

Determination of Refractive Index and Confinement Losses in Photonic Crystal Fibers Using FDFD Method: A Comparative Analysis

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Abstract— In this paper, we present a comparative numerical analysis to determine the refractive index of photonic crystal fibers (PCFs) by using FDFD method and used the results to evaluate the confinement losses of PCFs by considering the effects of air-hole rings in the cladding.

It is shown that by increasing the wavelength, the imaginary part of refraction index rises, resulting in increase of confinement losses nearly by order of 10. In lower wavelengths over the range of 0.2 to 1 μm , these losses were shown to be negligible. The obtained results show that as the number of air-hole ring in the cladding increases, the confinement losses over wavelengths would reduce. To show the effect of air-hole rings on confinement losses in PCFs, the FDFD method yielded accurate results that agree well with results of FEM method and source-model technique reported by others.

KEYWORDS: air-hole rings, confinement loss, finite difference frequency domain, refraction index, photonic crystal fibers

I. INTRODUCTION

Photonic crystal fibers (PCFs) are all-silica fibers that guide light by means of air-holes placed in the entire fiber length in the cladding [1]. The propagating modes of such fibers are leaky because the core refractive index is the same as the index beyond the finite cladding region. In addition to usual loss mechanisms as that of conventional fibers, there is loss mechanism peculiar to PCFs, known as confinement loss, which is due to presence of air-holes in their cladding [2-7]. The cladding of a PCF is usually comprised of hexagonally-packed rings of holes, and when the air-hole

spacing Λ becomes comparable to the wavelength, several rings of holes are required to reduce the confinement loss to a practical value [8-9]. The presence of air-holes in the cladding causes the average refractive index in this region to reduce. Therefore, in order to optimize the design of PCFs for a minimum confinement loss, it is necessary to study the loss characteristics for an optimum PCF structure [10, 11].

Generally, for analysis of PCFs, due to their complex structures, numerical methods are used [11, 12]. To study the influence of number of air-hole rings on confinement loss of PCFs, we utilized fully vectorial finite-difference frequency domain (FDFD) method. In this method PCFs are considered with a finite cladding region of circular holes and full-vector modal calculations are performed. This method gives the complex propagation constant from which imaginary part is used for calculation of the confinement losses.

II. THE ANALYSIS

Various methods are utilized for analysis of photonic crystal fibers (PCFs). In all the solving methods, the Maxwell's equations are solved and the electric (E) and magnetic (H) fields are resolved into longitudinal components in Cartesian coordinate as follows:

$$\xi(x, y, z, t) = \xi_r(x, y) + \xi_z(x, y) \exp[-j(\omega t - \beta z)] \quad (1)$$

where β indicates the propagation constant and ξ is either E or H field. By replacing (1) in

Maxwell's equations, we obtain Helmholtz vectorial equations as:

$$\begin{cases} (\nabla_t^2 + k_0^2 n^2 - \beta^2) E_t = -\nabla_t (E_t \cdot \nabla_t \ln n^2) \\ (\nabla_t^2 + k_0^2 n^2 - \beta^2) H_t = (\nabla_t \times \mathbf{H}_t) \nabla \times \nabla_t \ln n^2 \\ (\nabla_t^2 + k_0^2 n^2 - \beta^2) E_z = j\beta E_t \cdot \nabla_t \ln n^2 \\ (\nabla_t^2 + k_0^2 n^2 - \beta^2) H_z = (\nabla_t H_z + j\beta H_t) \cdot \nabla_t \ln n^2 \end{cases} \quad (2)$$

The above equations convert into eigen value problems for transmitting components of E and H fields. When the relative refractive index difference between core and cladding is small, the right hand side of (2) is negligible and can be approximated to zero, which is scalar approximation. Therefore, all the equations in (2) convert into one equation for either E or H field [11].

One point to note is that if the relative refractive index between core and cladding of PCF is high, the scalar approximation can be used to predict the propagation characteristics. Thus, a fully vectorial method is required for the analysis of PCFs. In this category, plane wave method (PWM), localized function method (LFM), finite difference time domain (FDTD) method, finite difference frequency domain (FDFD) method, multipole method (MPM) are relatively accurate methods for the analysis of PCFs [11].

