The Study of Variation of Photon Intensity Inside Biological Phantom by Green Theorem

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ABSTRACT— The Image reconstruction is an important problem in optical tomography. The process of the image processing requires the study of photon migration in biological tissues. There are several approaches to study and simulate propagation of photons in biological tissues. These approaches are categorized into stochastic and analytical groups. The Monte Carlo method as a stochastic method is widely used to simulate the propagation of photons in biological tissues which is time consuming. The diffusion equation however, can also be employed to simulate the migration of photons in tissues. In this study, a solution to diffusion equation based on Green theorem is applied to study the diffusion of photon in biological tissues. The precision and accuracy of this presented method is evaluated by comparing with that of Monte Carlo method and that of experimental study.

KEYWORDS: Diffusion equation, Green theorem, Boundary Integral method, Biological phantom.

I. INTRODUCTION

Diffuse optical tomography (DOT) is a relatively new non-invasive imaging method in which images of the objects within turbid medium are derived based on fluence measurement of visible or near-infrared light on the surface of the object. It has potential applications in medical imaging, such as in breast cancer detection, monitoring of infant brain tissue oxygenation level and functional brain activation studies [1-3]. After 1995 Jiang *et al*. demonstrated the possibility of diffuse optical tomography (DOT), and estimated the value of absorption and scattering coefficient of biological phantoms [1, 4]. In this regard many reports have been devoted to evaluate DOT [4-11]. Arridge in 1997 presented a valuable review of modelling and image reconstruction techniques which is still a useful introduction to the basic theoretical concepts [12]. Progress since 1997 has largely focused on developing more realistic and efficient models of light transport in tissues, and on solving the ill-posed inverse problem in an increasingly rigorous way. In particular, it is becoming increasingly common to include prior information of the anatomy and optical properties of the tissue in both the modelling and the image reconstruction [13].

Three procedures are essential to reconstruct an optical image. First, the migration of diffused photons in biological tissues must be modelled. Second, this model that must be applied to provide the distribution of photons in the object under examination called the forward problem, allows the measurements to be simulated from the model, and generates a sensitivity matrix (the Jacobian of the forward mapping) in an explicit or implicit manner relating the measurements to the internal optical properties. Finally, the image is reconstructed by inverting the Jacobian and solving the inverse problem [13]. The image processing in DOT is a complicated inverse problem. There are no analytic methods for the solution of this problem [1-2].

The image processing in DOT is quite complicated [2, 13]. That is due to various interactions of penetrating photons with biological tissues including absorption,

scattering, and fluorescence. Moreover, the biological tissues have anisotropic properties. The radiative transfer equation (RTE) is widely utilized as an accurate model for these purposes[4]. Generally, there is no analytic solution to RTE except for special cases and the numerical solutions with sufficiently dense discretizations lead to computationally demanding problems. But two different methods have been introduced to solve this equation. A stochastic method called Monte Carlo method (MC) and an approximated method, namely diffusion equation, have been widely applied to solve RTE. Monte Carlo method is frequently used to model light transport in turbid media [6]. In this method a large number of photons is traced in the sample until they are either absorbed inside the sample or transmitted through it. Although Monte Carlo method (MC) has sufficient accuracy, it is time consuming and it can not lend itself to image reconstruction [14-16]. However in 2006, Palmer and Ramanujam reported a MC-based forward and inverse model for extraction of absorption and scattering coefficients of turbid media from diffuse reflectance spectroscopy measurements. The accuracy of the MC model based on the above measurements was estimated to be better than 10% [14].

Diffusion equation can be numerically solved by finite element method (FEM) and finite difference method (FDM). FEM and FDM are faster than MC but their accuracy is usually less than MC. In 1995, Schweiger *et al.* extended their work on applying FEM to the propagation of light in tissues. They addressed herein the topics of boundary conditions and source specification for this method. They demonstrated that a variety of boundary conditions stipulated on the RTF can be implemented in a FEM approach, as well as the specification of a light source by a Neumann condition rather than an isotropic point source. They compared the results for a number of different combinations of boundary and source conditions under FEM, as well as the corresponding cases in a Monte Carlo model [17].

Since, the computational time of reconstruction procedure is a key parameter in DOT, so MC is not suitable for DOT and hence the finite element method is usually applied for image reconstruction [18-21].

