Statistics of Exciton Emission in a Semiconductor Microcavity: Detuning and Exciton-Exciton Effects

M. Bagheri Harouni and R. Roknizadeh*

Department of Physics, Quantum Optics Group, University of Isfahan, Hezar Jerib St. Isfahan 81746-73441, Iran

***Corresponding Author:** rokni@sci.ui.ac

ABSTRACT— We consider the interaction of quantum light with an ideal semiconductor microcavity. We investigate photon statistics in different conditions and the presence of detuning and exciton-exciton interaction. We show that in the resonant interaction and absence of the exciton-exciton interaction, the state of the whole system can be considered as $su(2)$ coherent state. According to our results, it turns out that photon statistics strongly depends on the initial state of the system. It is found that it is possible to generate squeezed light in the presence of the exciton-exciton interaction.

KEYWORDS: Generalized coherent states, Quantum well, Photon statistics, Non-classical states, Microcavity, Exciton

I. INTRODUCTION

The quantum state properties of the light emitted from the optical cavities containing one or more two-level atoms coupled to a single mode of electromagnetic field have been investigated in the context of cavity quantum electrodynamics (cavity QED) [1,2]. Beyond atomic physics, cavity QED is now a subject of much interest in solid state physics [3, 4] and references there in. Theoretical and experimental study in the context of solid state cavity QED brought the proof of squeezing in semiconductor microcavity with quantum well [5, 6].

Besides the coherent interaction between the excitons and photons, nonlinear interaction between the excitons plays an indispensable role on the coupled exciton-photon system. It is well-known that in the case of the low density, the excitons are approximately treated as ideal bosons which obey Bose statistics [7]. This regime is known as harmonic approximation, and in this regime the interaction of the exciton and electromagnetic radiation is completely linear. Theoretical calculation of the light intensity gives a good agreement with observed time-domain emission from the microcavity [8]. But when the density of excitons becomes relative higher, the excitons are no longer the ideal bosons. One way of dealing with this problem is to put these deviations from boson into effective interactions between the hypothetical ideal bosons as the exciton operators as still presented by bosonic operators [9]. These interactions consist of exciton-exciton interaction and phase space filling interaction. These interactions are naturally nonlinear, and it has been shown that these complex nonlinear interactions lead to the parametric amplification [10] and the generation of squeezed light in semiconductors [11].

By considering the exciton as ideal boson, we can consider the interaction of light with excitons in the framework of coupled-boson representation [12], which is a correspondence between two linear oscillators and an angular momentum oscillator. Coupled oscillators system has been extensively studied in quantum mechanics. We will investigate the interaction of the exciton and photon in a microcavity by this model. This model is extensively used for studying physical properties of exciton-photon interaction in semiconductor microcavity such as excitonic collapse and revival [13], and physical effects of some nonlinear interaction between excitons [14].

The effects of nonlinear interaction on fluorescence spectrum of exciton had been considered [14]. The deviation of excitons in high density regime from ideal bosons was investigated by the concept of q-deformed excitons [15]. Photon statistics of light in microcavity was considered by investigating damping effects [16].

In this paper we investigate the exciton-exciton and detuning effects on photon statistics of exciton emission, in the absence of damping effects in the context of coupled-boson representation. We show that in the case of the resonant interaction and absence of the exciton-exciton interaction, the dynamical state of the total system can be considered as an $su(2)$ coherent state. We illustrate that the dynamical behavior of this system depends strongly on the initial state of the system.

II. Theoretical Model

We consider a system consisting of a microcavity containing a semiconductor quantum well embedded in a high-Q cavity. We assume that the cavity and the quantum well are both ideal, and they are in an extremely low temperature situation, so that we can neglect all damping interaction due to the temperature fluctuations and phonons. The quantum well interacts with cavity field via exciton. The exciton and photon modes are quantized along the direction normal to the microcavity. Due to the translational invariance in the plane of microcavity, the photon only be dressed by the exciton with the same in-plane wave vector. In this model other exciton modes can be treated as a thermal bath for main exciton mode.