In PWM, the magnetic field is expanded into plane waves and the core of PCF is assumed a solid cylindrical structure surrounded by periodic air-holes placed infinitely in the cladding region. Since, practically, the numbers of air-holes in the cladding are limited; this method is not suitable for the analysis of PCF [12]. For solving waveguide problems, LFM is used numerically as well as vectorially. In this method, the orthogonal functions such as sinusoidal, Lagurre-Gaussian (for 1D waveguides), Hermite-Gaussian (for 2D waveguides), are utilized. By applying periodic air-holes condition, the unknown modes are approximated by localized modes and are resolved into plane wave components, resulting in eigen value equation [13].

The FEM method is a accurate tool for solving waveguides problems which is normally based on simple solution of Helmholtz equations in frequency domain by dividing air-holes into triple cells [11]. This method, in spite of being accurate, has a complex algorithm that is hard to solve. In FDM method, the Yee mesh is utilized for solving electromagnetic problems in either time domain (FDTD) or frequency domain (FDFD). In comparison with other methods, FDM is more accurate and simpler.

Another simple and less accurate method is effective index method (EIM) in which an averaged refractive index of cladding with periodic air-hole is considered and modeled as an equivalent conventional step index single-mode fiber. In this method, by assuming air-hole as a unit cell, the scalar wave equation is solved to obtain mode field [14].

In FDTD method, the eigen frequency is solved for the given propagation constant. By FDFD method, which is based on direct solution of either Maxwell's or Helmholtz's equations, the attenuation of PCF can be determined. This method does not require second differentiation and thus six field components can be determined separately.

A. FDFD method

To solve Maxwell's equations by FDFD method, we utilize Yee mesh, as shown in Fig. 1 [15]. Let the time dependence of the field be represented as $\exp[i(\beta z - \omega t)]$. Dividing E by $z_0 = \sqrt{\mu_0/\epsilon_0}$ in Maxwell's equations ($\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ and $\nabla \times \mathbf{H} = \epsilon \partial \mathbf{E}/\partial t$), we obtain:

$$\begin{cases} -ik_0 \epsilon E_x = \frac{\partial H_z}{\partial y} - i\beta H_y \\ -ik_0 \epsilon_r E_y = i\beta H_x - \frac{\partial H_z}{\partial x} \\ -ik_0 \epsilon_r E_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{cases} \quad (3)$$

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$$\begin{cases} -ik_0 \varepsilon E_x = \frac{\partial H_z}{\partial y} - i\beta H_y \\ -ik_0 \varepsilon_r E_y = i\beta H_x - \frac{\partial H_z}{\partial x} \\ -ik_0 \varepsilon_r E_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{cases} \quad (5)$$

$$\begin{cases} -ik_0 \varepsilon_{rx}(j,l)E_x(j,l) = \frac{H_z(j,l) - H_z(j,l-1)}{\Delta y} - i\beta H_y(j,l) \\ -ik_0 \varepsilon_{ry}(j,l)E_y(j,l) = \frac{H_z(j,l) - H_z(j,l-1)}{\Delta y} - i\beta H_y(j,l) \\ -ik_0 \varepsilon_{rz}(j,l)E_z(j,l) = \frac{H_y(j,l) - H_y(j-1,l)}{\Delta x} - \frac{H_x(j,l) - H_x(j,l-1)}{\Delta y} \end{cases} \quad (6)$$

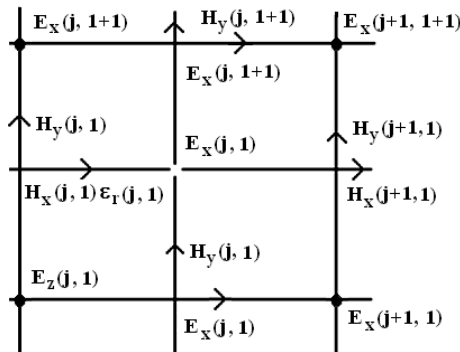


Fig. 1. Yee's mesh [15].

