

# Generation of Nonclassical States of the Radiation Field in the System of a Single Trapped Atom in a Cavity within the First Order of the Lamb-Dicke Approximation

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**Abstract**— In this paper, we propose a theoretical scheme for the generation of non-classical states of the cavity field in a system of a single trapped atom via controlling the Lamb-Dicke parameter. By exploiting the super-operator method, we obtain an analytical expression for the density operator of the system by which we examine the dynamical behaviors of the atomic population inversion, the phase-space Husimi Q-function as well as the von Neumann quantum entropy of the cavity-field. The results reveal that at certain periods of time one of the cavity-field quadratures may be squeezed and the two subsystems of the trapped atom and the cavity field are entangled. Moreover, we find that the atomic spontaneous emission and the cavity-field damping destroy the nonclassical characteristics of the cavity field.

**Keywords:** Quantum state engineering, non-classical states, trapped atom, Lamb-Dicke approximation.

## I. INTRODUCTION

The generation and manipulation of nonclassical quantum states of the radiation field and atomic systems continue to attract great theoretical and experimental interest. These states are of interest from the standpoint of quantum measurement concepts and may facilitate other measurements such as sensitive detection [1-3] or quantum computation [4,5]. The quantized motional states of trapped atoms or ions in confining potentials offer

interesting possibilities for a variety of applications, such as the storage and manipulation of quantum information (e.g. ‘qubits’), with particular reference to quantum logic operations and quantum computing[6-8].

In connection with quantum state engineering, the system of a single trapped atom strongly coupled to the electromagnetic field of a high- $Q$  optical cavity within the setting of cavity quantum electrodynamics (CQED) has attracted considerable attention[9]. In a cavity QED system both the mechanical effects of optical resonator on the quantized center-of-mass motion of the trapped atom [10] and the nonlinearity induced by the correlation between the cavity field and the motion of the trapped atom lead to some interesting physical features. In particular, the relative stability of a trapped atom system against decoherence and the possibility of achieving different regimes of atom-field coupling in such a system provide an efficient way to control the intensity of atom-field interaction [11]. These features make such systems preferable in comparison with other ones in CQED. Furthermore, the entanglement between the cavity field and the trapped atom [12] not only provides a test bench for exploring the effects of quantum statistics of the radiation field on the internal dynamics of the atom [13,14] but also makes possible to transfer of quantum coherence between the atomic vibrational motion and the cavity field [15].

The main purpose of the present contribution is investigation of the generation of non-classical state for the radiation field in a single-mode cavity where a harmonically trapped atom is coupled to the cavity mode. In order to make the model under consideration more realistic, we take into account the atomic spontaneous emission as well as the cavity field damping (photon leakage). The paper is structured as follows. In Section II, we introduce the physical model of the system. In Section III, by applying a unitary transformation, we first derive an effective Hamiltonian for the system under consideration within the first order of the Lamb-Dicke approximation, and then by exploiting the super-operator method, we obtain an analytical expression for the density operator of the system. In section IV, we analyze the dynamical behaviors of the atomic population inversion, the phase-space Husimi  $Q$  function as well as the von Neumann quantum entropy of the cavity-field. Moreover, the dynamical evolution of the cavity-field entropy is investigated. Finally, we summarize our conclusions in section V.

## II. PHYSICAL MODEL

As depicted in Fig. 1, we consider a  $\Lambda$ -type three-level atom of mass  $M$  confined in an optical resonator by an external harmonic potential with frequency  $\nu$ . The atomic dipolar transition  $|g_1\rangle \leftrightarrow |e\rangle$  is driven via the control laser of Rabi frequency  $\Omega_L$ , while the atomic transition  $|g_2\rangle \leftrightarrow |e\rangle$  couples to the optical resonator mode of frequency  $\omega_C$  which is pumped by a laser of frequency  $\Omega_P$  and the vacuum Rabi frequency  $g$  and decays with rate  $\kappa$ . The excited state with the natural line-width  $\gamma$  decays to the states  $|g_j\rangle (j=1,2)$  so that  $\gamma = \gamma_1 + \gamma_2$ . In order to have single-photon scattering in the system, prevent injection of additional noises of the pump laser, and simplify the calculations, we ignore the decay rate  $\gamma_1$ .