To simplify the model we will consider only one photon mode with wave vector $\vec{k}=0$ and frequency ω_c which interacts with the lowest exciton mode, i.e., $1s$ exciton. Combining the above considerations and neglecting all damping processes, we can write the following Hamiltonian for the coupled exciton-photon system [11, 17]:

$$\hat{H}=\omega_c\hat{a}^\dagger\hat{a}+\omega_{ex}\hat{b}^\dagger\hat{b}+g\left(\hat{a}^\dagger\hat{b}+\hat{b}^\dagger\hat{a}\right)+A\hat{b}^\dagger\hat{b}^\dagger\hat{b}\hat{b} \qquad (1)$$

where $\hat{b}\left(\hat{b}^\dagger\right)$ is annihilation (creation) operator of excitons with frequency ω_{ex}, $\hat{a}\left(\hat{a}^\dagger\right)$ is annihilation (creation) operator of the cavity-field. Both of these operators obey Bose statistics, i.e. $\left[\hat{b},\hat{b}^\dagger\right]=\left[\hat{a},\hat{a}^\dagger\right]=1$. As pointed out before, we describe deviation of excitons from bosonic commutation relation by an effective interaction between them. The third term in the Hamiltonian (1) stands for the exciton-photon interaction with coupling strength g which is assumed to be a real parameter. The last term describes the exciton-exciton interaction due to the Coulomb interaction. We have neglected photon-exciton saturation effect (phase-space filling) in this Hamiltonian. It is shown that this effect gives rise to small corrections as compared to exciton-exciton interaction [18]. In this Hamiltonian A is the nonlinear interaction coupling constant related to the exciton-exciton interaction. As is clear, the Hamiltonian (1) shows a coupled-boson model considered by Schwinger [12]. Dynamical evolution of this system can not be calculated in an exact way due to the presence of the nonlinear exciton-exciton interaction. According to the similarity of this model with coupled-boson model, we can study the dynamics of system by using Schwinger's angular momenta [12]. Angular momentum operators can be constructed as $\hat{j}_z=\frac{1}{2}\left(\hat{b}^\dagger\hat{b}-\hat{a}^\dagger\hat{a}\right)$, $\hat{j}_+=\hat{b}^\dagger\hat{a}$ and $\hat{j}_-=\hat{a}^\dagger\hat{b}$. The Casimir operator of the $su(2)$ algebra is $\hat{j}^2$ which can be written in this model as $\hat{j}^2=\frac{\hat{N}}{2}\left(\frac{\hat{N}}{2}+1\right)$, where the total excitation number operator $\hat{N}=\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b}$ is a constant of motion. As we know, the representation space of the $su(2)$ group is spanned by the simultaneous eigenstates of $\hat{j}^2$ and $\hat{j}_z$ which are given by:

$$|j,m\rangle = \frac{\left(\hat{b}^{\dagger}\right)^{j+m}\left(\hat{a}^{\dagger}\right)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|0\rangle, \quad (2)$$

which is the direct product of two number states with $j+m$ exciton in the quantum well and $j-m$ photons in the cavity-field. In terms of the angular momentum operators $\hat{j}_z$ and $\hat{j}_{\pm}$, the Hamiltonian (1) can be expressed as

$$\hat{H} = \frac{\hat{N}}{2}(\omega_c + \omega_{ex}) - \Delta\hat{j}_z + g\left(\hat{j}_- + \hat{j}_+\right) + A\left(\hat{j}_z^2 + \hat{j}_z\left(-2+\sqrt{1+4\hat{j}^2}\right) + \frac{\left(-1+\sqrt{1+4\hat{j}^2}\right)^2}{4} + \frac{1-\sqrt{1+4\hat{j}^2}}{2}\right), \quad (3)$$

where $\Delta = \omega_c - \omega_{ex}$ denotes the detuning between the photon frequency and the transition frequency of exciton system. As is clear, the nonlinear interaction is diagonal in the basis of $|j,m\rangle$. First, we assume that exciton system is so dilute that we can consider excitons as a free system and we neglect the interaction between excitons. Then we consider the following Hamiltonian:

$$\hat{H} = \frac{\hat{N}}{2}(\omega_c + \omega_{ex}) - \Delta\hat{j}_z + g\left(\hat{j}_- + \hat{j}_+\right). \quad (4)$$

In the interaction picture we have

$$\hat{H}_{\text{int}} = g\left(\hat{j}_- e^{i\Delta t} + \hat{j}_+ e^{-i\Delta t}\right). \quad (5)$$

In the following we shall consider the dynamical behavior of the system in the two different conditions: on-resonant and off-resonant interactions.

