and can be described by

$$\mathbf{B}_w=B_w\left(\hat{\mathbf{x}}\cos k_w z+\hat{\mathbf{y}}\sin k_w z\right), \tag{3}$$

and B_0 is the axial static magnetic field. Here, B_w denotes the wiggler amplitude, $k_w(=2\pi/\lambda_w)$ is the wiggler wave number, λ_w is the wiggler wavelength (period), n_i is the density of positive ions having charge *e*. The steady-state velocity of an electron moving in this field is

$$\mathbf{v}_0=v_w\left(\hat{\mathbf{x}}\cos k_w z+\hat{\mathbf{y}}\sin k_w z+v_{\|}\hat{\mathbf{z}}\right), \tag{4}$$

where $v_{\|}$ is the component in the positive *z* direction and

$$v_w=\frac{k_w v_{\|}^2\,\Omega_w}{\omega_i^2+k_w v_{\|}\left(\Omega_0-k_w v_{\|}\right)}, \tag{5}$$

is the signed magnitude of the wiggler-induced transverse velocity. $\omega_i=\left(2\pi n_i e^2/\gamma m\right)^{1/2}$, is the ion-channel frequency and $\Omega_{0,w}=eB_{0,w}/\gamma mc$, are the axial guide and wiggler magnetic field frequencies.

B. *Self-field calculation and steady-state orbits*

The self-electric and self-magnetic fields are induced by the steady-state charge density and current of the non-neutral electron beam. Solving Poisson's equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rE_r^{(s)}\right)=-4\pi e n_b(r), \tag{6}$$

yields the self-electric field in the form,

$$\mathbf{E}^{(s)}=-2\pi e n_b\, r\hat{\mathbf{r}}=-2\pi e n_b\left(x\hat{\mathbf{x}}+y\hat{\mathbf{y}}\right), \tag{7}$$

where n_b is the number density of the electrons.

The self-magnetic field is determined by Ampere's law,

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\mathbf{J}_b, \tag{8}$$

where $\mathbf{J}_b=-en_b\left(\mathbf{v}_\perp+v_{\|}\hat{\mathbf{e}}_z\right)$ is the beam current density and $\mathbf{v}_\perp$ is the transverse velocity. In cylindrical coordinates (r,θ,z), Ampere's law may be written in the form,

$$\frac{1}{r}\frac{\partial}{\partial\theta}B_z^{(s)}-\frac{\partial}{\partial z}B_\theta^{(s)}=-\frac{4\pi e n_b(r)v_\perp}{c}\cos\left(k_w z-\theta\right) \tag{9}$$

$$\frac{\partial}{\partial z}B_r^{(s)}-\frac{\partial}{\partial r}B_z^{(s)}=-\frac{4\pi e n_b(r)v_\perp}{c}\sin\left(k_w z-\theta\right), \tag{10}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\theta^{(s)}\right)-\frac{1}{r}\frac{\partial}{\partial\theta}B_r^{(s)}=-\frac{4\pi e n_b(r)v_{\|}}{c}, \tag{11}$$

where $v_\perp=v_w$. The solution of Eqs. (9)-(11) can be obtained by using the methods used in reference 10 (or references 12 and 13). This yields

$$\mathbf{B}_s^{(1)}=\mathbf{B}_{s\|}+\mathbf{B}_{sw}^{(1)}, \tag{12}$$

where

$$\mathbf{B}_{s\theta}=-2\pi e n_b\frac{v_{\|}}{c}r\hat{\mathbf{e}}_\theta=2\pi e n_b\frac{v_{\|}}{c}\left(y\hat{\mathbf{x}}-x\hat{\mathbf{y}}\right), \tag{13}$$

$$\mathbf{B}_{sw}^{(1)} = \frac{4\pi e n_b(r)}{k_w}\frac{v_w}{c^2}\left[\mathbf{e}_r \cos(k_w z - \theta) + \mathbf{e}_\theta \sin(k_w z - \theta)\right] = \frac{2\omega_b^2\left(v_\parallel^2/c^2\right)}{\omega_i^2 + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}\mathbf{B}_w, \tag{14}$$

and $\omega_b = \left(2\pi n_b e^2/\gamma m\right)^{1/2}$. The first term in Eq. (12) is due to axial velocity; the second term is due to the transverse velocity induced by the wiggler magnetic field. With inclusion of the self-magnetic field, the total magnetic field up to first-order correction, may be written as

$$\mathbf{B}^{(1)} = \mathbf{B}_w + \mathbf{B}_s^{(1)} + B_0\hat{\mathbf{e}}_z = \lambda^{(1)}\mathbf{B}_w + 2\pi e n_b \frac{v_\parallel}{c}\left(y\hat{\mathbf{x}} - x\hat{\mathbf{y}}\right) + B_0\hat{\mathbf{e}}_z, \tag{15}$$

where

$$\lambda^{(1)} = 1 + \frac{2\omega_b^2\left(v_\parallel^2/c^2\right)}{\omega_i^2 + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}, \tag{16}$$

is the first-order correction factor for the wiggler magnetic field.

