Hiding an Elephant into a Matchbox with Transformation Optics

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ABSTRACT— Based on transformation optics, we propose an illusion device that can make objects look much smaller and different than they actually are. In particular, the device has a capability to hide a large object (like an elephant) into a small one (like a matchbox). Compared to previous proposals for illusion devices, there is no requirement for negative refractive index or for speed of light going to infinity as in Euclidean invisible cloaks. We demonstrate the functionality of the device by full wave simulations.

KEYWORDS: Invisibility Cloak, Illusion Optics, Metamaterials, Transformation Optics

I. INTRODUCTION

Transformation optics (TO) [1]- [4] is a rapidly evolving field of optics that has led to designing many novel and fascinating devices, as well as constructing some of them experimentally employing metamaterials [5]. Invisibility cloak is the most prominent example of a device based on TO. Two archetypes of invisibility cloak were proposed simultaneously by Leonhardt [1] and Pendry *et al.* [2] to create a hollow region in space such that an object which is inserted in this region is hidden from an outside observer. Both of these cloaks contained zeroes of the refractive index, which makes it difficult to realize experimentally [6] and impossible in principle to work broadband [4]. This problem was eliminated by employing non-Euclidean geometry in designing the cloak [7]. Another strategy is to relax the demand of omnidirectional functionality, which led to designing a so-called carpet cloak [8]. Cloak at a distance or external cloak is another type of cloak which uses TO for hiding an object which is placed outside a negative index shell of the cloak [9]. This device uses folded geometry and negative index materials for creating the invisibility effect [10].

One can also use the properties of an external cloak to create an illusion device [9]. In the illusion devices, one object is hidden from the sight of the observer and another object (with different shape and material properties) is seen by the observer instead of the first object. Further illusion devices are described in [12-15]. TO has been used for designing also other devices such as concentrators [16], rotators [17] and superscatterers [18].

In this paper we use TO for designing a new type of an illusion device. With this device one can make a large object to appear much smaller and of a different shape, almost "like hiding an elephant into a matchbox". Our device works in a relatively simple way and its advantage is that it does not employ the negative refractive index like other illusion devices, and also the refractive index is not zero, so its functionality does not have to rely on resonant effects as in the Euclidean invisibility cloak.

The paper is organized as follows. In Section 2 we describe our device. In Section 3 we present the results of numerical simulations. The conclusion will be presented in Section 4.

II. THE ILLUSION DEVICE

According to the theory of TO, there are two spaces: virtual space where refractive index is unity and physical space with a non-unit index, and there is a particular geometrical mapping between the two spaces. If we denote the Cartesian coordinates in virtual space and physical space by (x'_1, x'_2, x'_3) and (x_1, x_2, x_3), respectively, then the relation between electromagnetic parameters of the two spaces are as follows [2, 4]:

$$\varepsilon = \frac{\mathbf{\Lambda}\varepsilon'\mathbf{\Lambda}^T}{\det\mathbf{\Lambda}}, \qquad \mu = \frac{\mathbf{\Lambda}\mu'\mathbf{\Lambda}^T}{\det\mathbf{\Lambda}} \tag{1}$$

where in $\varepsilon(\varepsilon')$ and $\mu(\mu')$ denote permittivity and permeability tensors in physical (virtual) space, and $\Lambda_{ij} = \partial x_i / \partial x'_j$.

The transformation relation for our device should be so that, it avoids having negative index material and also singularity in the material properties of the resulting device.