In addition, the Green theorem can also be used to convert diffusion equation into integral equation. This integral equation is defined on the surface of sample, so only the surface of sample is discretized leading to lower computational time. Therefore one can name two important advantages of this method over MC and FDM methods, namely lower computing times and comparable accuracies.

In this paper, reflection of and penetration to two different types of skins under incident beams of different wavelengths were calculated and the results were compared with those obtained using MC and FDM [22]. In this method, only the variation of photon intensity is used for imaging and other parameters such as coherency and polarization of photons are not used.

In this work an appropriate Green function for diffusion equation was introduced. Then, the differential form of the diffusion equation was converted to an integral equation by Green theorem. Finally, this integral equation was numerically solved. The method was adopted for simulating the migration of photons in breast tissue and the results was also verified by experiments. Moreover, the numerical results are also compared with those of MC method.

II. THEORETICAL BACKGROUND

The continuous-wave of most commonly used photon diffusion model can be expressed as [1, 23]:

$$D\nabla^2\varphi(\mathbf{r})+\mu_a\varphi(\mathbf{r})=S(\mathbf{r}) \qquad (1)$$

where, $\mathbf{r}$ denotes the tissue position, and $\varphi(\mathbf{r})$ is fluence rate defined as the energy flow per unit area per unit time. The parameter D is the diffusion coefficient that can be written as $D=1/3(\mu_a+\mu_s')$, where μ_a and

$\mu_s' = \mu_s(1-g)$ are absorption and reduced scattering coefficients, respectively. The anisotropic factor *g*, defined as $\langle\cos\theta\rangle$, has a value between -1 and 1. For most biological tissues, *g* is ~ 0.9 [24]. $S(\mathbf{r})$ is the isotropic source term at position $\mathbf{r}$.

It is very convenient to define an additional parameter, the optical albedo *a*, by:

$$a = \frac{\mu_s}{\mu_s + \mu_a} \tag{2}$$

The boundary condition owning to Fresnel reflections [24-26] is as follows:

$$\varphi(\vec{r}) - 2C_R D\frac{\partial\varphi(\mathbf{r})}{\partial n} = 0 \qquad \mathbf{r}\in\Gamma_s \tag{3}$$

in which $C_R = (1+R)/(1-R)$ where *R* can be estimated by Fresnel reflection coefficient. The appropriate Green function for the above mentioned Eq. 1 is [22]:

$$G(\mathbf{r},\mathbf{r}',\omega) = \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\exp\left(ik_x|\mathbf{r}-\mathbf{r}'|\right) \tag{4}$$

where $k_x = \sqrt{\frac{(\mu_a + i\,\omega/c)}{D}}$. For the case of continuous wave illumination, the modulation frequency ω equals zero. Applying boundary condition as in Eq. 3, and doing some mathematics, we obtain [21]:

$$\varphi(\mathbf{r}_{\partial\Omega}) = \int_\Omega G\;\tilde{S}\cdot d\mathbf{r} - \int_{\Gamma_s}\left(-G\frac{\partial\varphi}{\partial n} + \varphi\frac{\partial G}{\partial n}\right)d\mathbf{r}_s \tag{5}$$

where $\mathbf{r}_{\partial\Omega}$ is the observation point vector on the boundary Γ_s. Discretization of Γ_s called boundary element method (BEM), together with square elements and the fluence φ, and its normal derivative, $q = \partial\varphi/\partial n$, are approximated to:

$$\begin{cases}\varphi(\mathbf{r}) = \sum_{j=1}^{m} N_j(\mathbf{r})\varphi_j(\mathbf{r}) \\ q(\mathbf{r}) = \sum_{j=1}^{m} N_j(\mathbf{r})q_j(\mathbf{r})\end{cases} \tag{6}$$

where index *j* refer to the node j^{th} and $N_i(\mathbf{r}_k) = \delta_{ik}$ is Kronecker delta. If $\mathbf{r_k}$ spans all the nodes on the surface of the boundary, Eq. 1 can be modified as follows:

$$\begin{cases}H\psi + \Gamma\xi = \bar{S} \\ \xi = -R\psi + P\end{cases} \tag{7}$$