The equation of electron motion in the presence of self-fields may be written (in the scalar form):

$$\frac{d(\gamma m v_x)}{dt} = -2\pi e^2(n_i - n_b)x - \frac{2\pi n_b e^2}{c^2}v_\parallel v_z x + \frac{e v_z}{c}\lambda^{(1)}B_w \sin k_w z - \frac{eB_0}{c}v_x, \tag{17}$$

$$\frac{d(\gamma m v_y)}{dt} = -2\pi e^2(n_i - n_b)y - \frac{2\pi n_b e^2}{c^2}v_\parallel v_z y - \frac{e v_z}{c}\lambda^{(1)}B_w \cos k_w z + \frac{eB_0}{c}v_y, \tag{18}$$

$$\frac{d(\gamma m v_z)}{dt} = -\frac{e}{c}\left[\lambda^{(1)}B_w\left(v_x \sin k_w z - v_y \cos k_w z\right) - 2\pi n_b e\frac{v_\parallel}{c}\left(v_x x + v_y y\right)\right], \tag{19}$$

The steady-state solution of Eqs. (17) and (18) may be expressed in the form, $v_x = v_w' \cos k_w z$ and $v_y = v_w' \sin k_w z$, where

$$v_w' = \frac{k_w v_\parallel^2 \lambda^{(1)}\Omega_w}{\omega_i^2 - \omega_b^2\gamma_\parallel^{-2} + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}, \tag{20}$$

and $v_z \cong v_\parallel = \text{constant}$. Equation (20) describes the transverse velocity in the presence of the self-electric and first order self-magnetic field. Substituting new electron velocity components into Eq. (8), and using the method applied in reference 10 (or references 12 and 13), we obtain the wiggler-induced self-magnetic field up to second-order correction as

$$\mathbf{B}_{sw}^{(2)} = K\lambda^{(1)}\mathbf{B}_w, \tag{21}$$

where

$$K = \frac{2\omega_b^2\left(v_\parallel^2/c^2\right)}{\omega_i^2 + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}, \tag{22}$$

and $\gamma_\parallel = \left(1 - v_\parallel^2/c^2\right)^{-1/2}$. Using Eq. (21), the total magnetic field up to second-order correction is given by

$$\mathbf{B}^{(1)} = \lambda^{(2)}\mathbf{B}_w + 2\pi e n_b \frac{v_\parallel}{c}\left(y\hat{\mathbf{x}} - x\hat{\mathbf{y}}\right) + B_0\hat{\mathbf{e}}_z, \tag{23}$$

where

$$\lambda^{(2)} = 1 + K\lambda^{(1)}. \tag{24}$$

This process may be continued to find the higher order terms. We can write

$$\mathbf{B}_{sw}^{(n)} = K\lambda^{(n-1)}\mathbf{B}_w, \qquad n = 3, 4, 5, \tag{25}$$

where

$$\lambda^{(n)} = 1 + K\lambda^{(n-1)}, \qquad n = 2, 3, 4, \tag{26}$$

The total wiggler-induced magnetic field becomes

$$\mathbf{B}_{sw} = \lim_{n\to\infty}\mathbf{B}_{sw}^{(n)} = K\lambda\mathbf{B}_w, \tag{27}$$

where

$$\lambda = \lim_{n\to\infty} \lambda^{(n)}$$
$$= \lim_{n\to\infty} \sum_{i=0}^{n-1} K^i + \lim_{n\to\infty} \frac{2\omega_b^2\left(v_\parallel^2/c^2\right)}{\omega_i^2 + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)} K^n. \quad (28)$$

If the absolute value of K is less than one, then the series in Eq. (28) will converge to $1/(1-K)$, and the last term in the right-hand side will goes to zero. In this case, Eq. (28) may be expressed in the form

$$\lambda = \frac{\omega_i^2 - \omega_b^2 \gamma_\parallel^{-2} + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}{\omega_i^2 - \omega_b^2\left[1 + \left(v_\parallel^2/c^2\right)\right] + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}. \quad (29)$$