A. *On-resonant interaction of exciton and light*

If $\Delta = 0$, then the dynamical evolution of the system is determined by the following Hamiltonian

$$\hat{H}_{\text{int}} = g\left(\hat{j}_- + \hat{j}_+\right). \quad (6)$$

The time evolution of the above Hamiltonian is given by

$$\hat{U}(t) = e^{-igt\left(\hat{j}_- + \hat{j}_+\right)} = e^{\alpha\hat{j}_+ - \alpha^*\hat{j}_-}, \quad (7)$$

where $\alpha = -igt$ is a measure for the strength of the photon-exciton interaction. We assume at $t=0$, the cavity-field be in a number state $|n\rangle$, the quantum well be in its ground state, i.e. exciton system be in vacuum state $|0_{ex}\rangle$. The initial state can be written in terms of the eigenstates of $\hat{j}^2$ and $\hat{j}_z$ as $\left|\frac{n}{2},-\frac{n}{2}\right\rangle$. From the group theoretical point of view this state is a highest weight state for $su(2)$ representation, and the time evolution operator (7) is similar to the displacement operator of the $su(2)$ group. By the action of the time evolution operator (7) on the initial state of the system we can construct an $su(2)$ coherent state [19]

$$|\psi(t)\rangle = e^{\alpha\hat{j}_+ - \alpha^*\hat{j}_-}\left|\frac{n}{2},-\frac{n}{2}\right\rangle. \quad (8)$$

The generalized coherent states associated with the unitary representations of the $su(2)$ algebra or atomic coherent states, are parameterized by the two polar angles in the form $\xi = \frac{\theta}{2}e^{-i\phi}, (0 \le \theta \le \pi, 0 \le \phi \le 2\pi)$ [20], which determine the rotation of the state of the system on the Bloch sphere. In the present case these angles are determined by α which in turn the strength of interaction determines the rotation of system. Following the approach given in [21] for disentangling the time evolution operator, we can write the state (8) as:

$$|\psi(t)\rangle = \cos^n(gt)\, e^{-i\tan(gt)\hat{j}_+}\left|\frac{n}{2},-\frac{n}{2}\right\rangle. \quad (9)$$

Thus, the state of the total system is an $su(2)$ coherent state. This state contains information of exciton and photon system. In order to investigate the temporal behavior of the photon statistics, we should obtain the quantum state of the photon field. The density operator of the photon field is given by

$$\rho_f(t) = \cos^n(gt)\sum_{j=0}^{n}\tan^{2j}(gt)\binom{n}{j}|n-j\rangle\langle n-j|. \quad (10)$$

In the following we consider some features of quantum statistics properties of the photon field: the quadrature squeezing, photon counting statistics and purity. Squeezing is defined by reduced quantum fluctuations, below those for the vacuum state, in one quadrature phase amplitude of the field [22]. Quadrature operators are defined as:

$$\hat{X}_1 = \frac{1}{2}\left(\hat{a}e^{i\phi} + \hat{a}^{\dagger}e^{-i\phi}\right),$$
$$\hat{X}_2 = \frac{1}{2i}\left(\hat{a}e^{i\phi} - \hat{a}^{\dagger}e^{-i\phi}\right). \tag{11}$$