Here, $\psi_{1\times N}$, $\xi_{1\times N}$, and $\bar{S}_{1\times N}$ are column vectors containing the nodal values of the fluence φ, its normal derivative $q = \partial\varphi/\partial n$ and volume source *S*, respectively; and N is the number of nodes, and $R = C_R^{-1}$. The element of these vectors can be calculated as:

$$s_j = \int_\Omega G(\rho)\tilde{S}(\mathbf{r})\,d\mathbf{r} \tag{8}$$

and $\rho = |\mathbf{r}-\mathbf{r}'|$. The elements of matrices $H_{n\times n} = \{h_{i,j}\}$ and $\Gamma_{n\times n} = \{g_{i,j}\}$ are:

$$\begin{aligned} h_{i,j} &= \delta_{i,j}I + \int_{\partial\Omega}\frac{\partial G(\rho)}{\partial n}N_k(\mathbf{r})\,d\mathbf{r} \\ g_{i,j} &= -\int_{\partial\Omega} G(\rho)N_k(\mathbf{r})\,d\mathbf{r}\end{aligned} \tag{9}$$

By locating observation point on different nodes and using Eq. 6, and consideration of boundary condition, a system of equations are obtained giving the fluence at those surface points. By solving this set of equation, the fluence on the nodes can be found. To recover absorption and scattering coefficients, the measured data on the surface of sample is compared with results obtained by Green theorem method; this method is explained in [2].

III. RESULTS AND DOSCUSSION

As mentioned, the diffusion equation can also be solved by Green function method. In our

previous study, we have demonstrated that the computational time of Green function method is satisfactory when compared to FDM and MC methods [22]. Presented results show that computational time of MC and FDM are twenty-five times and four times longer than Green function method, respectively [22]. In addition, the accuracy of this method is better than FDM. So, in this study, we applied Green function method to simulate photon propagation in biological tissues.

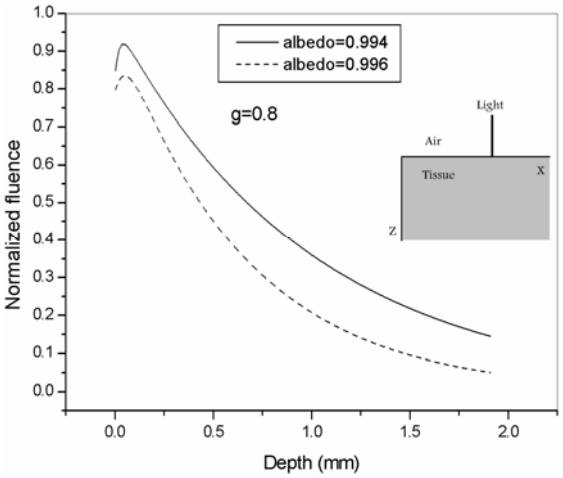


Fig. 1. The estimated fluence of penetrating photons inside biological phantom for different values of albedo.

In this study, first the ability of presented method is numerically evaluated, and then the precision of this method is evaluated by experimental method. The optical properties such as absorption and scattering coefficient can be estimated by DOT.

A. *Numerical Results*

To study the ability of presented method to simulate the photon migration inside biological tissues, the penetration of photon inside biological phantom with different albedo is studied (Fig. 1). The value of albedo of biological tissues is larger than 0.8, for example this value for aorta media, prostate and breast is 0.81, 0.996 and 0.998, respectively [3, 24]. Therefore, these values are important for cancer imaging. So, in this study, the penetration of light in the z direction for two different semi-infinite tissues with anisotropic factor of 0.8 and albedo of 0.994 and 0.996 is studied. The value of refractive index of tissue is equal to 1.4. One can see that the intensity of penetrating photons is decreased by increasing of depth. One can also see that the trend of numerical results is satisfied, because an increase in scattering coefficient results in a decrease in scattering length, $l_{scatt} = 1/\mu_s'$, so majority of photons are diffused in a short distance and therefore, the weight of transmitted photon at specified angle is reduced.