Substituting Eq. (29) into Eq. (27) and using Eq. (22), the total wiggler-induced self-magnetic field is given by

$$\mathbf{B}_{sw} = \frac{2\omega_b^2\left(v_\parallel^2/c^2\right)}{\omega_i^2 - \omega_b^2\left[1 + \left(v_\parallel^2/c^2\right)\right] + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}. \quad (30)$$

The transverse part of the steady-state helical trajectories of electrons, in the presence of self-fields, can be found as

$$v_w = \frac{k_w v_\parallel^2 \Omega_w}{\omega_i^2 - \omega_b^2\left[1 + \left(v_\parallel^2/c^2\right)\right] + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right)}. \quad (31)$$

Equation (31) shows resonant enhancement in the magnitude of the transverse velocity when

$$\omega_i^2 - \omega_b^2\left[1 + \left(v_\parallel^2/c^2\right)\right] + k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right) \approx 0, \quad (32)$$

Steady-state trajectories may be classified according to the type of guiding. There are three main categories:

1. When the value of axial magnetic field is zero $(\Omega_0 = 0)$; that is, a FEL with a helical wiggler and an ion-channel guiding. In this type of guiding Eq. (31) becomes,

$$v_w = \frac{k_w v_\parallel^2 \Omega_w}{\omega_i^2 - \omega_b^2\left[1 + \left(v_\parallel^2/c^2\right)\right] - k_w^2 v_\parallel^2}. \quad (33)$$

2. When the density of positive ions is zero $(\omega_i = 0)$; that is, a FEL with a helical wiggler and an axial magnetic field. In this type of guiding Eq. (36) becomes,

$$v_w = \frac{k_w v_\parallel^2 \Omega_w}{k_w v_\parallel\left(\Omega_0 - k_w v_\parallel\right) - \omega_b^2\left[1 + \left(v_\parallel^2/c^2\right)\right]}. \quad (34)$$

3. When both the ion-channel and the axial magnetic field are present; that is, a helical wiggler FEL with an ion-channel and an axial magnetic field. When both types of guiding are present, v_w is given by Eq. (31). This case contains three types: (a) two guiding frequencies are taken to be equal ($\Omega_0 = \omega_i$), (b) the ion-channel frequency is taken to be constant, (c) the axial magnetic field frequency is taken to be constant.

C. *Small signal gain*

Let electromagnetic radiation copropagate with the electron beam in the FEL interaction region, such as below

$$\begin{cases} \mathbf{E}_r = E_r\left(\hat{\mathbf{x}}\cos\zeta - \hat{\mathbf{y}}\sin\zeta\right) \\ \mathbf{B}_r = E_r\left(\hat{\mathbf{x}}\sin\zeta + \hat{\mathbf{y}}\cos\zeta\right) \end{cases} \quad (35)$$

where $\zeta = k_r z - \omega_r t + \phi$, $\omega_r (= k_r c)$ is frequency, and E_r is the amplitude of the wave. The electron equation of motion in the presence of the electromagnetic radiation and self-fields can be written in scalar form

$$\dot{\gamma}\beta_x + \gamma\dot{\beta}_x = +\lambda\frac{eB_w}{mc}\beta_z \sin k_w z - \frac{eB_0}{mc}\beta_x - \left[\frac{2\pi e^2(n_i - n_b)}{mc} + \frac{2\pi n_b e^2}{mc}\frac{v_\parallel^2}{c^2}\right]x - \frac{eE_r}{mc}\times (1-\beta_z)\cos\zeta, \quad (36)$$

$$\dot{\gamma}\beta_y + \gamma\dot{\beta}_y = -\lambda\frac{eB_w}{mc}\beta_z \cos k_w z + \frac{eB_0}{mc}\beta_y - \left[\frac{2\pi e^2(n_i - n_b)}{mc} + \frac{2\pi n_b e^2}{mc}\frac{v_\parallel^2}{c^2}\right]y + \frac{eE_r}{mc}\times (1-\beta_z)\sin\zeta, \quad (37)$$

$$\dot{\gamma}\beta_z + \gamma\dot{\beta}_z = \lambda\frac{eB_w}{mc}\left(\beta_y \cos k_w z - \beta_x \sin k_w z\right) - \frac{2\pi n_b e^2}{mc}\frac{v_\parallel}{c}\left(\beta_x x + \beta_y y\right) + \frac{eE_r}{mc}\left(\beta_y \sin\zeta - \beta_x \cos\zeta\right). \tag{38}$$

The last term in Eqs. (36) and (37), which represents the transverse optical force acting on the electron. Now, we consider radiation terms as a small external factor which produces small perturbation in the steady-state velocity components,

$$\beta_x = \beta_w \cos k_w z + \delta\beta_x = \beta_{x0} + \delta\beta_x, \tag{39}$$

$$\beta_y = \beta_w \sin k_w z + \delta\beta_y = \beta_{y0} + \delta\beta_y, \tag{40}$$

$$\beta_z = \beta_\parallel z + \delta\beta_z = \beta_{z0} + \delta\beta_z. \tag{41}$$