Then, the squeezing condition is defined as
$$S_l(\phi) = (\Delta X_l)^2 - \frac{1}{4} < 0 \ (l = 1 or\, 2), \tag{12}$$
where
$$S_1(\phi) = 2\left\langle \hat{a}^{\dagger}\hat{a}\right\rangle + 2\,\mathrm{Re}\left[\left\langle \hat{a}^2\right\rangle e^{2i\phi}\right] - 4\left(\mathrm{Re}\left[\left\langle \hat{a}\right\rangle e^{i\phi}\right]\right)^2,$$
$$S_2(\phi) = 2\left\langle \hat{a}^{\dagger}\hat{a}\right\rangle - 2\,\mathrm{Re}\left[\left\langle \hat{a}^2\right\rangle e^{2i\phi}\right] - 4\left(\mathrm{Im}\left[\left\langle \hat{a}\right\rangle e^{i\phi}\right]\right)^2, \tag{13}$$

The temporal evolution of the S_1 is plotted in Fig. 1. As shown, in this condition the radiation field does not exhibit squeezing. From a theoretical point of view squeezed states result from nonlinearities. The physical mechanism of their generation should therefore be sought in nonlinear interactions such as second [23] or higher harmonic generation [24] or parametric amplification [25].

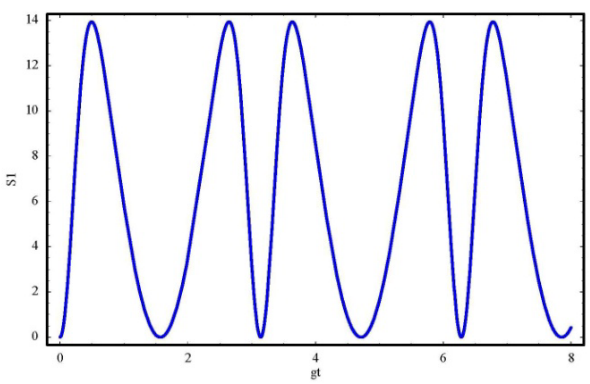

Fig. 1. Temporal evolution of S_1 versus g_t for initial photon number n=10.

Thus we expect that in this linear interaction between the exciton-photon, the cavity-field does not exhibit quadrature squeezing.

Sub-Poissonian statistics is measured with the Mandel parameter [26], that is defined as:

$$M = \frac{\left\langle \hat{n}^2\right\rangle - \left\langle \hat{n}\right\rangle^2}{\left\langle \hat{n}\right\rangle} - 1. \tag{14}$$

This parameter vanishes for the Poissonian distribution, is positive for the super-Poissonian distribution (photon bunching effect), and is negative for the sub-Poissonian distribution (photon anti-bunching effect). Fig. 2 shows the dynamical evolution of the Mandel parameter. As shown, the Mandel parameter exhibits sub-Poissonian statistics during the photon-exciton interaction. Apparently, this quantity oscillates between 0 and -1, the values of Poissonian statistics and number state, respectively. This shows that during the dynamics the field state will reduce to the number state.

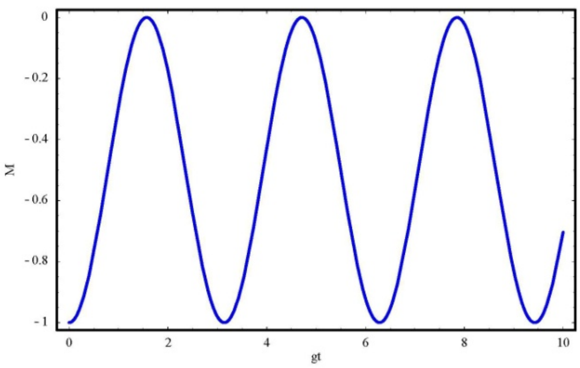

Fig. 2. Temporal evolution of Mandel parameter versus g_t for initial photon number $n = 10$.