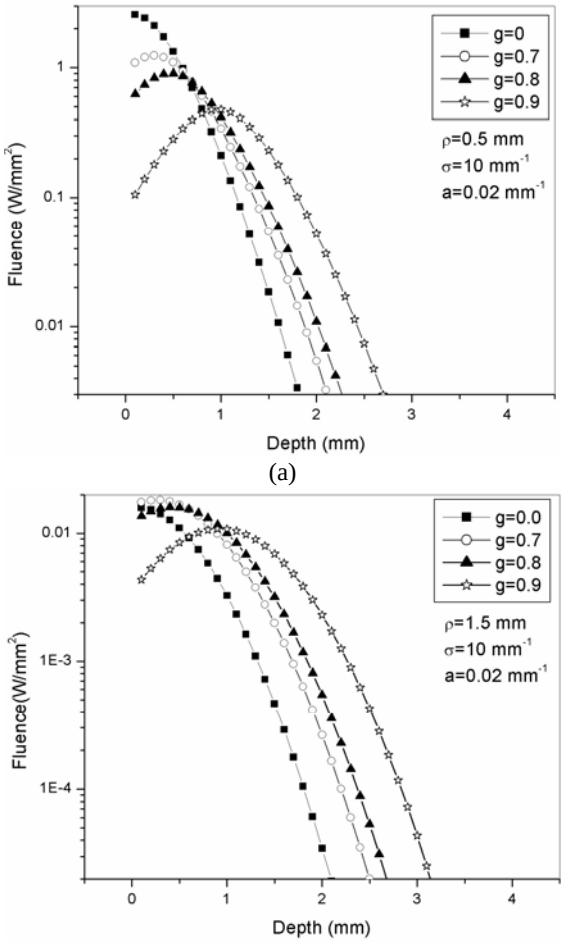


Fig. 2. The simulated fluence at different distance from location of illumination of laser for several value of anisotropic factor. (a) at distance of $\rho = 0.5\,\text{mm}$ and (b) $\rho = 1.5\,\text{mm}$

Fig. 1 depicts the fluence of photon penetrated in direction of illumination of light. Figs. 2 shows the variation of fluence of light for several values of anisotropic factors. As one can see from Figs. 2, an increase in anisotropic

factor results in a rise of scattering length, hence the diffused photon can travel further before first scattering event.

Previous figures show the penetration of light through biological tissue was studied. In DOT, the diffused reflectance is also important. So, the diffused reflectance from breast phantom for different wavelengths is simulated (Fig. 3). The values of optical properties are similar to those presented in [27-30]. The wavelength dependence of the maximum reflectance is a key point for image processing which must be considered in DOT. The wavelength dependence of optical properties of biological tissue results in a complicated process for image processing in DOT.

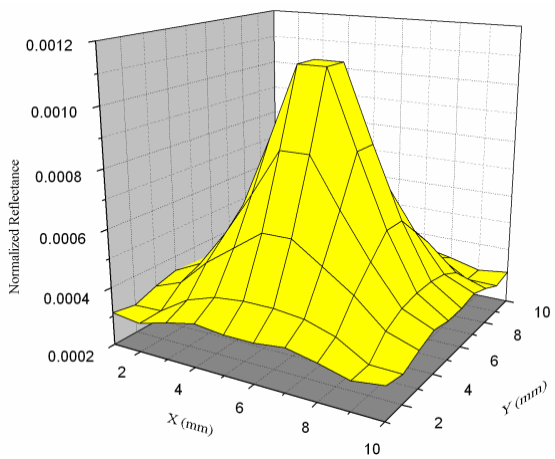
Table. 1. Comparison of normalized fluence obtained by green method, MC and experimental methods for different values of absorption coefficients [26-28].

Absorption coefficient (mm^{-1})	Green method	MC	Experimetal results
0.05	0.98	1.0	1.0
1.00	0.83	0.82	0.85
1.50	0.78	0.75	0.74
2.00	0.72	0.71	0.68
2.50	0.63	.61	0.60
3.00	0.61	0.58	0.50
3.50	0.51	0.55	0.42

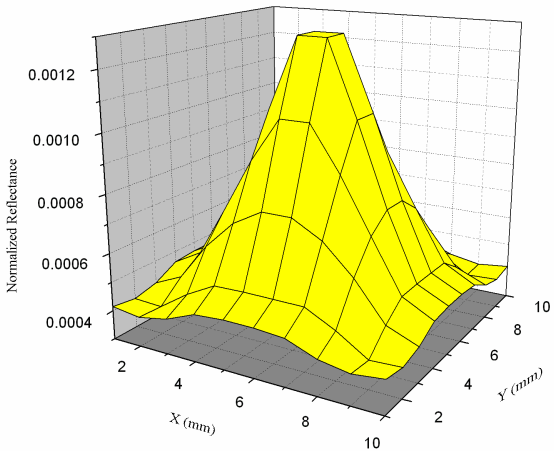
Figures 1-3 confirms the Green theorem application to the photon migration studies inside biological tissues.