Then, rewrite Eqs. (36)–(38) to first order in perturbed quantities and their derivatives,

$$\dot{\gamma}\beta_{x0} + \gamma_0\delta\dot{\beta}_x + \delta(\gamma)\dot{\beta}_{x0} = \lambda\gamma_0\Omega_w\delta\beta_z \times \sin k_w z - \gamma_0\Omega_r(1-\beta_{z0})\cos\zeta, \tag{42}$$

$$\dot{\gamma}\beta_{y0} + \gamma_0\delta\dot{\beta}_y + \delta(\gamma)\dot{\beta}_{y0} = -\lambda\gamma_0\Omega_w\delta\beta_z \times \cos k_w z + \gamma_0\Omega_r(1-\beta_{z0})\sin\zeta, \tag{43}$$

$$\dot{\gamma}\beta_{z0} + \gamma_0\delta\dot{\beta}_z = \lambda\gamma_0\Omega_w\left(\delta\beta_y \cos k_w z - \delta\beta_x \times \sin k_w z\right) + \gamma_0\Omega_r\left(\beta_{y0}\sin\zeta - \beta_{x0}\cos\zeta\right), \tag{44}$$

where $\Omega_r = \dfrac{eE_r}{\gamma mc}$, $\gamma = \gamma_0 + \delta(\gamma)$, and

$$\dot{\gamma} = \delta(\dot{\gamma}) = \gamma_0^3\left(\beta_{x0}\delta\dot{\beta}_x + \beta_{y0}\delta\dot{\beta}_y + \beta_{z0}\delta\dot{\beta}_z + {} + \delta\beta_x\dot{\beta}_{x0} + \delta\beta_y\dot{\beta}_{y0}\right) \tag{45}$$

Now, multiply Eqs. (42) and (43) by β_{x0}/γ_0 and β_{y0}/γ_0, respectively, and add the results to deduce

$$\frac{\dot{\gamma}}{\gamma_0}\left(\beta_{xo}^2 + \beta_{yo}^2\right) + \beta_{x0}\delta\dot{\beta}_x + \beta_{y0}\delta\dot{\beta}_y = -\Omega_r\beta_w \times (1-\beta_{z0})\cos(k_w z + \zeta). \tag{46}$$

The relations $\beta_{x0}\delta\dot{\beta}_x + \beta_{y0}\delta\dot{\beta}_y = \dot{\gamma}/\gamma_0^3 - \beta_{z0}\delta\dot{\beta}_z - \delta\beta_x\dot{\beta}_{x0} - \delta\beta_y\dot{\beta}_{y0}$, which can be obtained from Eq. (45), and $\beta_{x0}^2 + \beta_{y0}^2 = 1 - \beta_{z0}^2 - 1/\gamma_0^2$, may be used in Eq. (46) to obtain

$$\frac{\dot{\gamma}}{\gamma_0}\left(1-\beta_{z0}^2\right) - \left(\delta\beta_x \sin k_w z - \delta\beta_y \cos k_w z\right) \times ck_w\beta_w\beta_{z0} - \beta_{z0}\delta\dot{\beta}_z = -\Omega_r\beta_w\cos(k_w z + \zeta) \times (1-\beta_{z0}), \tag{47}$$

Now the term $\left(\delta\beta_y \cos k_w z - \delta\beta_x \sin k_w z\right)$ may be eliminated between Eqs. (44) and (47) to obtain

$$\frac{\dot{\gamma}}{\gamma_0}\left[\lambda\Omega_w\left(1-\beta_{z0}^2\right) - ck_w\beta_w\beta_{z0}^2\right] - \delta\dot{\beta}_z\left(\lambda\Omega_w \times \beta_{z0} + ck_w\beta_w\beta_{z0}\right) = -\Omega_r\beta_w\left[\lambda\Omega_w(1-\beta_{z0}) - ck_w\beta_w\beta_{z0}\right]\cos(k_w z + \zeta). \tag{48}$$

This equation is an important result which relates $\dot{\gamma}$ to $\delta\dot{\beta}_z$ in the presence of radiation. Now a relation for $\dot{\gamma}$ can be derived as follows:

$$\dot{\gamma} = -\frac{e}{mc}(\boldsymbol{\beta}\cdot\mathbf{E}_r) = -\frac{eE_r}{mc}\beta_w\cos(k_w z + \zeta) = -\gamma_0\Omega_r\beta_w\cos(k_w z + \zeta). \tag{49}$$