To describe the system, two essential assumptions are considered. First, we assume that the atom is tightly confined in the trap, such that the atomic wave packet width is much smaller than the lasers wavelengths, i.e.,  $k\Delta x \ll 1$  (where  $k$  is the laser wave number). Under such a condition, the so-called Lamb-Dicke regime holds [17],

$$\eta\sqrt{2\langle m \rangle + 1} \ll 1, \quad (1)$$

where  $\langle m \rangle$  is the mean vibrational phonon number, and  $\eta$  is the Lamb-Dicke parameter by which the effect of dynamics of light on the atomic motion is often well described [18].

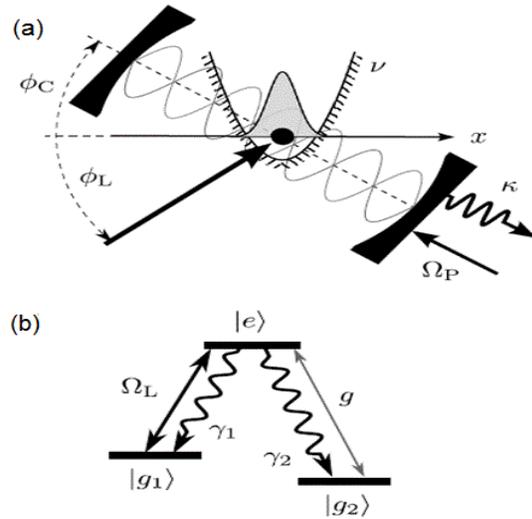


Fig. 1. (a) Setup of the system. An atom is confined inside an optical resonator by an external harmonic trap of frequency  $\nu$ . The angles  $\eta_L$  and  $\eta_C$  give, respectively, the orientations of the laser and the cavity wave vectors with respect to the axis of the atomic motion. (b) Relevant electronic transitions [16].

Second, we suppose the initial mean number of photons in the cavity is much smaller than unity to ensure that pure quantum effects occur. Therefore, the Hamiltonian of the system can be expanded by means of perturbation theory. Expanding to the first order in the Lamb-Dicke parameter  $\eta$  such that only single-phonon transitions for external atomic degrees of freedom occur, the total Hamiltonian of the system is given by:

$$\hat{H} = \hat{H}^{(0)} + \hat{H}^{(1)} + O(\eta^2), \quad (2)$$

where

$$\begin{aligned} \hat{H}^{(0)} &= \hbar g \cos \varphi (\hat{\sigma}_{eg_2} \hat{a} + \hat{a}^\dagger \hat{\sigma}_{g_2e}) + \\ &\frac{\hbar \Omega_L}{2} (\hat{\sigma}_{eg_1} + \hat{\sigma}_{g_1e}), \\ \hat{H}^{(1)} &= \frac{\hbar \Omega_P}{2} (\hat{a} + \hat{a}^\dagger) - \hbar g \sin \varphi \eta \cos \phi_C (\hat{b} + \hat{b}^\dagger) \times \\ &(\hat{\sigma}_{eg_2} \hat{a} + \hat{a}^\dagger \hat{\sigma}_{g_2e}) + i \frac{\hbar \Omega_L}{2} \eta \cos \phi_L (\hat{b} + \hat{b}^\dagger) (\hat{\sigma}_{eg_1} - \hat{\sigma}_{g_1e}), \end{aligned} \quad (3)$$

are the Hamiltonians corresponding to the zeroth and first orders in  $\eta$ , respectively, with  $\hat{a}$  ( $\hat{a}^\dagger$ ) and  $\hat{b}$  ( $\hat{b}^\dagger$ ) the annihilation (creation) operators of photons and phonons (atomic vibrational motion). The operator  $\hat{\sigma}_{ij} = |i\rangle\langle j|$  describes the transition between internal atomic states  $|i\rangle$  and  $|j\rangle$ ,  $\phi_L$  and  $\phi_C$  denote, respectively, the angles between the cavity and laser wave vectors and the axis of the atomic motion, while  $\varphi$  accounts for the displacement of the trap center with respect to the origin (Fig. 1). At zero order in the Lamb-Dicke parameter, the internal and external atomic degrees of freedom are decoupled and the system is found to be in a stable state. However, in the first order of  $\eta$ , the atomic center of mass motion couples to the laser which leads to the occurrence of entanglement between the cavity field and the trapped atom.