By using the following notations:

$$\begin{cases} \varepsilon_{rx}(j,l) = [\varepsilon_r(j,l) + \varepsilon_r(j,l-1)]/2 \\ \varepsilon_{ry}(j,l) = [\varepsilon_r(j,l) + \varepsilon_r(j,l) - \varepsilon_r(j-1,l)]/2 \\ \varepsilon_{rz}(j,l) = [\varepsilon_r(j,l) + \varepsilon_r(j-1,l-1) + \varepsilon_r(j,l-1) + \varepsilon_r(j-1,l)]/4 \end{cases} \quad (7)$$

the expressions (3) to (6) can be simplified as:

$$ik_0 \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} 0 & -i\beta I & U_y \\ i\beta I & 0 & -U_x \\ -U_y & U_x & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (8)$$

$$-ik_0 \begin{bmatrix} \varepsilon_{rx} & 0 & 0 \\ 0 & \varepsilon_{ry} & 0 \\ 0 & 0 & \varepsilon_{rz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 & -i\beta I & V_y \\ i\beta I & 0 & -V_x \\ -V_y & V_x & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \quad (9)$$

where I is a square unit matrix, and ε_{rx} , ε_{ry} , ε_{rz} are diagonal matrices determined by (7). The square matrices U_x , U_y , V_x , and V_y are evaluated under boundary condition. For example, if the values of window edge components in Yee's mesh tend to zero, the results are obtained as following matrices [15]:

$$U_x = \frac{1}{\Delta x} \begin{bmatrix} -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & \\ & & & & -1 & 1 \\ & & & & & -1 \end{bmatrix}$$

$$U_y = \frac{1}{\Delta y} \begin{bmatrix} -1 & & & & & \\ & -1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & 1 \\ & & & & & & -1 & & \\ & & & & & & & 1 \end{bmatrix} \quad (10)$$

$$V_x = \frac{1}{\Delta x} \begin{bmatrix} 1 & & & & & \\ -1 & 1 & & & & \\ & -1 & \ddots & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & \\ & & & & -1 & 1 \\ & & & & & -1 & 1 \end{bmatrix}$$

to 2 μm range. In Table 1, the values of effective refractive indices obtained by various methods are compared for $\Lambda=2.3 \mu\text{m}$ at $\lambda=1.55 \mu\text{m}$. Among the results of compared methods, the results of MPM and FDFD methods almost tally with each other.

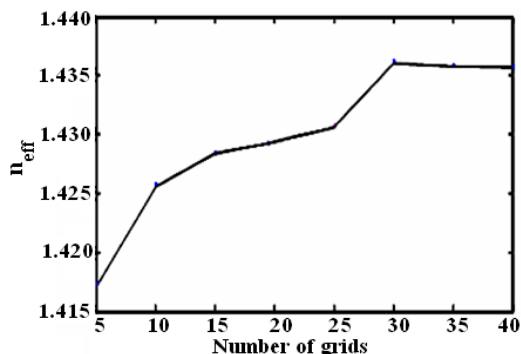


Fig. 3. Variation of effective refractive index vs. grid numbers.

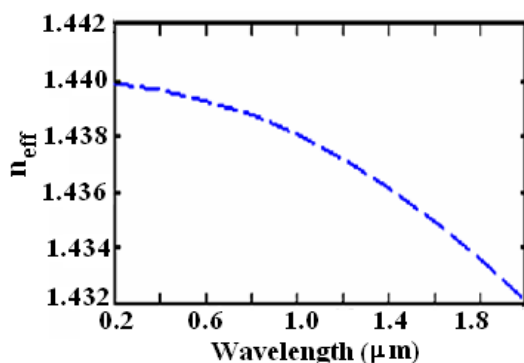


Fig. 4. Spectrum of the effective refractive index.

Table I. Comparison of effective indices obtained by different methods

Methods	FDFD	MPM	PWM	LFM	EIM
		[11]	[12]	[13]	[14]
n_{eff}	1.4354	1.4353	1.4270	1.4230	1.4286

In Fig. 5, we have shown the real and imaginary parts of the refractive index versus wavelength. By increasing the wavelength, the imaginary part of refractive index rises while the real part lowers. In other word, when the wavelength increases, more fraction of light energy is confined in the cladding, thus resulting in more propagation loss, which is termed as confinement losses.

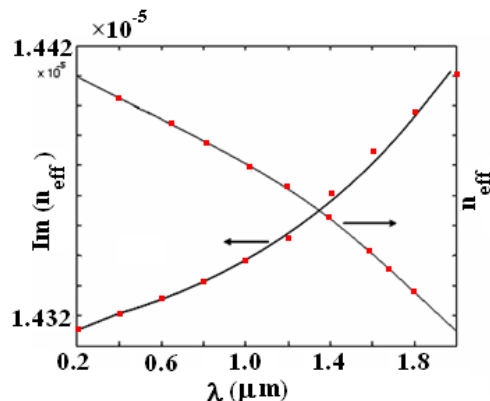


Fig. 5. Real and imaginary parts of the effective refractive indices.