Substitution of this value in Eq. (48) with some algebra will deduce $\dot{\beta}_z$ as

$$\dot{\beta}_z = \delta\dot{\beta}_z = -\Omega_r\beta_w\cos(k_w z + \zeta). \tag{50}$$

Using a first-order approximation for $z = v_\parallel t = c\beta_\parallel t$, Eqs. (49) and (50) may be written in the following form:

$$\dot{\gamma} = -\gamma_0\Omega_r\beta_w\cos(\Omega t + \phi), \tag{51}$$

$$\dot{\beta}_z = \delta\dot{\beta}_z = -\Omega_r\beta_w\cos(\Omega t + \phi), \tag{52}$$

where $\Omega = (k_r + k_w)v_\parallel - \omega_r$.

The phase ϕ in the above equation determines the initial position of the electron relative to the optical wave. Averaging $\dot{\gamma}$ over all phases yields

$$<\dot{\gamma}>_{\phi}=\frac{1}{2\pi}\int_{0}^{2\pi}\dot{\gamma}\,d\phi=0\,. \tag{53}$$

Therefore, to first order there is no net transfer of power between the electron beam and the optical wave. The second-order correction will consist of accounting for the fact that as an individual electron (with a phase ϕ) gains or loses in energy, its position relative to the unperturbed position $(z=c\beta_{\|}t)$ is advanced or retarded. Therefore, the unperturbed position, $z=c\beta_{\|}t$, must be replaced by

$$z(t)=c\beta_{\|}t+c\int_{0}^{t}\Delta\beta_{z}(t')dt' \tag{54}$$

where $\Delta\beta_{z}(t)=\int_{0}^{t}\dot{\beta}_{z}(t')dt'$ is the change of β_{z} relative to the unperturbed state. The substitution of $\dot{\beta}_{z}$ in Eq. (54) will yield

$$z(t)=c\beta_{\|}t+\frac{cD}{\Omega}\left[\Omega t\sin\phi+\cos(\Omega t+\phi)-\cos\phi\right] \tag{55}$$

where $D=\Omega_{r}\beta_{w}(1-\beta_{z0})/\Omega$. Substitution of Eq. (55) in Eq. (49) will yield

$$\dot{\gamma}=-\Omega_{r}\gamma_{0}\beta_{w}\cos\{(\Omega t+\phi)+(\omega_{r}+\omega_{w})D\Omega^{-1}\times \\ [\Omega t\sin\phi+\cos(\Omega t+\phi)-\cos\phi]\} \tag{56}$$

where $\omega_{w}=k_{w}c$. Comparison of Eq. (56) with Eq. (51) reveals that the perturbed state consists of a phase slippage,

$$\Delta\phi=(\omega_{w}+\omega_{r})D\Omega^{-1}[\Omega t\sin\phi+\cos(\Omega t+\phi)- \\ \cos\phi], \tag{57}$$

Since D is proportional to γ^{-1} and E_{r}, $\Delta\phi$ can be made arbitrarily small. Therefore, expanding the cosine term in Eq. (55) for $\Delta\phi<<\pi$ leads to

$$\dot{\gamma}=-\Omega_{r}\gamma_{0}\beta_{w}\{\cos(\Omega t+\phi)-\sin(\Omega t+\phi)(\omega_{r}+ \\ \omega_{w})\Omega^{-1}[\Omega t\sin\phi+\cos(\Omega t+\phi)-\cos\phi]\}. \tag{58}$$

Averaging over phase ϕ yields

$$\langle\dot{\gamma}\rangle_{\phi}=\Omega_{r}\gamma_{0}\beta_{w}\frac{(\omega_{r}+\omega_{w})}{2}(\Omega t\cos\Omega t-\sin\Omega t) \tag{59}$$

Now, integrating Eq. (59) over the electron transit time through the wiggler interaction length yields the average change in γ per electron:

$$<\Delta\gamma>_{\phi}=\int_{0}^{T=L/c\beta_{z}}<\dot{\gamma}>_{\phi}dt= \\ -\Omega_{r}\gamma_{0}\beta_{w}\frac{(\omega_{r}+\omega_{w})D\Omega T^{3}}{2}g(\Omega T), \tag{60}$$

where

$$g(\Omega T)=\frac{(2-2\cos\Omega T-\Omega T\sin\Omega T)}{\Omega^{3}T^{3}}, \tag{61}$$

and where L is the FEL interaction length.