### III. EFFECTIVE HAMILTONIAN AND DENSITY OPERATOR OF THE SYSTEM

The quantum dynamics of the system under consideration within the first order of the Lamb-Dicke parameter  $\eta$  can be described by an effective Hamiltonian. In the rotating wave approximation, and by using the unitary transformation  $\hat{U}(t, 0) = \text{Exp} \left[ -\frac{i}{\hbar} \int_0^t \hat{H}_{\text{int}}^{(1)}(t') dt' \right]$ , the effective Hamiltonian in the interaction picture is obtained as [19]:

$$\hat{H}_{\text{eff}}^{(1)} = \frac{\hbar \Omega_P g \sin \varphi}{2\Delta} \eta \cos \varphi_C (\hat{b} + \hat{b}^\dagger) (\hat{a}^2 \hat{\sigma}_{eg_2} + \hat{a}^{\dagger 2} \hat{\sigma}_{g_2e}) +$$

$$\frac{\hbar \Omega_P \Omega_L}{4\delta_1} \eta \cos \varphi_L (\hat{b} + \hat{b}^\dagger) (\hat{a} + \hat{a}^\dagger) (\hat{\sigma}_{eg_1} + \hat{\sigma}_{g_1e}) +$$

$$\frac{\hbar g^2 \sin^2 \varphi}{\delta_{C2}} \eta^2 \cos^2 \varphi_C (\hat{b} + \hat{b}^\dagger)^2 \hat{a}^\dagger \hat{a} (\hat{\sigma}_{ee} - \hat{\sigma}_{g_2g_2}) +$$

$$\frac{\hbar \Omega_L^2}{4\delta_1} \eta^2 \cos^2 \varphi_L (\hat{b} + \hat{b}^\dagger)^2 (\hat{\sigma}_{ee} - \hat{\sigma}_{g_1g_1}), \quad (5)$$

where  $\Delta = \omega_p - \omega_c$  is the cavity-probe detuning,  $\delta_1 = \omega_L - (\omega_e - \omega_{g_1})$  is the detuning between the pump laser and the atomic dipole transition  $|g_1\rangle \leftrightarrow |e\rangle$ , and  $\delta_{C2} = \omega_c - (\omega_e - \omega_{g_2})$  is the detuning between the cavity mode and the atomic dipole transition  $|g_2\rangle \leftrightarrow |e\rangle$ . Comparing Eqs. (4) and (5) explicitly shows that increasing the contribution of the Lamb-Dicke parameter gives rise to the enhancement of nonlinear effects in the atom-photon interaction.

The nonlinear terms in the effective Hamiltonian of Eq. (5) appear as two-photon and two-phonon transitions as well as the anti-Jaynes-Cummings terms. The density operator of the system obeys the master equation

$$\frac{d}{dt} \hat{\rho}^{(1)} = \frac{1}{i\hbar} [\hat{H}_{\text{eff}}^{(1)}, \hat{\rho}] + \hat{L} \hat{\rho}, \quad (6)$$

in which the dissipations due to the cavity-field decay at rate  $\kappa$  and the atomic spontaneous emission at rate  $\gamma$  appear in the Lindblad super-operator  $\hat{L}$ . Here, we assume that initially the cavity field is prepared in a coherent state, and the atom is in the internal state  $|g_2\rangle$  and vacuum vibrational state  $|m=0\rangle$ . By applying the super-operator method [20] and after some lengthy but straightforward calculation, we obtain the following analytical expression for  $\hat{\rho}^{(1)}(t)$

$$\hat{\rho}^{(1)}(t) = e^{\hat{A}t} e^{\hat{B}t} e^{\hat{C}t} e^{\hat{D}t} e^{\hat{E}t} e^{\hat{J}t} e^{\hat{K}t} \hat{\rho}(0), \quad (7)$$

