Use of a Two-Channel Moiré Wavefront Sensor for Measuring Topological Charge Sign of the Vortex Beam and Investigation of Its Change Due to an Odd Number of Reflections

Mohammad Yeganeh[a] and Saifollah Rasouli[a,b,*]

[a]**Department of Physics, Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan, Iran**
[b]**Optics Research Center, Institute for Advanced Studies in Basic Sciences (IASBS), Zanjan, Iran**

[*]**Corresponding Author Email:** rasouli@iasbs.ac.ir

ABSTRACT— **One of the solutions of the Helmholtz equation is the vortex beams. In the recent decades, production and applications of these types of beams have found serious attentions. Determination of the vortex beam topological charge and its sign are very important issues. Odd number of reflections of the vortex beam changes its vorticity. In this paper, we have used a q-plate to generate a vortex beam from a plane wave of a He-Ne laser beam and a two-channel moiré based wavefront sensor is used to measure wavefront gradient of the vortex beam. In two different arrangements the vortex beam experience odd and even number of reflections, respectively, and from the moiré pattern deformations the topological charge of the vortex beam and its sign are determined.**

KEYWORDS: moiré deflectometry, vortex beam, topological charge, reflection effect on the topological charge.

I. INTRODUCTION

Moiré technique is a conventional method for the measurement of displacements and for the measuring deflection angle of light beams passing through phase objects [1]-[3]. In recent years, various applications of the moiré technique for study of the phase media are presented. Recently, we have introduced two-channel wavefront sensor based on moiré deflectometry and we have used it for evaluation of the atmospheric turbulence parameters [4]-[8]. In addition, reconstruction of vortex beam with two-channel wavefront sensor was done [9]. In this work, we have used two channel moiré deflectometry for determining the sign of topological charge of a vortex beam and investigation of the effect of reflection on it.

When a plane-wave passes through a spiral phase plate, which imposes a phase proportional to the azimuth angle, ϕ, the wavefront shape converts to a spiral shape. The optical path imposed by the phase plate in one circuit, should be equal to an integer coefficient of the wavelength. Wavefront slopes of these kinds of beams have an azimuth component. Laguerre–Gaussian (LG) beam is one example of these waves. Quantum communications and quantum cryptography, optical tweezers and optical spanners are applications of these beams [10].

There are several ways to produce a LG beam. Spiral phase plate, forked gratings, and spatial light modulators (SLM) are common tools to generate vortex beams [11]. One of the other important tools to convert a plane wave to a LG beam is q-plate which made of a planar slab of uniaxial birefringent medium, having an homogeneous birefringent phase retardation of π (half-wave plate) across the slab and an inhomogeneous orientation of the fast (or slow) directions lying parallel to the slab planes [12].

It is very important to determine the sign of topological charge of a vortex beam. For example, when a light beam from a monochromatic point source propagates through strong atmospheric turbulence or an optical field turns into a speckle field. Such fields are characterized by the presence of isolated dark points which named brunch points and spiral phase structure will appear around these points [13]. The sign and topological charge corresponding to these points have very important roles in the reconstruction and correction of the wavefront of the optical fields. In addition, determination of the vortex beams sign is important in the optical communications, where we refer only up and down digits corresponding to the plus and minus signs to topological chare of vortex beam.

II. THEORY

One of the solutions of the paraxial wave equation in cylindrical coordinates is LG beams. The light in these beams has amplitude that depends on three main terms:

$$E(r) \propto r^{|l|} L_p^l \left(\frac{2r^2}{\omega^2}\right) \exp(il\phi), \quad (1)$$

where L_p^l is an associate Laguerre polynomial of the transverse coordinate *r*, which has *p* radial zero crossings. When *p = 0*, the shape of the light beam is a single ring of radius proportional to $\sqrt{l}$ giving the beam doughnut shape. For a beam propagating along the *z*-direction, the phase of the field in cylindrical coordinates is

$$\varphi(r,\theta,z) = kz + \frac{kr^2}{2R(z)} + l\phi - \Psi(z) = const., \quad (2)$$

where *k* is the wave-number, *R* is the radius of curvature of the wavefront, and ψ(z) is the Gouy phase [11]. For *l > 0*, the vorticity is clockwise as moving through beam's propagation direction. In sufficiently large distance *z,* in which second and fourth terms can be ignored, the gradient of the beam will be:

$$\begin{aligned}\overrightarrow{\nabla\varphi}_{xyz} &\approx \frac{l}{r}\hat{\phi}_0 + k\,\hat{z}_0 \\ &= \frac{l}{r}\left(-\sin\phi\,\hat{x}_0 + \cos\phi\,\hat{y}_0\right) + k\,\hat{z}_0,\end{aligned} \quad (3)$$

where $\hat{x}_0, \hat{y}_0, \hat{z}_0$ are unit vectors in x, y, z directions respectively and $\hat{\phi}_0$ is unit vector in azimuth angle.