B. *Experimental Results*

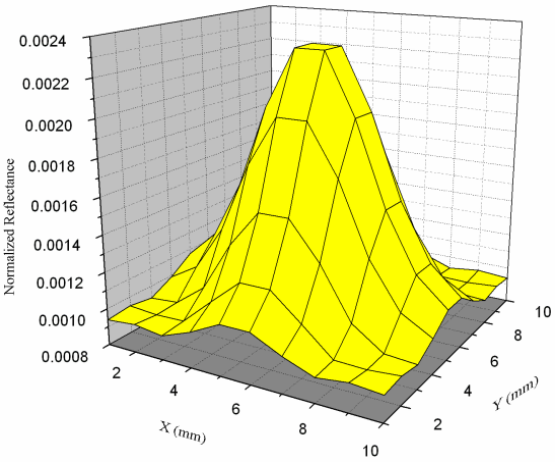
For experimental study of precision of this method, several phantoms like biological tissues were prepared and the diffusion of photons inside them were studied. The phantom like biological tissues are made by Intralipid and Indian ink, the optical properties of phantoms are controlled by indirect method [27-30]. These phantoms have cylindrical shape with diameter of 3.0 and height of 8 cm. In DOT the variation of absorption coefficient and the depth of lesion are important.



(a)



(b)



(c)

Fig. 3. Normalized reflectance from breast phantom for a) incident beam wavelength= 400 nm, b) 500 nm, and c) 650 nm.

First, the phantoms with different absorption coefficients are prepared. A fiber laser of 532 nm wavelength was incident the phantom and the diffused reflectance on surface of phantom is measured at angle of 90 degrees. One can

see that the results obtained by Green theorem are in good agreements with results obtained by MC method (Table.1).

The depth of lesions is an important parameter in DOT, so the effect of depth of lesion on the diffused reflectance was studied. Figure 4 shows the effect of lesion on reflectance fluence. The numerical results obtained by Green method are compared with MC and experimental data. The trend of numerical methods is similar, but the precision of MC is better than our Green method. However the computational time of MC's code is twenty-five times longer than that of Green method's code.

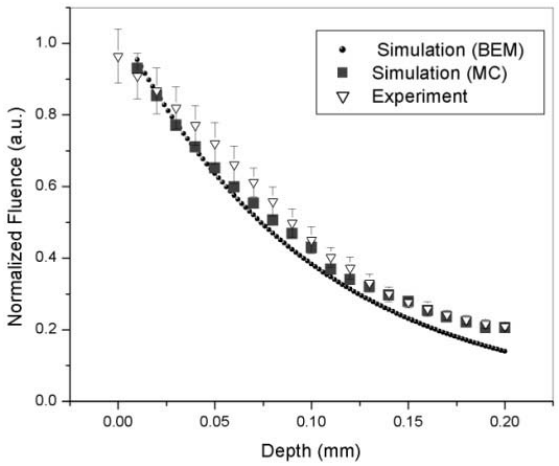


Fig. 4. Calculated normalized fluence by MC and Green theorem based on Green method for different depths and compared with experimental value of depths.

IV. CONCLUSION

In this study, the diffusion equation was converted to integral equation by Green theorem and the resultant integral was numerically solved by presented Green method. We have studied the diffusion of light in phantom like tissues. The numerical results obtained by presented method are compared with another numerical methods and experimental results. In this regard, we prepared biological breast phantoms made by intralipid and Indian ink. We have shown that the results obtained by presented method are in good agreement with experimental results and MC. It is noted however, that that results obtained by MC are more accurate than those obtained by FDM and FEM. The results of this work can well be applied to photo migration type problems suggesting shorter computing times for computer routines with rather satisfactory precision.

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