Now, we consider the purity of photon field during its time evolution. We consider the linear entropy as a measure of the stability of the initial pure state, i.e.,

$$P = 1 - \mathrm{Tr}\left(\rho_f^2(t)\right). \tag{15}$$

In this sense, we say that an initial pure quantum state is stable if $\left(\mathrm{Tr}\left(\rho_f^2(t)\right) = 1\right)(P = 0)$ for all times. This parameter can be considered as a measure for quantum entanglement. In fact, the time evolution of the field entropy reflects the time evolution of the degree of entanglement between the cavity-field and the exciton [27]. It is clear that $\mathrm{Tr}\left(\rho_f^2(t)\right)$ takes on a maximum value of 1 if the sub-system cavity field is in a pure state; it then follows that

$P=0$ for a simple product (disentangled) state. Fig. 3 shows the temporal behavior of this parameter. It is clear that purity of the system will appear periodically during the time evolution. As shown, by increasing the n, the number of initial photons, this parameter will increase. The parameter $\mathrm{Tr}\left(\rho_f^2(t)\right)$ takes on its minimum value $1/M$, where M is a number of accessible orthogonal states of the cavity-field. Then increasing the number of the initial photons causes the purity P increased. In this case the cavity-field will recovered its initial purity during the interaction periodically.

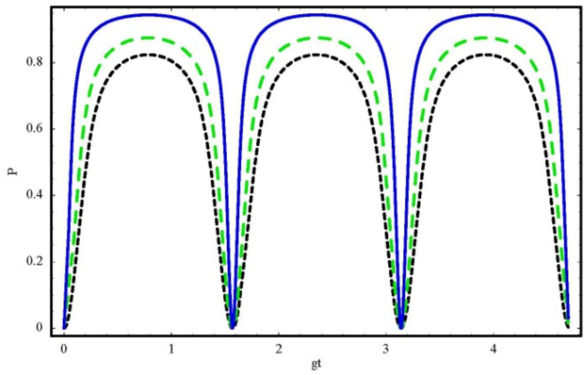

Fig. 3. Temporal evolution of purity parameter versus g_t for different initial photon number. Dashed line, long dashed and solid line ,respectively, correspond to $n=10,\ 20$ and $n=100$.

As shown, the photon counting statistics and purity of cavity-field have an oscillatory time evolution. The initial purity of the cavity field is achieved when the Mandel parameter correspond to a coherent state $(M=0)$ or a number state $(M=-1)$ of the field. According to the physical model, a linear interaction between the exciton-photon without damping effects is considered. Conformably to this model the quantum well and cavity field exchange their excitations periodically and hence, the exciton population will experience an oscillatory evolution. The oscillations in the photon counting statistics relates to the periodic evolution of the exciton population.

III. Off-Resonant Interaction of Excitons with Light

In the case of $\Delta \neq 0$, the Hamiltonian in the interaction picture is given by (5). Due to the explicit time dependence of Hamiltonian, and the fact that Hamiltonians at different times do not commute with each other, the time evolution operator may be written as:

$$\hat{U}(t)=\hat{T}\exp\left(-i\int_0^t \hat{H}_{\mathrm{int}}(t')dt'\right), \quad (16)$$

where $\hat{T}$ is the time ordering operator. We use a perturbation approach to consider the dynamics of the system under consideration. By using the Feynman disentanglement theorem [28], we can write the time evolution operator (16) as:

$$\hat{U}(t)=\exp\left(-ig\beta(t)\,\hat{j}_+\right)\hat{T} \exp\left(-ig\int_0^t\left(\hat{j}_- +2ig\beta(t')\,\hat{j}_z-\left(ig\beta(t')\right)^2\hat{j}_+\right)e^{i\Delta t'}dt'\right), \quad (17)$$

where $\beta(t)=\frac{i}{\Delta}\left(e^{-i\Delta t}-1\right)$. Now we expand the time ordered exponential up to third order of coupling constant. For determination of the quantum state of the system at any time t, we consider the action of time evolution operator $\hat{U}(t)$ on an initial state of the system. We choose the initial state of the system as before: the cavity-field in number state $|n\rangle$ and the exciton system in vacuum state. Then the state of the total system at any time $t>0$ is given by:

$$|\psi(t)\rangle=c\left[1-\frac{ng^2}{\Delta^2}\left(1+i\Delta t-e^{i\Delta t}\right)\right]e^{-ig\beta(t)\hat{j}_+}\left|\frac{n}{2},-\frac{n}{2}\right\rangle+ c\,\frac{2ig^3}{\Delta^3}\left[i\sin(\Delta t)-\Delta t\right]\sqrt{n}e^{-ig\beta(t)\hat{j}_+}\left|\frac{n}{2},-\frac{n}{2}+1\right\rangle, \quad (18)$$

where c is the appropriate normalization constant of this state. In this case, due to the absence of nonlinearity in the dynamics as pointed out before, this state does not exhibit quadrature squeezing.