In this work, we are interested to measure the effect of a reflection from a plane mirror on the wavefront gradient of the vortex beam. We consider incident of the vortex beam on the mirror surface with an incident angle of *θ.* Three coordinate systems are selected as shown in Fig. 1, in which z axis in the first one coincides with the propagation direction of the incident beam, z' axis is in the opposite direction of the normal vector of the mirror, and z'' axis in the third one coincides with the opposite direction of the propagation vector of the reflected beam from the mirror. Now, normal vector of the mirror can be written as $\hat{n} = \sin\theta\,\hat{x}_0 - \cos\theta\,\hat{z}_0$. Effect of a rotation around y axis by angle $(-\theta)$ on the incident beam is calculated using

$$\left(\overrightarrow{\nabla\varphi}\right)_{x'y'z'} = \mathrm{R}(-\theta, y)\cdot\left(\overrightarrow{\nabla\varphi}\right)_{xyz}, \quad (4)$$

where **R** is rotation operator. We know that due to the reflection from the mirror surface, sign of the perpendicular component of the wavefront gradient is changed. Now, effect of reflection from the mirror is considered by a reflection operator $\mathrm{P}_{z'}$ on the beam:

$$\left(\overrightarrow{\nabla\varphi}\right)^{\mathrm{Re}}_{x'y'z'} = \mathrm{P}_{z'}\cdot\left(\overrightarrow{\nabla\varphi}\right)_{x'y'z'}. \quad (5)$$

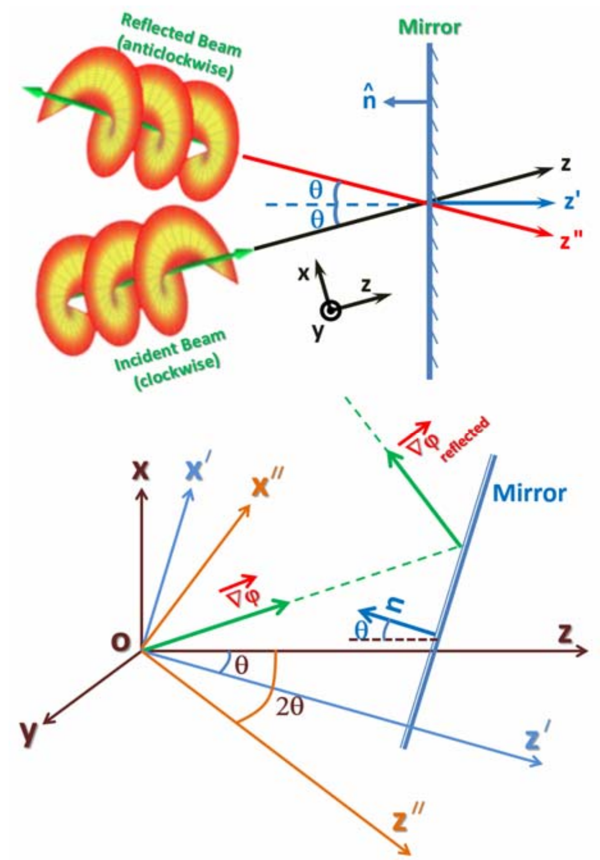

Fig. **1** Representation of a typical incident and reflected vortex beam from a mirror and all of the selected coordinate systems.