where

$$\begin{aligned}
\hat{A}\hat{\rho} &= -i\xi[\eta(\hat{b} + \hat{b}^\dagger)(\hat{a}^2\hat{\sigma}_{eg_2} + \hat{a}^{\dagger 2}\hat{\sigma}_{g_2e}), \hat{\rho}], \\
\hat{B}\hat{\rho} &= -iv[\eta(\hat{b} + \hat{b}^\dagger)(\hat{a}\hat{\sigma}_{eg_1} + \hat{a}^\dagger\hat{\sigma}_{g_1e}), \hat{\rho}], \\
\hat{C}\hat{\rho} &= -iv[\eta(\hat{b} + \hat{b}^\dagger)(\hat{a}\hat{\sigma}_{g_1e} + \hat{a}^\dagger\hat{\sigma}_{eg_1}), \hat{\rho}], \\
\hat{D}\hat{\rho} &= -i\chi[\eta^2(\hat{b} + \hat{b}^\dagger)^2(\hat{\sigma}_{ee} - \hat{\sigma}_{g_1g_1}), \hat{\rho}], \\
\hat{E}\hat{\rho} &= -i\beta[\eta^2(\hat{b} + \hat{b}^\dagger)^2\hat{a}^\dagger\hat{a}(\hat{\sigma}_{ee} - \hat{\sigma}_{g_2g_2}), \hat{\rho}], \\
\hat{J}\hat{\rho} &= 2\kappa\hat{a}\hat{\rho}\hat{a}^\dagger, \\
\hat{K}\hat{\rho} &= -\kappa\hat{a}^\dagger\hat{a}\hat{\rho} - \kappa\hat{\rho}\hat{a}^\dagger\hat{a}.
\end{aligned} \tag{8}$$

$$\begin{aligned}
\xi &= \frac{\hbar\Omega_P g \sin \varphi}{2\Delta} \eta \cos \varphi_C, \quad v = \frac{\hbar\Omega_P \Omega_L}{4\delta_1} \eta \cos \varphi_L, \\
\chi &= \frac{\hbar\Omega_L^2}{4\delta_1} \eta^2 \cos^2 \varphi_L, \quad \beta = \frac{\hbar\Omega_L^2}{4\delta_1} \eta^2 \cos^2 \varphi_L.
\end{aligned} \tag{9}$$

#### IV. QUANTUM DYNAMICAL PROPERTIES OF THE SYSTEM

In this section, we study the temporal evolution of the atomic population inversion and the dynamics of the phase space  $Q$ -function, to investigate the possibility of quadrature squeezing of the cavity field. We also examine the time evolution of the von Neumann quantum entropy of the cavity field to investigate the generation of atom-field entanglement.

##### A. Temporal evolution of the atomic population inversion

Here, by using  $\hat{\rho}^{(1)}(t)$  given by Eq. (7), we can examine the time evolution of the atomic population inversion which is defined as  $W(t) = \langle \hat{\sigma}_z \rangle = Tr(\hat{\rho}^{(1)}(t)\hat{\sigma}_z)$ . In Figs. 2 (a) and 2 (b) we have plotted  $W(t)$  as a function of the scaled time  $gt$  in the zeroth- and first- order of the Lamb-Dicke approximation, respectively. In this figure and all the subsequent figures, we analyze our results based on the experimentally feasible parameters given in Refs. [16, 21]. To understand the behaviors of the population inversion in the zeroth- and first- order of the parameter  $\eta$  we consider the collapse and revival times [22]. The time required for the first collapse and the following revival of the Rabi oscillations,

denoted by  $t_c$  and  $t_r$ , respectively, can be estimated by the following analytical expressions:

$$t_c \approx \frac{\sqrt{4\eta^2 g^2 \sin^2 \varphi (\bar{n} + 1) + (\delta_{c2} + \Omega_L)^2}}{2\eta g^2 \sin^2 \varphi \sqrt{\bar{n}}}, \tag{10}$$

and

$$\begin{aligned}
t_r &\approx \frac{2\pi}{\sqrt{4\eta^2 g^2 \sin^2 \varphi (\bar{n} + 2) + (\delta_{c2} + \Delta - \delta_1)^2}} \times \\
&\frac{1}{1 - \frac{\sqrt{4\eta^2 g^2 \sin^2 \varphi (\bar{n} + 1) + (\delta_{c2} + \Delta - \delta_1)^2}}{\sqrt{4\eta^2 g^2 \sin^2 \varphi (\bar{n} + 2) + (\delta_{c2} + \Delta - \delta_1)^2}}},
\end{aligned} \tag{11}$$

where  $\bar{n}$  is the mean number of photons of the initial cavity field. It should be noted that the first revival of the oscillations occur if at least the terms oscillating with the greatest weights in  $W(t)$  acquire a phase difference of  $2\pi$ . Subsequent revivals occur at the phase differences being multiplicities of  $2\pi$ . Moreover, for the inversion to collapse, the oscillations associated with different values of photon number should be uncorrelated.

