Impact of Fourth-Order Dispersion Coefficient on the Gain Spectrum and the Saturation Behavior of One-Pump Fiber Optical Parametric Amplifiers

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ABSTRACT— **In this paper, the gain spectrum and the saturation behavior of one-pump fiber optical parametric amplifiers (1-P FOPAs) are investigated by taking into account the fourth-order dispersion coefficient β_4 in the analysis. The results show that it is necessary to consider β_4 in the analysis when the wavelength difference between the signal and pump waves is large enough and/or whenever the pump wavelength approaches to the zero-dispersion wavelength (ZDW) of the fiber. Also, it is shown that by increasing the value of β_4, the gain value is increased and the saturation power is decreased. Finally, the simulation results are compared with the available experimental data and a very good agreement is obtained.**

KEYWORDS: Parametric amplification, optical fiber, gain spectrum, saturation regime, fourth-order dispersion coefficient.

I. INTRODUCTION

Parametric amplification which is based on the four-wave mixing (FWM) process in optical fibers can be utilized to develop a new type of optical amplifiers, known as fiber optical parametric amplifiers (FOPAs) [1, 2]. As it is shown in Fig. 1, in one-pump (1-P) FOPAs two pump photons at the same angular frequency of ω_p are combined together with the signal wave via a fiber coupler (FC) at the fiber input [3]. At the fiber output, the signal wave at the angular frequency of ω_s is amplified and a new frequency component, the so-called idler is generated at the angular frequency of ω_i; so that the energy is conserved among the interacting photons as $2\omega_p = \omega_s + \omega_i$. Finally, all the interacting waves can be monitored at the fiber output using an optical spectrum analyzer (OSA). When the pump wave is strong enough and the phase-matching condition which is the main feature of parametric processes is fulfilled, a significant gain is obtained from the FOPA [4].

One of the main advantages of FOPAs over the Erbium-doped fiber amplifiers (EDFAs) is the arbitrary gain wavelength so that the gain can be easily centered on the desirable wavelength, provided that the optical fiber has a zero-dispersion wavelength (ZDW) near the pump wavelength λ_p [2]. This feature has an important application for generating parametric gain over the wavelength ranges where the commercial amplifiers do not exist. In addition, FOPAs have attracted great interest owing to their great potential in various areas of science and technology such as wavelength conversion, signal processing, short pulse generation, noise suppression, etc. [5-10].

The FOPA gain depends on several parameters such as the fiber dispersion and the fiber loss α. In many works, only the dispersion up to the third order (β_3) has been included in the simulations and in a few works, the forth-order dispersion coefficient (β_4) is considered but for the small-signal approximation and a lossless fiber (α=0) [11, 12]. It should be noted that the small-signal approximation is only applicable for very weak signals and it does not hold for the saturation regime where the signal power is high compared to the small signal regime. In this paper however, we investigate the gain spectrum and the saturation behavior of the FOPAs by taking into the account both β_4 and α and improve the accuracy of the modeling of FOPAs. Also, we verify our model by comparing its results with the available experimental data in the field.

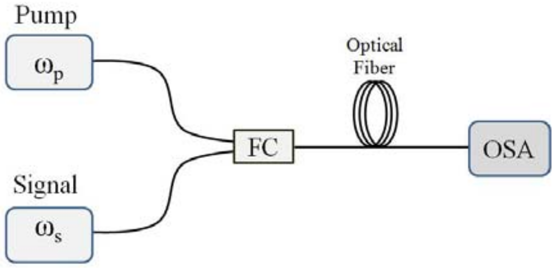

Fig. 1 Schematic illustration of the one-pump fiber optical parametric amplifier (1-P FOPA). FC is fiber coupler and OSA is optical spectrum analyzer.

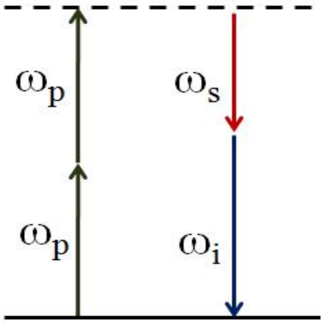

Fig. 2 Schematic illustration of the parametric amplification from the quantum mechanics point of view. ω_p is the pump frequency and ω_s and ω_i are the signal and the idler frequencies, respectively.

This paper is organized as follows. In Section II, the theory of 1-P FOPAs is presented where the dispersion coefficients and the coupled amplitude equations are introduced. Then, the simulation results including the gain spectra and the saturation curves of 1-P FOPAs are given in Section III for different values of β_4 and the role of pump wavelength on the gain spectrum and its bandwidth is discussed. Furthermore, the simulation results are verified by comparing with the available experimental data. Finally, the paper is concluded in Section IV.