Finally, we need to determine the gradient components in $x''y''z''$ coordinate system. For this reason, we act rotation operator around y axis thorough angle $(-\theta)$:

$$\left(\overrightarrow{\nabla\varphi}\right)^{\mathrm{Re}}_{x''y''z''} = \mathrm{R}(-\theta, y)\cdot\left(\overrightarrow{\nabla\varphi}\right)^{\mathrm{Re}}_{x'y'z'}$$
$$= \frac{l}{r}\left(-\sin\phi\,\hat{x}''_0 + \cos\phi\,\hat{y}''_0\right) - k\,\hat{z}''_0 \qquad (6)$$
$$= -\left(-\frac{l}{r}\hat{\phi}''_0 + k\,\hat{z}''_0\right),$$

where $\hat{x}''_0, \hat{y}''_0, \hat{z}''_0$ are unit vectors in the rotated Cartesian coordinate system and $\hat{\phi}''_0$ is unit vector in cylindrical coordinate in same system. Comparing Eq. (6) and Eq. (3), a minus sign appears before $\hat{z}''_0$ that was expected because of the unit vector $\hat{z}''_0$ is in the opposite direction of the propagation direction of the reflected beam. Also, inverse of Eq. (2), in the recent equation, the sign of two components in cylindrical coordinates are not same. This means that the vorticity of the beam is changed. It is equal to change of the sign of the topological charge.

In this work, we used moiré deflectometry for investigation the change in the sing of the vortex beam topological charge due an odd number of reflections.

The wavefront gradient $\overrightarrow{\nabla\varphi}$ can be expressed in term of the relative moiré fringe displacements in the two arms of our apparatus:

$$\overrightarrow{\nabla\varphi} = \frac{kd}{z_k}\left[\left(\frac{\delta y_m}{d_{ym}}\right)\hat{x}_0 + \left(\frac{\delta x_m}{d_{xm}}\right)\hat{y}_0\right], \qquad (7)$$

where d, d_{xm} and d_{ym} are the gratings and moiré fringe spacings in x and y directions respectively, and $\delta x_m, \delta y_m$ are the moiré fringe shifts in the x and y directions, respectively [6].

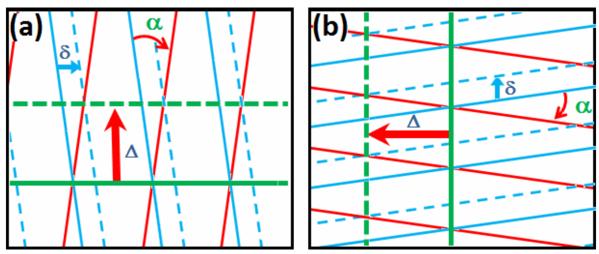

Fig. **2** Representation of the moiré pattern displacements induced by the relative displacements of the gratings, the first gratings (front gratings) of the moiré deflectometers are shown by the blue lines and the second (back) gratings are shown with red lines. The second gratings in both of the figures are rotated by a clockwise angle α relative to the first gratings. A small displacement of the first gratings or their self-images in the moiré deflectometers in the horizontal (a) (in the vertical (b)) direction causes a large displacement of the moiré pattern to the upward (a) (left (b)).

The relation between the displacement direction of the moiré fringes and the incident beam's slope depends on the relative angle of the first and second gratings' lines in each channel. As shown in Fig. 2, when the first and second gratings in a moiré deflectometer are almost vertical (horizontal), and the second (back) grating rotated clockwise relative to the first (front) one, inclination of the beam to the right (upward) causes a movement of the moiré pattern to the upward (left) side.

III. Experiments

In the experiment, a collimated plane wave of a He–Ne laser ($\lambda = 632.8\,nm$, $P = 30\,mW$) is converted to an optical vortex beam with $l = 20$ by means of an electrically controlled q-plate with $q = 10$ [14].

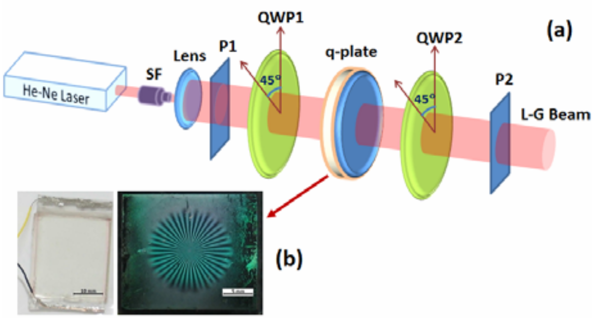

Fig. **3** (a): Schematic of the experimental setup is used for generation of the LG beam from a q-plate: SF is spatial filter, P1, P2 are polarizers, and QWP1 and QWP2 are the first and second quarter-wave plates, (b): left: q-plate, right: q-plate between two crossed polarizers.