As is seen in Fig. 2 (a), in the weak cavity-field limit (low-photon number) and in the zeroth order of the Lamb-Dicke parameter, the Rabi oscillations can be identified and the atomic population inversion shows the collapse-revival repeatedly. However, in the first-order of the Lamb-Dicke approximation (Fig. 2(b)), as time goes on, the population inversion oscillates so drastically that the phenomenon of collapse-revival is not so clear. Furthermore, comparing the two figures reveals that for the both cases the atomic inversion oscillates around zero, which means that there is a same tendency of the cavity field and the atom to store the energy. Besides, in the first-order of approximation the collapse and revival times decrease in comparison with those correspond to the zeroth-order of approximation. In this manner subsequent revivals overlap and ultimately an irregular behavior of the time evolution of  $W(t)$  occurs. As we will show in the next sections, the nonclassical behaviors of the cavity field occur

in the time interval between the first collapse and the subsequent revival of the Rabi oscillations.

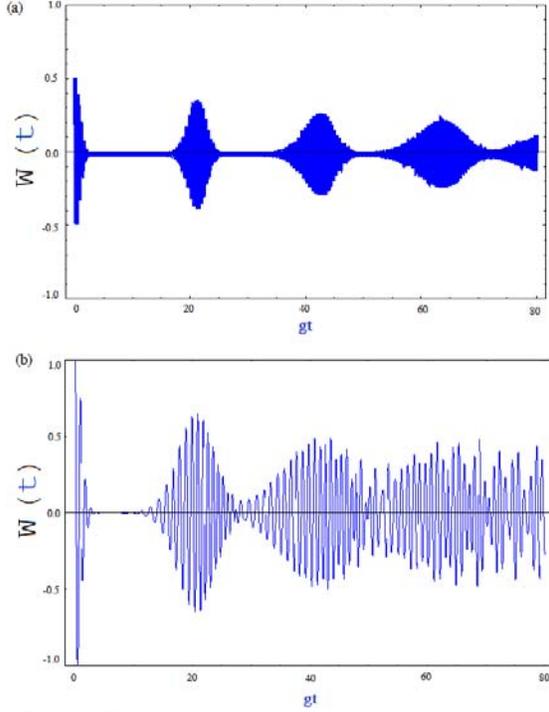


Fig. 2. Time evolution of the atomic population inversion versus the scaled time  $gt$  in the zeroth- and first-order of  $\eta$ . Here we have set: (a)  $\varphi = \frac{\pi}{3}$ ,  $\phi_L = \phi_C = 0$ ,  $\Omega_L = 2.8\text{MHz}$ ,  $\Omega_p = 2.8\text{MHz}$ , and  $\bar{n} = 0.08$ ,  $\Delta = 6\text{MHz}$ ,  $\delta_1 = 20\text{MHz}$ ,  $\delta_{c_2} = 12\text{MHz}$ ,  $g = 3.6\text{MHz}$ ,  $\kappa = 0.4\text{MHz}$ ,  $\gamma = 2.6\text{MHz}$ . (b) with the same corresponding data used in Figure 2 (a) and for  $m = 0.07$ ,  $\eta = 0.1$ .

### B. Phase-space Distribution function $Q(\alpha, \alpha^*, t)$ of the cavity field

The quasi-probability distribution functions are important tools to give insight on the statistical description of quantum dynamics. They have become customary tools of analyzing experimental results in detecting quantum states of systems like an atom oscillating in a harmonic trap or for a mode oscillator. The quasi-probability distribution function is a c-number function that allows one to calculate the expectation values of a quantum system. The calculations of the quasi-probability functions, given a density matrix, are often tedious task that involves integration over phase space variables.