II. DISPERSION COEFFICIENTS AND COUPLED AMPLITUDE EQUATIONS

As it is shown in Fig. 2, parametric amplification can be viewed from quantum mechanical picture. Here, the degenerated parametric amplification leads to the annihilation of two pump photons at frequency ω_p and the creation of two photons at the signal frequency ω_s and the idler frequency ω_i. The linear wave-vector mismatch between the interacting waves: pump, signal and idler, can be written as [4]:

$$\Delta\beta = \beta(\omega_s) + \beta(\omega_i) - 2\beta(\omega_p) \tag{1}$$

where $\beta(\omega_l)$ is the mode propagation constant of each wave calculated at its center frequency ω_l $(l = s, i\, and\, p)$. β can be expanded in a Taylor series about the zero-dispersion frequency of the fiber ω_0 as follow [4]:

$$\begin{aligned}\beta(\omega) = \beta_0 + (\omega-\omega_0)\beta_1 + \frac{1}{2!}(\omega-\omega_0)^2\beta_2 \\ + \frac{1}{3!}(\omega-\omega_0)^3\beta_3 + \frac{1}{4!}(\omega-\omega_0)^4\beta_4\end{aligned} \tag{2}$$

where β_1, β_2, β_3, and β_4 are the first-, second-, third-, and the fourth-order dispersion coefficients of the fiber calculated at ω_0. These dispersion coefficients which depend on the fiber design can be calculated through the dispersion curve of the fiber provided by fiber manufacturers [4]. In many works the dispersion coefficients up to the third order are considered in the Taylor expansion series. Here however, we take into account the fourth-order dispersion coefficient to improve the accuracy of the simulations. From Eqs. (1) and (2) one obtains the following expression for the linear wave-vector mismatch as [13]:

$$\Delta\beta = -\left\{ \beta_3(\omega_p - \omega_0) + \frac{\beta_4}{2}\left[(\omega_p - \omega_0)^2 \times (\omega_p - \omega_s)^2 + \frac{1}{6}(\omega_p - \omega_s)^2 \right] \right\} \tag{3}$$

The coupled amplitude equations which describe the propagation of the three interacting waves in 1-P FOPA are given by [1-3]:

$$\frac{\partial A_P}{\partial z} = i\gamma\left(|A_P|^2 + 2|A_s|^2 + 2|A_i|^2 \right)A_P + 2i\gamma\, A_s A_i A_p^* \exp(i\Delta\beta\, z) - \frac{1}{2}\alpha A_P \tag{4}$$

$$\frac{\partial A_s}{\partial z} = i\gamma\left(|A_s|^2 + 2|A_i|^2 + 2|A_P|^2 \right)A_s + i\gamma\, A_i^* A_p^2 \exp(-i\Delta\beta\, z) - \frac{1}{2}\alpha A_s \tag{5}$$

$$\frac{\partial A_i}{\partial z} = i\gamma\left(|A_i|^2 + 2|A_s|^2 + 2|A_P|^2 \right)A_i + i\gamma\, A_s^* A_p^2 \exp(-i\Delta\beta\, z) - \frac{1}{2}\alpha A_i \tag{6}$$

where A_p, A_s, and A_i are the pump, signal, and idler amplitudes, respectively; and α is the fiber loss and γ is the nonlinear parameter. Sign * represents the complex conjugate and the linear wave vector mismatch $\Delta\beta$ is given by (3).

III. RESULTS AND DISCUSSION

The FOPA gain (in dB units) is defined as $G = 10\log\left(\frac{P_{s,\ out}}{P_{s,\ in}} \right)$, where $P_{s,in}$ and $P_{s,out}$ are the signal powers at the fiber input and output, respectively. The output power of each wave is determined when Eqs. (4)-(6) are solved numerically, for example using the Runge-Kutta method, with the known powers at the fiber input. Simulation parameters adopted from [14] are as follows. A 160-m long highly nonlinear fiber (HNLF) with a nonlinear parameter of $\gamma = 13.5 W^{-1}km^{-1}$ and ZDW of $\lambda_0 = 1563.2 nm$. The dispersion coefficients of the fiber calculated at λ_0 are respectively $\beta_3 = 0.038\ ps^3/km$ and $\beta_4 = 2.3\times10^{-5}\ ps^4/km$. The fiber loss is assumed to be $\alpha = 0.5 dB/km$ and the pump wavelength is $\lambda_p = 1564.2 nm$.