The change in radiation power in one transit is

$$\Delta P=-\frac{I}{e}mc^{2}<\Delta\gamma>_{\phi}, \tag{62}$$

where $I=n_{b}ev_{\|}\pi r_{b}^{2}$ is the average electron beam current. By using Eqs. (60) and (62), under the assumption that the electrons are near resonance with the wave, i.e., $\Omega=(k_{r}+k_{w})c\beta_{\|}-\omega_{r}\approx 0$, the gain equation in the presence of the self-fields can be rewritten as

$$G_{s}=\frac{\Delta P}{P}=\omega_{b}^{2}\omega_{w}\left(\frac{L}{c\beta_{\|}}\right)^{3}\beta_{w}^{2}\beta_{\|}\,g(\Omega T), \tag{63}$$

where $P\equiv c\pi r_{b}^{2}(E_{r}^{2}/4\pi)$.

In the limit eliminating self-fields, the maximum gain Eq. (63) reduces to

$$G_{s0}=\omega_{b}^{2}\omega_{w}\left(\frac{L}{c\beta_{\|0}}\right)^{3}\beta_{w0}^{2}\beta_{\|0}\,g(\Omega T), \tag{64}$$

where β_{w0} and $\beta_{\|0}$ are the wiggler-induced transverse velocity and the axial velocity in the absence self-fields, respectively. If ω_i and Ω_0 are both set equal to zero (eliminating the ion-channel and the axial magnetic field), Eq. (63) reduces to

$$G_0 = \omega_b^2 \omega_w \left(\frac{L}{c\beta_0} \right)^3 a_w^2 \beta_0 g(\Omega T) , \tag{65}$$

where $a_w = -eB_w / \gamma m k_w c^2$ and $\beta_0 = (1 - a_w^2 - \gamma^{-2})^{1/2}$.

III. Numerical Results

A numerical study of the self-fields effects on gain in a helical wiggler free-electron laser with axial magnetic field and ion-channel guiding have been made. The parameters that are used in this section are $k_w = 3.14 cm^{-1}$, $n_b = 8 \times 10^{12} cm^{-3}$, and $B_w = 1kG$. The Lorentz relativistic factor γ was taken to be 30, and the normalized axial velocity $\beta_\parallel$ was taken to be close to 1. The FEL interaction length L was taken to be $100\lambda_w$.

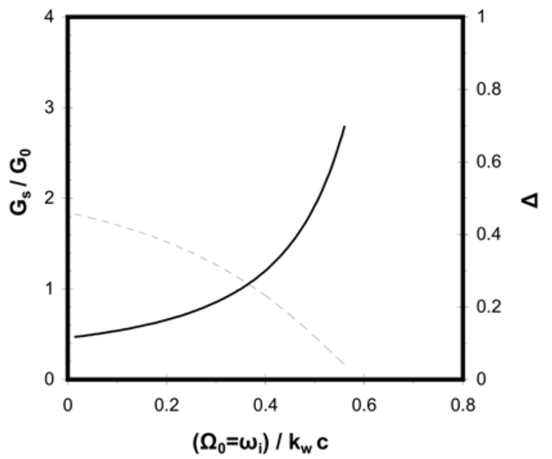

Fig. 1 The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) vs. the normalized axial magnetic field frequency that is equal to the ion-channel frequency, for group I orbits. The parameters are $k_w = 3.14 cm^{-1}$, $n_b = 8 \times 10^{12} cm^{-3}$, and $B_w = 1kG$.

Figure 1 shows the variation of the normalized gain (G_s/G_0) (solid lines) and value of the ratio of gain in the presence of the self-fields to the gain in the absence of the self-fields $\Delta(\equiv G_s/G_{s0})$ (dotted lines) versus either normalized guiding frequency with the two taken to be equal $(\omega_i/k_w c = \Omega_0/k_w c)$, for group I orbits. Fig. 1 shows that the normalized gain increases with increase the normalized guiding frequency. As shown in Fig. 1, Δ is less than 1 and therefore the gain decrement is obtained due to the self-fields. Figure 2 shows the variation of the normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) versus either normalized guiding frequency with the two taken to be equal $(\omega_i/k_w c = \Omega_0/k_w c)$, for group II orbits.

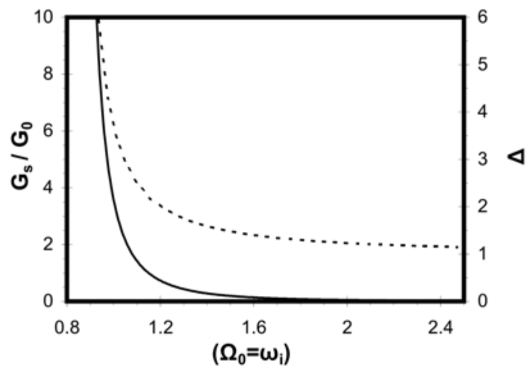

Fig. 2 The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) vs. the normalized axial magnetic field frequency that is equal to the ion-channel frequency, for group II orbits. The parameters are $k_w = 3.14 cm^{-1}$, $n_b = 8 \times 10^{12} cm^{-3}$, and $B_w = 1kG$.