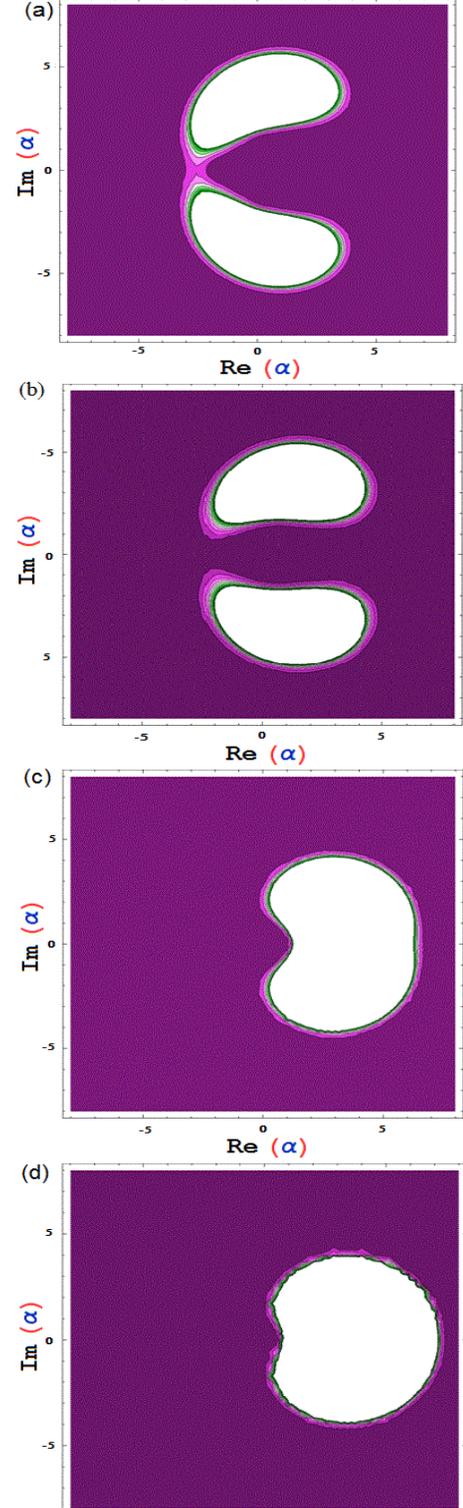


Fig. 3. Temporal evolution of the  $Q$ -function in the  $(\alpha, \alpha^*)$  complex plane with the same corresponding data used in Figure 2: (a)  $t=1/4t_r$ , (b)  $t=1/2t_r$ , (c)  $t=3/4t_r$ , and (d)  $t=t_r$ .

The exception is the  $Q$ -function, which is simply expressed as the coherent expectation

values of the field density matrix and is therefore widely adopted to describe field dynamics in situations where the density matrix is easily computed. This distribution function has no singularity problems at all. It exists for any density matrix, is bounded, and is even greater than or equal to zero. It is defined by [22]

$$Q(\alpha, \alpha^*, t) = \frac{1}{\pi} \langle \alpha | \hat{\rho}_f^{(1)}(t) | \alpha \rangle, \quad (12)$$

where  $|\alpha\rangle$  is the Glauber coherent state and  $\hat{\rho}_f^{(1)}(t) = Tr_{atom}(\hat{\rho}^{(1)}(t))$  is the reduced density matrix of the cavity-field in the first-order of the Lamb-Dicke approximation. In Fig. (3) we have sketched the density plot of the  $Q$ -function of the cavity field in the system under consideration for different times. As is seen, the  $Q$ -function splits into two blobs at  $1/4t_r$  (Fig. 3a), which represents the presence of quantum coherence in the system. Note that the cross sections of the two blobs have been squeezed along the  $Im\alpha$  axis. At  $t = 1/2t_r$  the two blobs have the same amplitude but opposite phase that demonstrates the cavity-field is in the coherent superposition of two squeezed states (Fig. 3b). At  $t = 3/4t_r$  two blobs are extended (Fig. 4c), and finally at  $t=t_r$  the two blobs join together completely which denotes that the state vector for the system cannot be written in a factored form.

### C. Quadrature squeezing of the cavity field

One of the obvious features of the non-classical radiation field is the quadrature squeezing. Squeezed light has less noise in one of the field quadratures than the vacuum level and an excess of noise in the other quadrature such that the Heisenberg uncertainty principle is satisfied. Therefore we define two quadrature  $\hat{X}_1$  and  $\hat{X}_2$  such that

$$\hat{X}_1 = \frac{\hat{a} + \hat{a}^\dagger}{2}, \quad \hat{X}_2 = \frac{\hat{a} - \hat{a}^\dagger}{2i}, \quad (13)$$

where  $[\hat{X}_1, \hat{X}_2] = i/2$ . In this case, the squeezing would occur in one of the quadratures if [22, 23]

$$S_i(t) \equiv 4\langle (\Delta X_i)^2 \rangle - 1 < 0 \quad (i=1 \text{ or } 2), \quad (14)$$

$$\text{where } \langle (\Delta X_i)^2 \rangle = \langle \hat{X}_i^2(t) \rangle - \langle \hat{X}_i(t) \rangle^2 \quad (i=1,2).$$