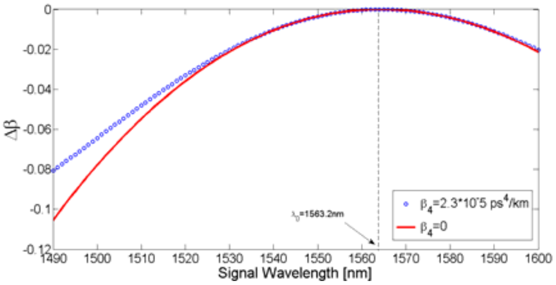

Fig. 3 Linear wave vector mismatch $\Delta\beta$ as a function of signal wavelength λ_s for a the fixed pump wavelength of $\lambda_p = 1564.2 nm$. The solid line is for the case when the forth-order dispersion coefficient is neglected ($\beta_4 = 0$) and the open circles are for the case when the forth-order dispersion coefficient is included $\beta_4 = 2.3\times10^{-5}\ ps^4/km$.

Fig. 3 shows the linear wave vector mismatch $\Delta\beta$ as a function of signal wavelength λ_s for a the fixed pump wavelength of $\lambda_p = 1564.2 nm$. As it can be seen, around the wavelength region of λ_0 ($1540 nm < \lambda_s < 1580 nm$), presence of β_4 has no effect on $\Delta\beta$ and both curves match with each other. This is because when the pump and the signal are close to each other, $(\omega_p - \omega_s)$ in Eq. (3) is small enough so that the related term of β_4 will be very small compared with that of β_3. On the other hand, when the signal wavelength deviates from the λ_0 the β_4 term in Eq. (3) becomes comparable to the β_3 term and therefore the difference between two curves increases. In other words, as it is evident from Fig. 3 the role of β_4 is important when the difference between the pump and the signal wavelengths are greater than 40 nm.

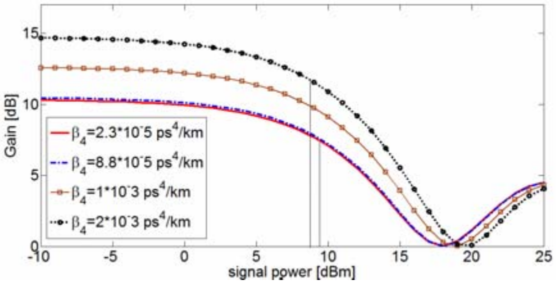

Fig. 4 Saturation curves of the 1-P FOPA for different values of β_4 and a fixed $\beta_3 = 0.038\ ps^3/km$. The pump power and the signal wavelength are 3W and 1550 nm, respectively. The vertical lines indicate the saturation power of each curve.

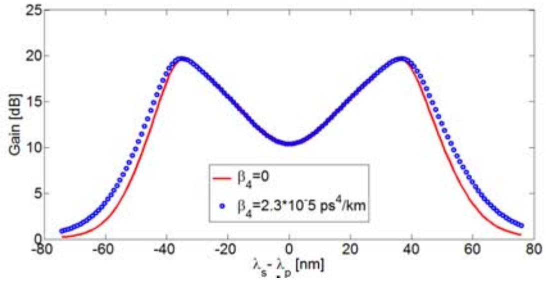

Fig. 5 Gain spectra of the 1-P FOPA for two cases, with (open circles) and without (solid line) including β_4. The signal power is $P_s = 12dBm$ which is located in the saturation regime.

The saturation curves of the FOPA are shown in Fig. 4 where the gain is depicted versus the input signal power for different values of β_4 and a fixed $\beta_3 = 0.038\ ps^3/km$. As it is shown, for a fixed pump power ($P_p = 3W$), as the input signal increases the parametric gain is gradually decreases so that at a specific power which is called the saturation power P_{sat}, the gain is reduced by 3 dB. Then, by further increase of signal power, the gain reduction is increased. The gain and the saturation power of the FOPA depends on the β_4 so that for higher values of β_4, the gain value is higher and the saturation power is lower. This is shown by vertical lines at which the saturation powers for the highest and lowest β_4 are indicated respectively as $P_{sat} = 9dBm$ and $P_{sat} = 9.5dBm$. This feature of the FOPA in the saturation regime has important applications in signal processing and noise suppression [8, 15 and 16].

In Fig. 5, gain spectra of the 1-P FOPA for two cases (with and without including β_4) are shown for a signal power of $P_s = 12dBm$ which located in the saturation regime (see Fig. 4). As it is seen, the presence of β_4 is not significant in the middle of the spectrum, whereas the situation is opposite on either sides of the spectrum where the difference between the signal and pump wavelengths is greater than 40 nm. This result is in agreement with that of Fig. 3.