The dynamical evolution of the Mandel parameter is shown in Fig. 4. As is clear in all times the cavity-field obey the sub-Poissonian distribution. This plot shows that field as a sub-system will pass through some number states periodically. On the other hand, for the large values of detuning in comparison with the coupling constant, field statistics is sub-Poissonian. With the decreasing of the detuning the field statistics approaches to the Poissonian statistics and at specific times the super-Poissonian statistics.

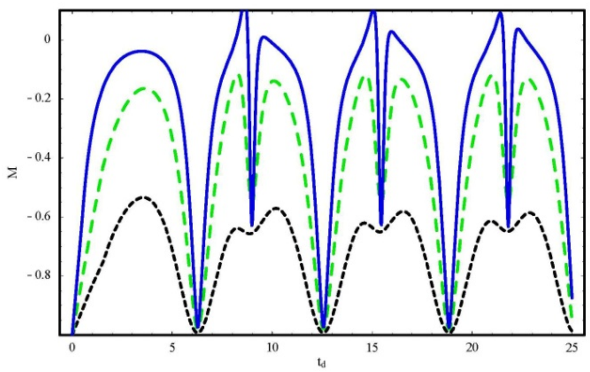

Fig. 4. Mandel parameter in the presence of detuning versus $\Delta t = t_d$. Dashed line, long dashed line and solid line, respectively, correspond to $g/\Delta = 0.5$, $g/\Delta = 1$ and $g/\Delta = 2$. For all figures $n = 10$.

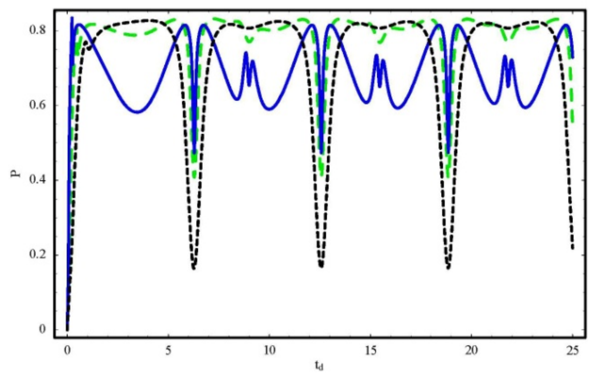

Fig. 5. Purity parameter in the presence of detuning versus $\Delta t = t_d$. In this plot $n = 10$. Dashed line, long dashed line and solid line, respectively, correspond to $g/\Delta = 0.5$, $g/\Delta = 1$ and $g/\Delta = 2$.

The plots of purity parameter (15) in this case are depicted in Fig. 5. As shown, for the small values of g/Δ it is more probable for the photon field to obtain its initial purity, while for finite vales of this ratio, the purity never becomes zero. This intimates that when the detuning is large, there is a tendency of the cavity-field to preserve its initial purity. On the other hand, when the ratio g/Δ is large, the oscillatory behavior of purity becomes more apparent.