Schematic diagram of the experimental setup to generation LG beam from a q-plate is shown in Fig. 3. The laser beam passes through a spatial filter, SF, and is collimated by a lens. A polarizing beam splitter or a polarizer sheet (P1) associated a quarter-wave plate (QWP1) which its optical axis has an angle of ±45° related to transition axis of first polarizer are generate circular polarized beam. Then, q-plate convert plan circular polarized light to LG beam. The second quarter-wave plate (QWP2) must be kept at the same angle of the first one. QWP2 and second polarizer (P2) are used for filtrate remanded circular polarized light except vortex beam. The transition axis of the P2 must be perpendicular to first one.

For generating the LG beam, q-plate needs to apply an alternative square voltage with the frequency and peak-to-peak voltage of about 4KHz and 4V respectively on it. The q-plate is not sensitive to frequency, but the voltage on it should be adjusted to find the desired doughnut shape of the beam. After generating the LG beam and aligning the set up, QWP2 and P2 can be removed from setup.

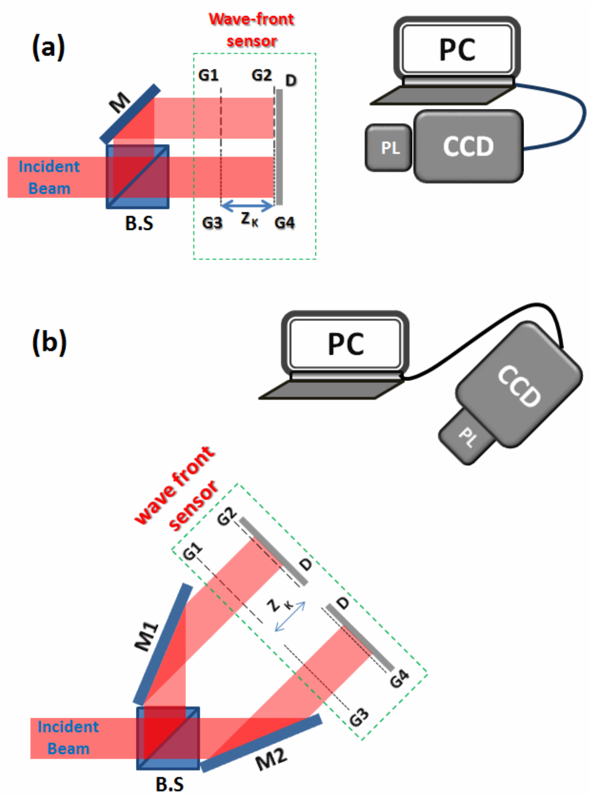

Fig. **4** Schematic of two different arrangements of two-channel moiré based wavefront sensor: B.S is beam splitter, M, M1, M2 are mirrors, G1-G4 are Ronchi gratings and Pl is collimated lens that projects the moiré pattern produced on the diffuser D on the CCD camera.

For investigation of the effect of an odd or even numbers of reflections on the sign of the topological charge, we have used two different setups as shown in Fig. 4. In Fig. 4(a), a mirror reflects the second beam into a direction parallel to the first beam's propagation direction, and the two beams pass through a pair of moiré deflectometers. In the second setup, both of beams, the transmitted and the reflected beams from the beam splitter, reflect from two mirrors to align them parallel with a 45° tilt relation to incident beam. Reflected beams pass through a pair of moiré deflectometers similar to the previous setup.