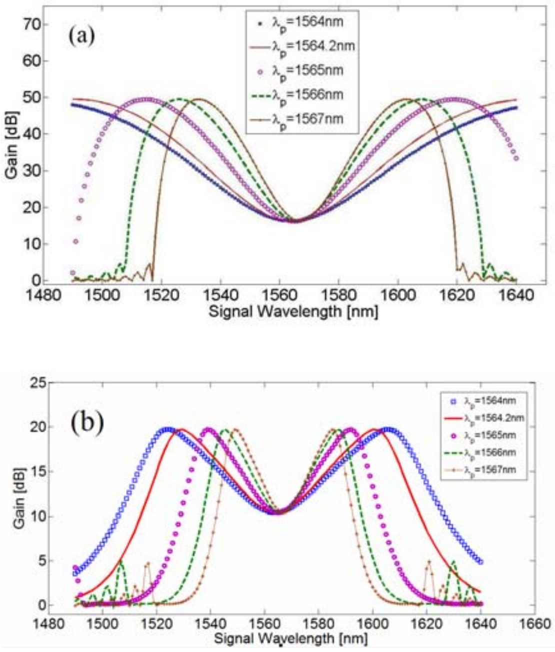

Fig. 6 Gain spectra of the 1-P FOPA for different pump wavelengths in (a) the small-signal regime with $P_s = -30dBm$ and in (b) the saturation regime with $P_s = 12dBm$. The fourth-order dispersion coefficient is $\beta_4 = 2.3\times10^{-5}\ ps^4/km$ and the ZDW of the fiber is $\lambda_0 = 1563.2nm$.

Fig. 6 shows the gain spectra of the 1-P FOPA in (a) the small-signal regime with $P_s = -30dBm$ and (b) the saturation regime with $P_s = 12dBm$ for different pump wavelengths, both in presence of the fourth-order dispersion coefficient $\beta_4 = 2.3\times10^{-5}\ ps^4/km$. As is shown, for a wider spectrum, the pump wavelength should be closer to ZDW which is $\lambda_0 = 1563.2nm$. As a result, for a FOPA operating in small-signal or saturation regime with a wide gain spectrum it is necessary to include β_4 in the analysis. In contrast, when λ_p is far from λ_0, the gain bandwidth is narrow and hence the inclusion of β_4 is not necessary.

To validate our model, we compare the simulation results with the available experimental data as well as the approximate model presented by M. E. Marhic *et al.* [17]. This is shown in Fig. 7 where the dashed line corresponds to our simulation results with including β_4 and the solid line refers to the approximate model of [17] without including β_4 and the fiber loss. Parameters of the FOPA are the same as those presented in [17], i.e., L=500m, $\gamma = 17W^{-1}km^{-1}$, $\lambda_0 = 1561.1nm$, $\beta_3 = 0.05\ ps^3/km$ and $\beta_4 = -5\times10^{-5}\ ps^4/km$. Pump and signal powers are 25 dBm and -30 dBm, respectively.

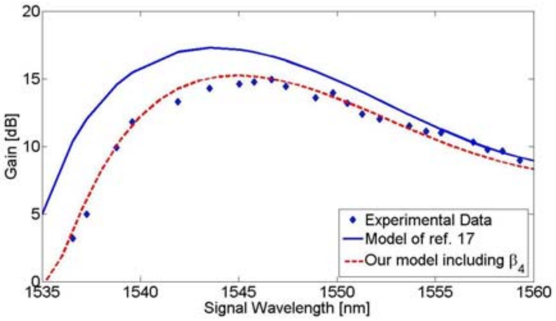

Fig. **7** Experimental gain spectrum of the 1-P FOPA together with two different simulation results. The solid line refers to the model of [17] without taking into account β_4, and the dashed line is based on our model which includes β_4. Experimental data are taken from [17].

As it is evident, our model has a very good agreement with the experimental results especially for signal wavelengths which are far from the pump wavelength of $\lambda_p = 1562.5nm$. Therefore, it is necessary to include fourth-order dispersion coefficient β_4 in the theory for exact modeling of FOPAs particularly for the regions which are far from the pump wavelength.

IV. CONCLUSION

We have investigated the gain spectrum and the saturation behavior of one-pump fiber optical parametric amplifiers (1-P FOPAs) by taking into account the fourth-order dispersion coefficient β_4 in the coupled amplitude equations. It was shown numerically and verified experimentally that it is necessary to include β_4 for either sides of the gain spectrum where the difference between the signal and pump wavelengths is large enough, e.g., larger than 40 nm. In contrast, for the middle region of the gain spectrum where the pump and signal wavelengths are close to each other, the role of β_4 is negligible. In addition, it was shown that when the pump wavelength is very close to the zero-dispersion wavelength (ZDW) of the fiber, the FOPA exhibits a wide gain spectrum where β_4 may play a significant role. Also, in the saturation regime, where the input signal power is high enough, a larger value of the β_4 leads to a higher gain and a lower saturation power. This feature of the FOPA in the saturation regime has important applications in signal processing and noise suppression.

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