Fig. 2 shows that the normalized gain decreases with increase the normalized guiding frequency. For group II orbits if $(\omega_i/k_w c = \Omega_0/k_w c) < 1$, the condition $\beta_\parallel$ close to 1, as required for derivation of the gain equation, breaks down. For group II orbits because Δ is greater than 1, the gain enhancement is obtained due to the self-fields.

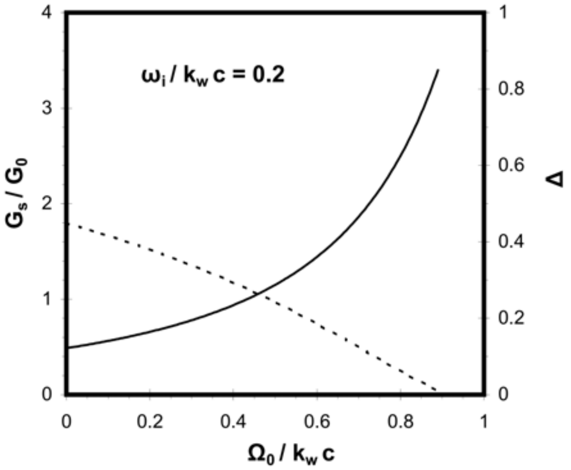

Figure 3. The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) vs. the normalized axial magnetic field frequency $\Omega_0/k_w c$ in the presence of an ion-channel, with $\omega_i/k_w c = 0.2$, for group I orbits. The parameters are $k_w = 3.14\,cm^{-1}$, $n_b = 8\times10^{12}\,cm^{-3}$, and $B_w = 1kG$.

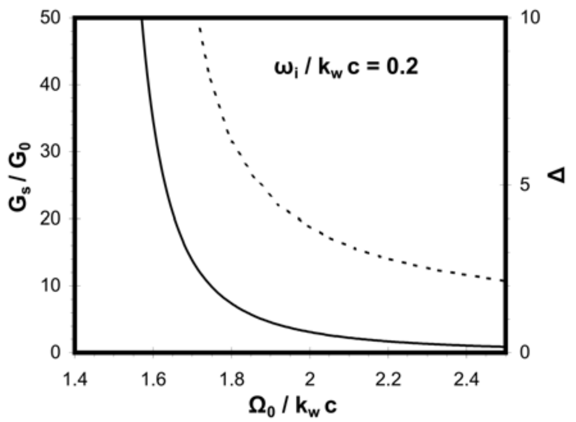

Figure 4. The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) vs. the normalized axial magnetic field frequency $\Omega_0/k_w c$ in the presence of an ion-channel, with $\omega_i/k_w c = 0.2$, for group II orbits. The parameters are $k_w = 3.14\,cm^{-1}$, $n_b = 8\times10^{12}\,cm^{-3}$, and $B_w = 1kG$.

Fig. 3 shows the normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) versus the normalized axial magnetic field frequency $\Omega_0/k_w c$ in the presence of an ion-channel when it is held constant with $\omega_i/k_w c = 0.2$, for group I orbits. Fig. 3 shows that Δ is less than 1 and the gain decreases with considering the self-fields. The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) versus the normalized axial magnetic field frequency $\Omega_0/k_w c$ in the presence of an ion-channel when it is held constant with $\omega_i/k_w c = 0.2$, for group II orbits, are shown in Fig. 4.

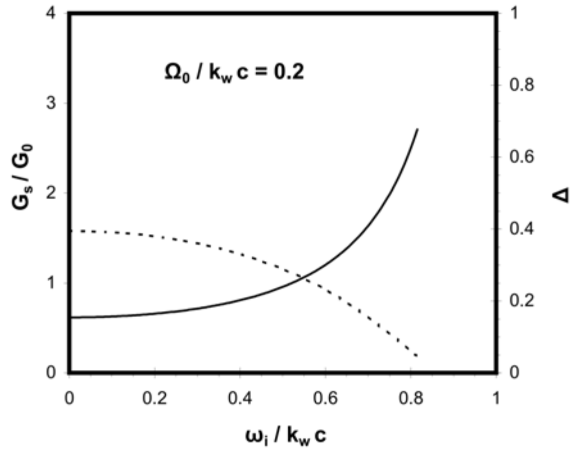

Figure 5. The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) vs. the normalized ion-channel frequency $\omega_i/k_w c$ in the presence of an axial magnetic field, with $\Omega_0/k_w c = 0.2$, for group I orbits. The parameters are $k_w = 3.14\,cm^{-1}$, $n_b = 8\times10^{12}\,cm^{-3}$, and $B_w = 1kG$.