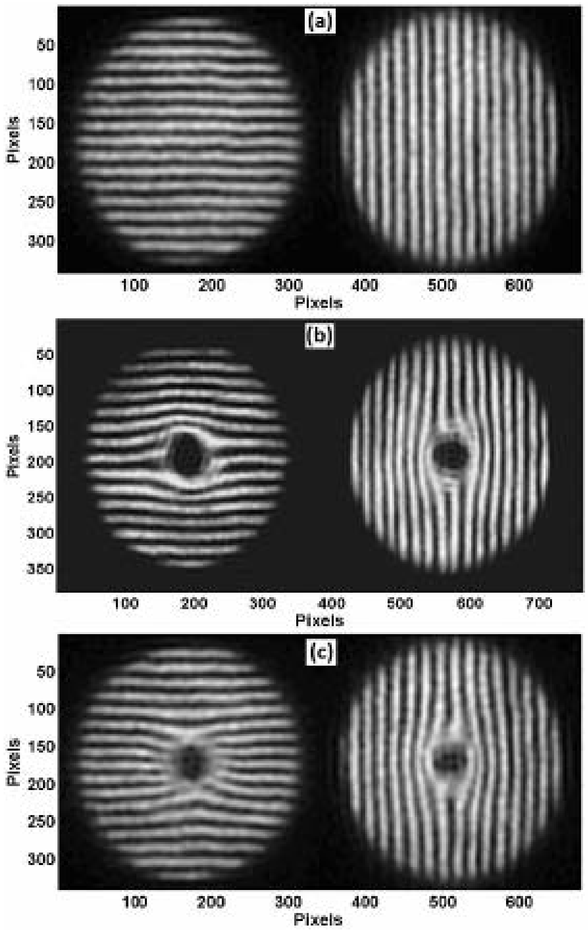

Fig. **5** Typical recorded frames of moiré patterns for three different cases of the incident beam: (a): plane wave, (b): vortex beam, recorded by the set-up of Fig. 4(a), (c): same beam by the set-up of Fig. 4(b).

The moiré deflectometers are installed quite close to each other. Directions of the gratings' rulings of the moiré deflectometers, G2 and G4, are installed slightly rotated corresponding first gratings G1 and G3 respectively, but are perpendicular in the two beams to form rotation moiré patterns. Moiré patterns are formed at a plane where the second gratings of the moiré deflectometers and a diffuser are installed. The moiré patterns from both the beams are projected on a CCD camera. Moiré patterns are recorded by the CCD camera and transferred to a computer, to be measured highly accurately. For two cases of the incident wave, plane wave and vortex beam, moiré fringes are recorded. For the case of vortex beam we have used two different set-ups as shown in Fig. 4. Deformations of the moiré fringes in the recorded patterns for two different cases of the vortex beam respect to the moiré pattern corresponding to the case of the plane have been deduced and the evolution of the wavefront gradient is determined.

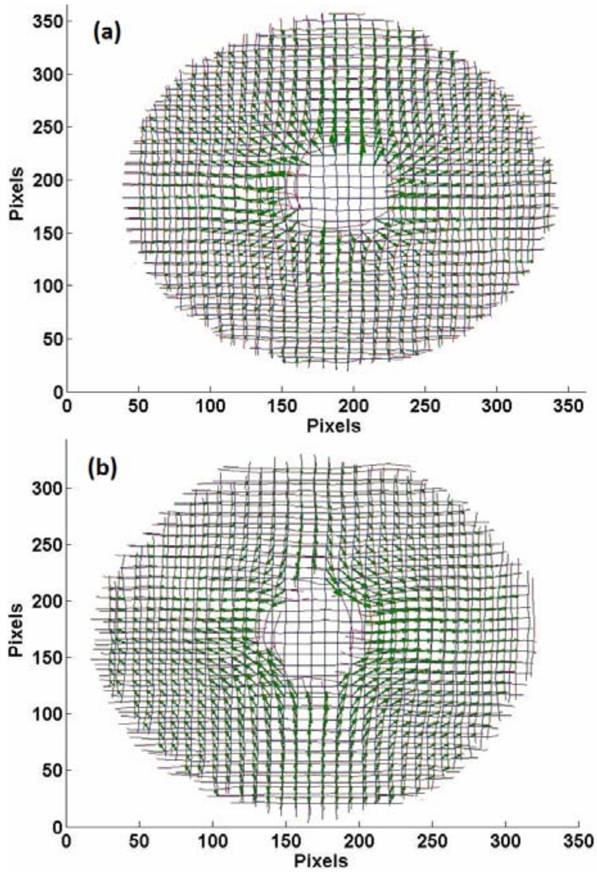

Fig. **6** Two-dimensional diagrams of the displacement vectors of the moiré patterns corresponding to the moiré patterns of (a): Fig. 5(b) and (b): Fig. 5(c).

The gratings had equal periods of $d = 0.1mm$ and the used Talbot distance was set at $z_k = 6.34cm$ corresponding to second-order. The moiré patterns from both arms, which consist of 317×352 pixels each, were finally acquired by the CCD camera for further processing [4].