Now, we consider the temporal behavior of  $S_1(t)$  within the first order of the Lamb-Dicke approximation, which gives information on squeezing of  $\hat{X}_1(t)$ . Numerical result is presented in Fig. 4. We find that the quadrature squeezing occurs for short time in the very beginning of the interaction. In fact, within the first order of the Lamb-Dicke parameter, the system represents a non-linear characteristic so that by controlling the parameter  $\eta$  it is possible to achieve quadrature squeezing of the cavity field. It is interesting to note that the extending of blobs in plot of  $Q$ -function at  $t = 3/4 t_r$  and after that time (Figs. 4c, 4d) can be attributed to the effect of decoherence (the cavity field damping and atomic spontaneous emission).

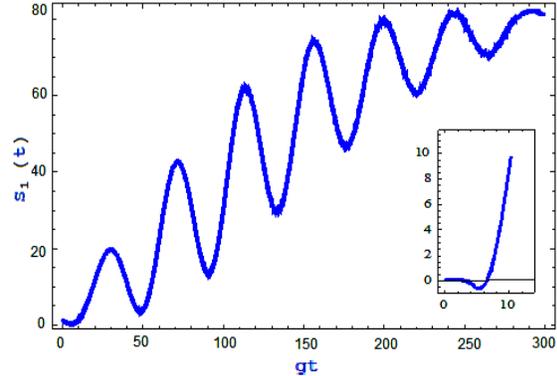


Fig. 4. Temporal evolution of the squeezing parameter  $S_1(t)$ , corresponding to the squeezing of the quadrature component  $X_1$  versus the scaled time  $gt$ , with the same corresponding data used in Figure 2. The inset shows the data at shorter times.

### D. Time evolution of the cavity-field entropy

Now, we consider the cavity-field entropy and examine the effect of the Lamb-Dicke parameter on its dynamical behavior. We use the field entropy as a measure for the degree of entanglement between the cavity field and the trapped atom. The quantum dynamics described by the effective Hamiltonian (5) leads to an entanglement between the cavity field and the trapped atom, which will be

quantified by the field entropy. As shown by Phoenix and Knight [24] the von Neumann quantum entropy is a convenient and sensitive measure of the entanglement of two interacting subsystems

$$s = -\text{Tr}(\hat{\rho} \text{Ln} \hat{\rho}), \quad (15)$$

where  $\hat{\rho}$  is the density operator for a given quantum system and we have set the Boltzmann constant  $k_B = 1$ . If  $s(t)$  takes its minimum value to be zero, the cavity field and the atom are disentangled while if  $s(t)$  takes its maximum value to be 1, the cavity field and the atom are in maximal entangled state. Araki and Lieb [25] showed that these entropies for a composite system satisfy the triangle inequality  $|s_a - s_f| \leq s \leq s_a + s_f$ . In our model, the initial state is prepared in a pure state, so the whole atom-field system remains in a pure state at any time  $t > 0$  and its entropy is always zero. However, due to the entanglement of the atom and the cavity field at  $t > 0$ , both the atom and the field are generally in mixed states, although at certain times the field and the atomic subsystems are almost in pure states. The entropies of the trapped atom and the field, are defined through the corresponding reduced density operators by

$$s_{a(f)} = -\text{Tr}_{a(f)}(\hat{\rho}_{a(f)} \text{Ln} \hat{\rho}_{a(f)}), \quad (16)$$

provided we treat both separately. Now we examine numerically the dynamics of the field entropy. The numerical results of the evolution of the field entropy versus the scaled detuning  $\delta_1/\nu$  are shown in Fig. 5 with the same corresponding data used in Fig. 2. It is seen from Fig. 5(a) that the cavity field entropy evolves periodically at  $t = 1/2t_r$ . This periodic evolution can be attributed to the periodicity of the atom-field coupling. In addition, the minimum value of the cavity-field entropy at this time is zero. Comparing results with Fig. 4(b) in which the  $Q$ -function of the field mode bifurcates two blobs having the same amplitude but opposite phase, expresses that the cavity-field is in the coherent superposition of two squeezed states.