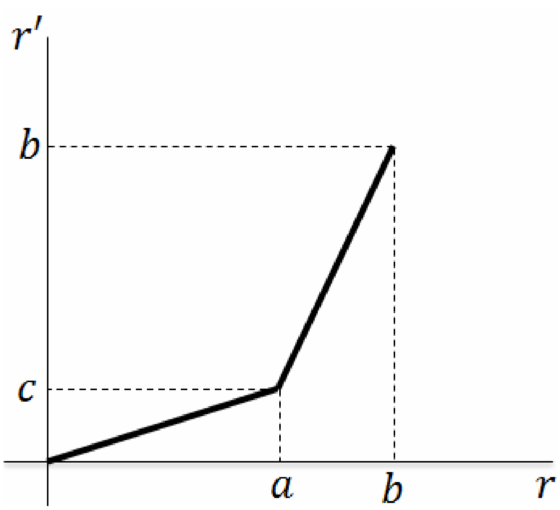


Fig. 1. Graph of the function $f(r')$ in Eq. (2)

We now employ the following transformation in the cylindrical coordinate system: $r = f(r'), \theta = \theta', z = z'$ with the function f defined by:

$$f = \begin{cases} \left(\dfrac{a-b}{c-b}\right)r' + b\left(\dfrac{c-a}{c-b}\right) & for \quad c \le r' \le b \\ (a/c)r' & for \quad 0 \le r' \le c \end{cases} \tag{2}$$

where in $f(c) = a$ and $f(b) = b$. These transformations compress a cylindrical shell of inner (outer) radius $a(b)$ in physical space into another cylindrical shell with inner (outer) radius $c(b)$, $c < a < b$ in virtual space. Fig. 1 shows the graph of the above transformation. As we expected above, all parts in the graph of Fig. 1 has positive slope; so the resulting materials will have positive index. In case of $c = 0$ the above transformation would be exactly the transformation function for the invisible cavity cloak [1, 2] which has singular material parameter. So, this case is not considered in our calculations.

The transformation matrices for the core $(0 \le r \le a)$ and the shell $(a \le r \le b)$ in cylindrical coordinate are as follows:

$$\mathbf{\Lambda}_{core} = \begin{pmatrix} a/c & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

$$\mathbf{\Lambda}_{shell} = \begin{pmatrix} \dfrac{(a-b)}{(c-b)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

According to Eq. (1), the material tensors for the core/shell of the device are:

$$\varepsilon_{core} = \mu_{core} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (c/a)^2 \end{pmatrix}, \tag{5}$$

$$\varepsilon_{shell} = \mu_{shell} = \begin{pmatrix} \left(\frac{a-b}{c-b}\right)\frac{r'}{r} & 0 & 0 \\ 0 & \left(\frac{c-b}{a-b}\right)\frac{r}{r'} & 0 \\ 0 & 0 & \left(\frac{c-b}{a-b}\right)\frac{r'}{r} \end{pmatrix}, \quad (6)$$

where virtual space is supposed to be empty space with $\varepsilon' = \mu' = 1$. It should be noted that, inasmuch as Eq. (1) can just be applied for Cartesian coordinate, for calculating the permittivity and permeability tensors in Eqs. (5) and (6) from Eq. (1), the transformation matrix which should be used, have to be of the form [19]

$$\mathbf{\Lambda}_{xx'} = \mathbf{\Lambda}_{xr}\mathbf{\Lambda}_{rr'}\mathbf{\Lambda}_{r'x'}. \quad (7)$$

where $\mathbf{\Lambda}_{xr}$ ($\mathbf{\Lambda}_{r'x'}$) is the transformation matrix for the transformation from Cartesian (cylindrical) coordinate to cylindrical (Cartesian) coordinate in physical (virtual) space and are equal to

$$\mathbf{\Lambda}_{xr} = \begin{pmatrix} x/r & -y & 0 \\ y/r & x & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

$$\mathbf{\Lambda}_{r'x'} = \begin{pmatrix} x'/r' & -y'/r' & 0 \\ -y'/r'^2 & x'/r'^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

and $\mathbf{\Lambda}_{rr'}$ would be the transformation matrix from cylindrical coordinate in physical space to cylindrical coordinate in virtual space. $\mathbf{\Lambda}_{rr'}$s are expressed by Eqs. (3) and (4) for the core and shell, respectively. Thus after some simple calculations and using the fact that $x'/r' = x/r$ and $y'/r' = y/r$ it yields