Fig. 5 shows the recorded frames of two moiré patterns on the CCD camera for three different cases: (a) plane wave (when the q-plate is totally untuned, (b) LG beam having OAM *l=20* through Fig. 4(a) setup, and (c) same beam through Fig. 4(b) setup, respectively. Each frame consists of two moiré fringe patterns along *x* and *y* directions corresponding to the beams in the two arms of the apparatuses. In the first case, the wavefront of the plane TEM00 beam was planar. Therefore, the moiré fringe patterns on both arms were uniformly spaced linear dark and bright fringes. In the last two cases, where beams experiences helical phase front, displacements in the moiré fringes have been observed in both channels.

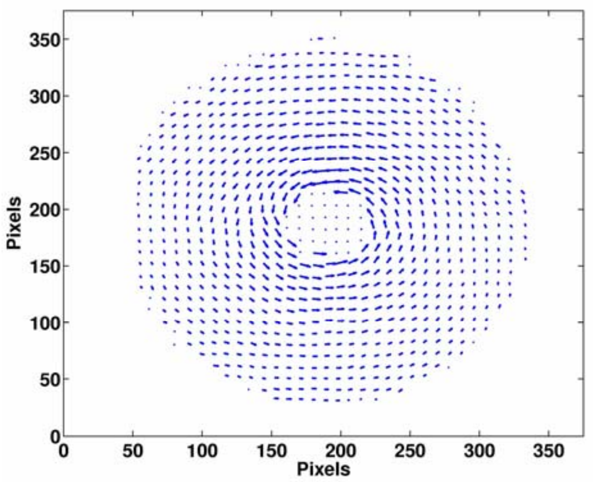

Fig. **7** Diagram of the calculated gradients vectors for LG cross section for *l=20*.

Relative displacement vectors of the moiré fringes are depicted in Fig. 6. In Fig. 7, diagram of the calculated gradients vectors for LG cross section for *l=20* are presented. Comparing moiré patterns corresponding to the plane and vortex beams shows that in the cases of vortex beam the moiré fringes diverging or converging around the singular point of the beam. Depending on the relative rotation angle of gratings, it depends on the right or left handed vorticity of the beam. However, for Fig. 5(b), in which, in both channels of the moiré deflectometers the light beams suffered even numbers of reflections, have shown same behaviors. Also, for Fig. 5(c) in the first channel, in which, the light beam is not suffered any reflection, the moiré pattern is converged, while in the other channel it suffered two reflections was diverged.

It is clear that one of the beams in Fig. 4(a) undergoes two reflections (in beam splitter and mirror) and other one not undergoes any reflections. Then, we except that no changes should be happen in the sign of the topological charge of both beams. Moreover, one of beams in Fig. 4(b) undergoes two reflections (from the beam splitter and mirror) and other one undergoes one reflection (only by the mirror). As a result, it is expected that the sign of the topological charge of these beams to be different.

IV. CONCLUSION

In this paper, we used a q-plate to generate a vortex beam from a plane wave of a He-Ne laser beam. A two-channel moiré based wavefront sensor is used to measure wavefront gradient of the vortex beam. In two different arrangements the vortex beam experiences odd and even number of reflections, respectively, and from the moiré patterns deformation, the sign of the topological charge were determined. We demonstrated theoretically and experimentally that odd numbers of reflections causes change in vorticity of the vortex beam which is corresponding to the change of topological charge's sign.

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Mohammad Yeganeh is currently a PhD Student of Physics in the Department of Physics of the Institute for Advanced Studies in Basic Sciences (IASBS). He received his BSc and MSc degrees from University of Tabriz, Tabriz, Iran, in 2003 and 2006, respectively. He currently works on the application of the moiré deflectometry on the Laguerre–Gaussian beams characterization. His research interests are the optics of liquid crystals, moiré techniques, atmospheric turbulence, laser spectroscopy, and geotechnical tests.

Saifollah Rasouli is an Associate Professor of Physics in the Department of Physics of the Institute for Advanced Studies in Basic Sciences (IASBS). His research interests include moiré technique applications, interferometry, optical metrology, atmospheric turbulence, and wavefront sensing. Dr. Rasouli is an associate member of the Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy. He received the 2009 ICO/ICTP Gallieno Denardo Award. He is a Senior Member of SPIE, a Regular Member of OSA, PSI, and OPSI.

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