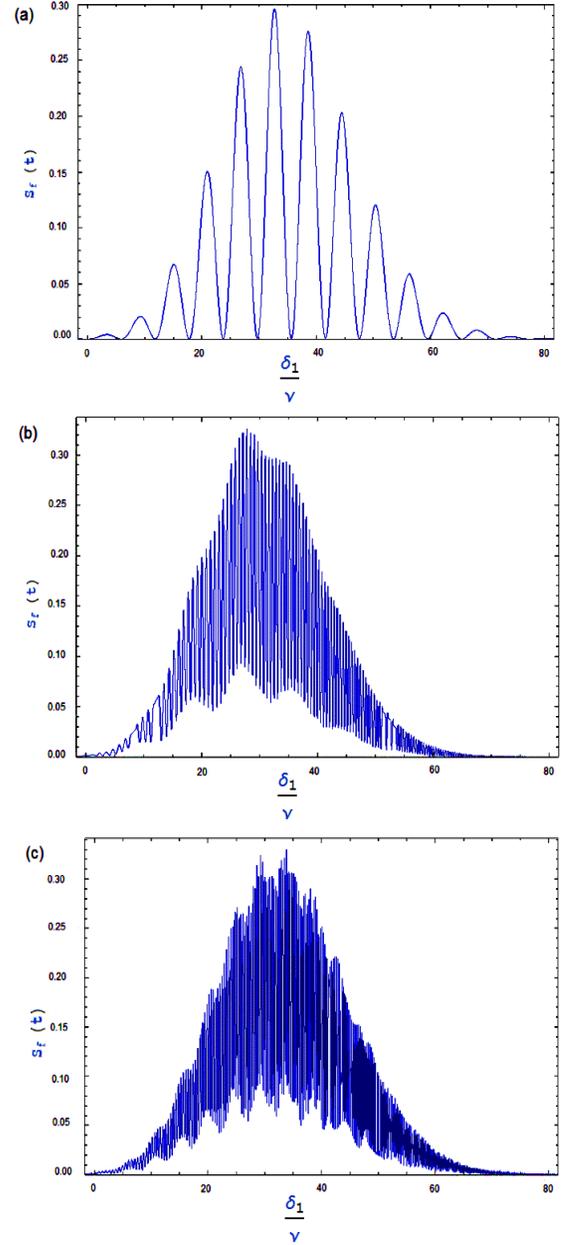


Fig. 5 Temporal evolution of the cavity-field entropy in the first order of the Lamb-Dicke parameter: (a)  $t = 1/2t_r$ , (b)  $t = 3/4t_r$ , and (c)  $t = t_r$ .

At  $t = 3/4t_r$  and  $t = t_r$  the minimum value of the entropy does not reach zero. Also, increasing the  $\eta$  parameter results in not only increasing the amplitude of the field entropy but also occurring fast oscillations in the course of time evolution of the field entropy. Physically, this is in agreement with Figs. 3(c) and 3(d) in which two blobs mixed together denoting the occurrence of decoherence in the system. In the system under consideration, both cavity-

field decay and atomic spontaneous emission are the decoherence factors which destroy the generation of non-classical state of the field.

## V. CONCLUSION

In this paper, we investigated the generation of nonclassical state for the cavity-field in a system of a single trapped atom by controlling the Lamb-Dicke parameter. We showed that although there is no apparently nonlinear characteristic in the Hamiltonian of the system of a trapped atom in the cavity, within the first order of the Lamb-Dicke approximation, the system exhibits an inherent nonlinearity. Analyzing the atomic population inversion within the first order of the Lamb-Dicke approximation indicated that the collapse and revival times decrease in comparison with those correspond to the zeroth-order of approximation. By examining the dynamical evolution of the phase space Q-function we found that the cavity-field state evolves toward a coherent superposition of two squeezed state at  $1/2\tau$ . Moreover, we showed that the quadrature squeezing occurs for short time in the very beginning of the interaction. Analyzing the entropy of the cavity field subsystem revealed that at certain times the subsystems are almost in pure states, and they are entangled at some other times. The results also indicated that the atomic spontaneous emission and the cavity-field decay destroy the nonclassical properties of the cavity field.

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