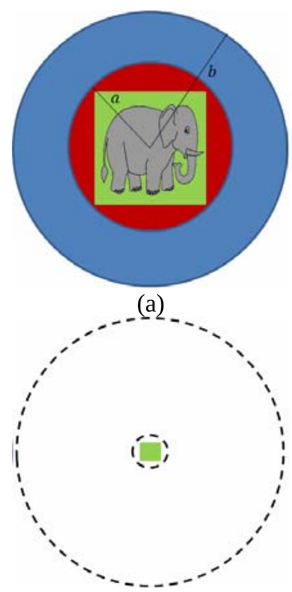
$$\det \mathbf{\Lambda}_{xx'} = \left(\frac{a-b}{c-b}\right)\frac{r}{r'}, \quad (10)$$

for the shell and

$$\det \mathbf{\Lambda}_{xx'} = \left(\frac{a}{c}\right)\frac{r}{r'}, \quad (11)$$

for the core. Finally, the permittivity and permeability tensor in Eqs. (5) and (6) can be determined by applying the transformation matrix $\mathbf{\Lambda} = \mathbf{\Lambda}_{xx'}$ and Eqs. (10) and (11), in Eq. (1). More details about the above calculations can be found in [2, 4, 19].

Now suppose that $c << a$, which means that the size of the core region $(0 \le r' \le c)$ in virtual space is much smaller than the size of the corresponding region $(0 \le r \le a)$ in physical space. If we insert a large object into a conductor box and put this box in the core part of our illusion device, this box looks much smaller than its real physical size. This way, with a little bit of exaggeration we can say that an elephant can be hidden in a matchbox. In reality the elephant would be hidden in a box larger than the elephant, but because of the illusion, this box would look much smaller than it would actually be.



(a)

(b)

Fig. 1. A schematic diagram for the illusion device. (a) Physical space: a large object (elephant) is inserted in a box and the box is placed in the core of the device. (b) Virtual space: the observer sees a box (matchbox) that is much smaller than the real box in physical space and the object inside. The object is not visible.

Fig. 1 (a) shows a schematic diagram of our illusion device in physical space and Fig. 1 (b) shows the observer's imagination when he/she looks at the device. Our device has an advantage over the previously reported illusion devices [11, 12]. For them, the position of the second object (which is going to be seen) is different from the position of the hidden object. In our device both of the objects are at the same place. And more importantly, the previous illusion devices employed negative refracting materials while our device does not. This is a great advantage in terms of possible experimental realization. Our device also does not require speed of light to be infinite as is the case of the Euclidean invisibility cloak.

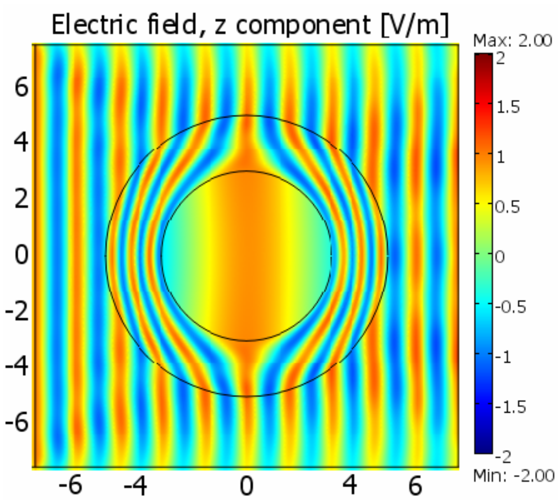


Fig. 2. Full wave simulation of the z-component of the electric field for a TE plane wave with $a = 2\lambda = 2$, $b = 5$ and $c = 0.5$ incident on the empty illusion device.