Fig. 5 shows the normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) versus the normalized ion-channel frequency $\omega_i/k_w c$ in the presence of an axial magnetic field when it is held constant with $\Omega_0/k_w c = 0.2$, for group I orbits. For group II orbits, the normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) versus the normalized ion-channel frequency $\omega_i/k_w c$ in the presence of an axial magnetic field when it is held constant with $\Omega_0/k_w c = 0.2$, are shown in Fig. 6. It should be noted that the foregoing result has been shown that the Budker condition eliminates the group II orbit in a FEL with ion-channel guiding. While, when both the ion-channel and the axial magnetic field are present, there are two groups of electron trajectories, because electron beam will be guided by the axial magnetic field.

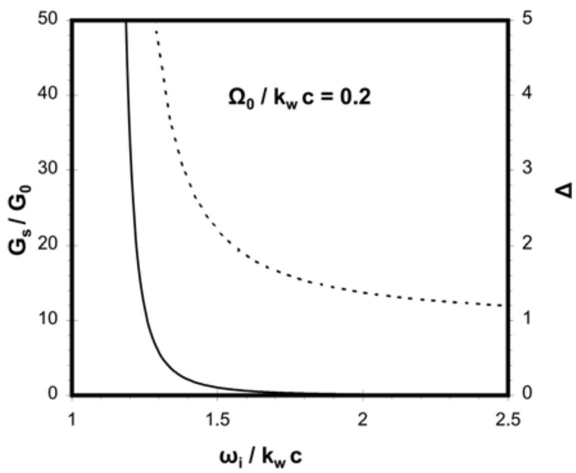

Figure 6. The normalized gain (G_s/G_0) (solid lines) and the gain ratio $\Delta(\equiv G_s/G_{s0})$ (dotted lines) vs. the normalized ion-channel frequency $\omega_i/k_w c$ in the presence of an axial magnetic field, with $\Omega_0/k_w c = 0.2$, for group II orbits. The parameters are $k_w = 3.14\,cm^{-1}$, $n_b = 8\times10^{12}\,cm^{-3}$, and $B_w = 1\,kG$.

IV. CONCLUSION

In this work, the effects of self-fields on gain in a helical wiggler free electron laser with axial magnetic field and ion-channel guiding is investigated. The self-electric field is derived from Poisson's equation and the self-magnetic field is obtained from Ampere's law by a self-consistent method. A detailed analysis of electron interaction with radiation field in the presence of self-fields is presented.

It should be noted that the wiggler-induced self-magnetic field decreases the effective wiggler magnetic field for group I orbits, whereas for group II orbits it increases the effective wiggler magnetic field. Therefore, we expect that in the presence of self-fields the gain decreases for group I orbits and increases for group II orbits.

The results of the present one-dimensional analysis are valid for $v_w/v_\parallel \ll 1$. When v_w is large the radial variation of the wiggler field can no longer be neglected and the present results are not valid. The radial variation of the wiggler field is also negligible for thin beams for which the beam radius is much smaller than the wiggler wavelength. It should be noted that in the limiting case $\omega_i \to 0$, our results are in agreement with Ref. 19, which investigates gain in a one-dimensional FEL with axial magnetic field. In the limiting case $\Omega_0 \to 0$, our results are not in perfect agreement with Ref. 20 because, in Ref. 20 a more realistic (three-dimensional) helical wiggler is employed.

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Hasan Ehsani Amri received his PhD degree from the Science and Research Branch of Islamic Azad University in February 2011. He is now with the Physics Department of the Islamic Azad University of Nour Branch as an assistant professor. His research areas are laser spectroscopy and photonics and free electron laser.

Taghi Mohsenpour received his PhD degree from the Amirkabir University of Technology in February 2010. He is now with the Physics Department of the University of Mazandaran as an assistant professor. His research areas are free electron laser and quantum plasmas.

Scope

Original contributions relating to advances, or state-of-the-art capabilities in the theory, design, applications, fabrication, performance, and characterization of: Lasers and optical devices; Laser Spectroscopy; Lightwave communication systems and subsystems; Nanophotonics; Nonlinear Optics; Optical Based Measurements; Optical Fiber and waveguide technologies; Optical Imaging; Optical Materials; Optical Signal Processing; Photonic crystals; and Quantum optics, and any other related topics are welcomed.

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