In Fig. 6, the confinement losses for one ring of air-hole as a function of wavelength is depicted for $d/\Lambda=0.8$. In shorter wavelengths over the range of 0.2 to 1 μm , this loss is negligible, but above this range, confinement losses increase rapidly, by order of 10. This result agrees well with that of [18] and [19], where FEM method was used. In [18], the confinement losses of a PCF with one air-hole ring in its cladding for $\Lambda=2.3 \mu\text{m}$ at $\lambda=1.55 \mu\text{m}$, was of the order of 10^4 dB/m .

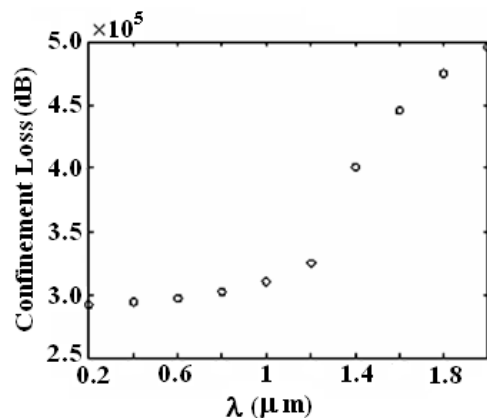


Fig. 6. Confinement losses spectrum for $d/\Lambda=0.8$.

By increasing the number of air-hole rings in the cladding of PCF, the confinement losses would decrease in such a way that it can be neglected in relative to other loss mechanisms in the PCF. In Fig. 7, we plotted the variations of confinement losses versus the ratio d/Λ for various number of air-holes in the cladding for $\Lambda=2.3 \mu\text{m}$ at $1.55 \mu\text{m}$. We observe that confinement losses have decreasing nature with respect to the numbers air-hole rings. The

reason for reduction of confinement losses is that when the number of air-holes in the cladding of PCF increases, the effective refractive index of cladding will decrease, making the relative index difference between core and cladding to increase. As a result, more light energy will concentrate into core region, which in turn causes the confinement loss to decrease. Under this condition, mode guiding in PCF is carried out by total internal reflection rather than photonic band gap phenomenon.

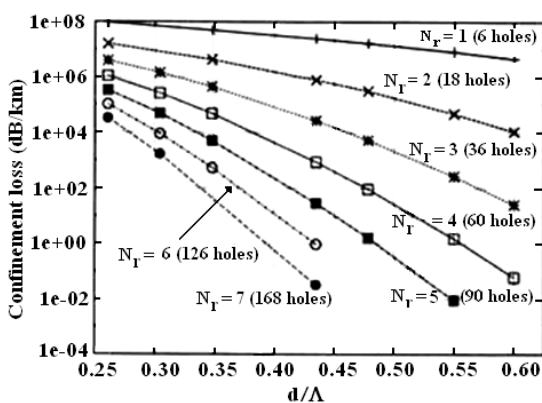


Fig. 7. Confinement loss versus d/Λ for different number of air-hole rings.

IV. CONCLUSION

A comparative numerical analysis, based on FDFD method, is presented to determine the refractive index of PCF and the obtained results are used to evaluate the confinement losses of PCFs by considering the effects of air-hole rings in the cladding.

Although in analytical effective index method sufficient design information can be obtained, the numerical methods are more accurate. The localized function method, which is based on direction solution of Maxwell's equations, the evaluation process leads to some complex integral equations. As well, the FEM method, in spite of being a accurate method, has the same problem complexity. The FDM method has the same accuracy as that of the FEM with a simpler mathematical operations. The frequency domain version of FDM, i.e., FDFD method has the advantage of direct solutions of Maxwell's or Helmholtz's equations to

determine the losses in PCF. Moreover, in FDFD method performing second differentiations is not required and by using sparse technique the calculation time can be reduced.

On confinement losses analysis of PCFs. it is shown that by increasing the wavelength, the imaginary part of refraction index rises, resulting in increase of confinement losses nearly by order of 10. In lower wavelengths over the range of 0.2 to 1 μm , these losses were shown to be negligible, but above this range, confinement losses increases rapidly.

The variation of confinement losses over wavelengths for different number of air-hole rings have shown that as the number of air-holes rings increases, the confinement losses would reduce. In analyzing the effect of air-hole rings on confinement losses in PCFs, the FDFD method yielded accurate results that agree well with results of FEM method and source-model technique reported by others.

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