III. NUMERICAL SIMULATION

In order to simulate wave propagation in the illusion device, we first calculate the Cartesian elements of the permittivity tensor from the above cylindrical elements by the relations [5]

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_r \cos^2\theta + \varepsilon_\theta \sin^2\theta, \\ \varepsilon_{xy} &= \varepsilon_{yx} = (\varepsilon_r - \varepsilon_\theta)\cos\theta\sin\theta, \\ \varepsilon_{yy} &= \varepsilon_r \sin^2\theta + \varepsilon_\theta \cos^2\theta, \end{aligned} \tag{12}$$

and similarly for μ. Fig. 3 shows the full wave simulation (COMSOL Multiphysics) of a TE plane wave incident on this illusion device from the left when the core region still does not contain the box.

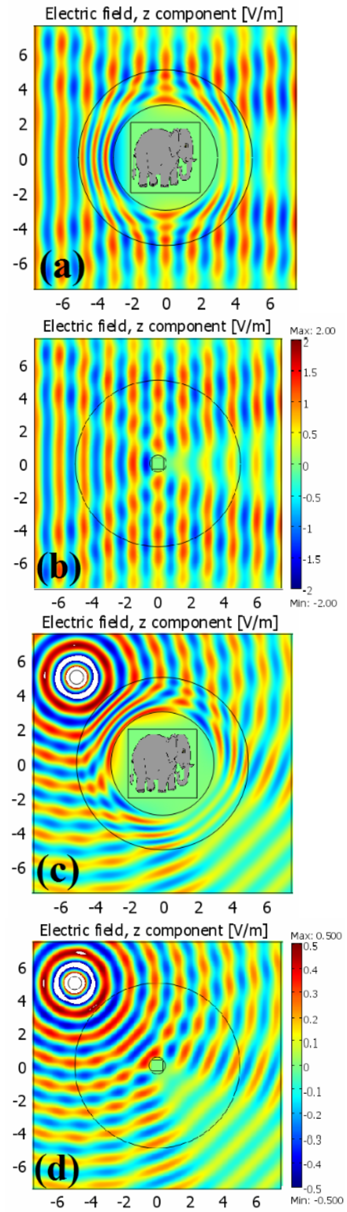


Fig. 3. (a) Scattering pattern for a TE plane wave with $a = 2\lambda = 3, b = 5$ and $c = 0.5$ unit incident from the left on the illusion device with a perfect electric conductor (PEC) square box with side length 4 in its core; an object (elephant) is also placed inside the box. The circles mark the boundary of the device at $r = b$ and the boundary of the core at $r = a$. (b) The equivalent virtual space with a much smaller PEC box which is a coordinate transform of the box in (a). The inner circle $r' = c$ is much smaller than the corresponding one in (a). (c) and (d) show the same, but for illumination by a point source located at (-5, 5) instead of the plane wave.

Fig. 4 (a) shows the simulation for the case which the box is present in the core region, again for a TE plane wave. Fig. 4 (b) shows the corresponding field pattern in virtual space. Figs. 4 (c) and (d) show similar patterns for illumination by a point source placed at (-5, 5). The parameters $a = 2\lambda = 3, b = 5$ and $c = 0.5$ were used. By comparing Figs. 4 (a) and (b) or Figs. 4 (c) and (d), we see that the scattering pattern of the large box in physical space is equivalent to scattering on a much smaller box in empty space, which demonstrates the functionality of our device.

IV. CONCLUSION

We have introduced a new illusion device that could hide a large object into an apparently much smaller box. We used the concept of transformation optics for designing the device and calculating the geometric and dielectric properties of the device. The operation of the device was demonstrated by wave simulations in 2D. Our device can be generalized to other shapes and coordinate systems. This illusion device can provide a similar effect to the device reported in [11], but, in contrast, it is built from realizable positive index metamaterials and also does not require singularities in its material parameters so the speed of light need not to go to infinity.

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