IV. Photon Statistics in the Presence of Exciton-Exciton Interaction

In this section we shall consider the influence of exciton-exciton interaction on the photon statistics. By taking into account the exciton-exciton interaction, the total Hamiltonian of the system is given by (3). The last term in this Hamiltonian relates to the exciton-exciton interaction with coupling constant A. This nonlinear interaction is diagonal in the basis of $|j,m\rangle$ (direct product of exciton and photon number states). Due to this property we consider an interaction picture with respect to the free part of the Hamiltonian and diagonal exciton-exciton interaction. Therefore we obtain the exciton-photon interaction Hamiltonian in the interaction picture with respect to the exciton-exciton induced evolution

$$\hat H_{\text{int}} = e^{i\hat H_0 t}\hat H' e^{-i\hat H_0 t}, \qquad (19)$$

where $\hat H' = g\left(\hat j_+ + \hat j_-\right)$ and

$$\hat H_0 = \frac{\hat N}{2}(\omega_c+\omega_{ex}) - \Delta\hat j_z + A\left(\hat j_z^2 + \hat j_z\left(-2+\sqrt{1+4\hat j^2}\right) + \frac{\left(-1+\sqrt{1+4\hat j^2}\right)^2}{4} + \frac{1-\sqrt{1+4\hat j^2}}{2}\right). \qquad (20)$$

By using the $su(2)$ algebra the above Hamiltonian in the interaction picture is written as

$$\hat H_{\text{int}}(t) = g\left(\hat j_+ e^{iAt\left(1+2\hat j_z\right)} e^{iAt\left(-2+\sqrt{1+4\hat j^2}\right)} e^{-i\Delta t} + \hat j_- e^{iAt\left(1-2\hat j_z\right)} e^{-iAt\left(-2+\sqrt{1+4\hat j^2}\right)} e^{i\Delta t}\right). \qquad (21)$$

In this equation $\hat j^2$ is the Casimir operator of the $su(2)$ algebra, and $\Delta = \omega_c - \omega_{ex}$ is the detuning parameter. This Hamiltonian can be written in the form of $\hat H_{\text{int}} = g\left(\hat j'_+ e^{-i\Delta t} + \hat j'_- e^{i\Delta t}\right)$, where we introduce the generalized $su(2)$ ladder operators as:

$$\hat{j}'_+ = \hat{j}_+ e^{-iAt\left(1-2\hat{j}_z\right)} e^{+iAt\left(-2+\sqrt{1+4\hat{j}^2}\right)},$$
$$\hat{j}'_- = \hat{j}_- e^{+iAt\left(1-2\hat{j}_z\right)} e^{-iAt\left(-2+\sqrt{1+4\hat{j}^2}\right)}, \qquad (22)$$

These operators are similar to the definition of generalized f-deformed operators [29], which are constructed from the ordinary ladder operators and a multiplicative function of constants of motion. In our case the operator $\hat{j}^2$ is a constant of motion. However, the generalized operators (22) have the same commutation relation as ordinary $su(2)$ ladder operators:

$$\left[\hat{j}'_+, \hat{j}'_-\right] = 2\hat{j}_z. \qquad (23)$$

The time dependence of the Hamiltonian (21) is due to the presence of the detuning and the exciton-exciton interaction. By neglecting the detuning $(\Delta = 0)$ and the exciton-exciton interaction $(A = 0)$, this Hamiltonian reduces to the Hamiltonian (6). To consider the dynamics of the system, we use a perturbation expansion of the corresponding time evolution operator. Up to the second order of perturbation the time evolution operator can be written as:

$$\hat{U}(t) = 1 - i\int_0^t \hat{H}_{\text{int}}(t')\,dt' - \int_0^t \hat{H}_{\text{int}}(t')\,dt' \int_0^{t'} \hat{H}_{\text{int}}(t'')\,dt''. \qquad (24)$$

We choose the initial state of the system as before: the cavity-field is prepared in a Fock state $|n\rangle$ and the quantum well in its excitonic ground state. The quantum state of the system at time t is given by:

$$|\psi(t)\rangle = c\left(A_1(t)\left|\frac{n}{2}, -\frac{n}{2}\right\rangle + A_2(t)\left|\frac{n}{2}, -\frac{n}{2}+1\right\rangle + A_3(t)\left|\frac{n}{2}, -\frac{n}{2}+2\right\rangle \right), \qquad (25)$$

where c is a normalization constant, $|c|^2 = \left(|A_1(t)|^2 + |A_2(t)|^2 + |A3(t)|^2\right)^{-1}$, and the coefficients $A_1(t)$, $A_2(t)$ and $A_3(t)$ are, respectively, given by:

$$A_1(t) = 1 - n\frac{g^2}{A^2}\frac{iAt\, f\left(-\frac{n}{2}+1, l\right) - e^{iAt\, f\left(-\frac{n}{2}+1, l\right)} + 1}{f^2\left(-\frac{n}{2}+1, l\right)},$$

$$A_2(t) = \sqrt{n}\,\frac{g}{A}\frac{e^{-iAt\, f\left(-\frac{n}{2}+1, l\right)} - 1}{f\left(-\frac{n}{2}+1, l\right)},$$

$$A_3(t) = \sqrt{2n(n+1)}\,\frac{g^2}{A^2}\Bigg[f\left(-\frac{n}{2}+1, l\right) + f\left(-\frac{n}{2}+2, l\right) e^{-iAt\left(f\left(-\frac{n}{2}+2, l\right) + f\left(-\frac{n}{2}+1, l\right)\right)} + \left(f\left(-\frac{n}{2}+2, l\right) + f\left(-\frac{n}{2}+1, l\right)\right) e^{iAt\, f\left(-\frac{n}{2}+2, l\right)} \Bigg] \Big/ \left(\left(f\left(-\frac{n}{2}+2, l\right) + f\left(-\frac{n}{2}+1, l\right)\right) f\left(-\frac{n}{2}+1, l\right) \times f\left(-\frac{n}{2}+2, l\right)\right),$$

which $f(x, l) = 2 - 3x - l + \Delta/A$, where l is the number of photons in the initial state. By using this state we can determine photon statistics.

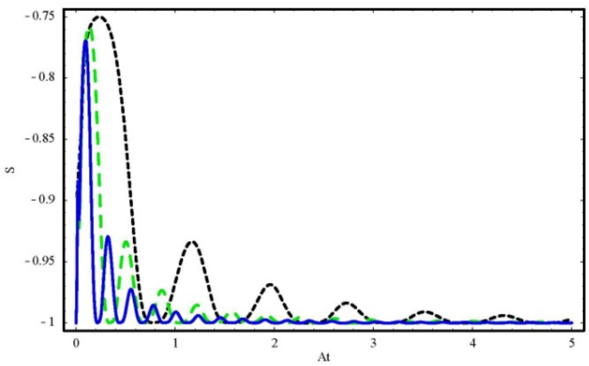

Fig. 6. Squeezing in the presence of detuning and exciton-exciton interaction versus A_t. In this plot $n = 10$ and $g/A = 10$. Dashed line and solid line, respectively, correspond to $\Delta/A = 20$ and $\Delta/A = 40$.

Fig. 6 represents the quadrature squeezing in the photon field. As shown, in this condition the state does not show quadrature squeezing. However, there is a difference between this result and previous one. In the absence of exciton-exciton interaction, Fig. 1, the quadrature squeezing exhibits an oscillatory behavior, while in the presence of the exciton-exciton interaction the variation of squeezing parameter becomes stable. It is apparent that the change of detuning parameter will alter the period of oscillations of quadrature squeezing.

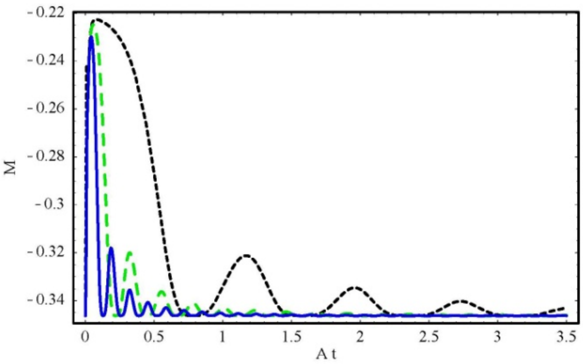

Fig. 7. Mandel parameter in the presence of detuning and exciton-exciton interaction versus A_t. In this plot $n=10$ and $g/A=10$. Dashed line, long dashed line and solid line, respectively, correspond to $\Delta/A=10$, $\Delta/A=20$, and $\Delta/A=30$.

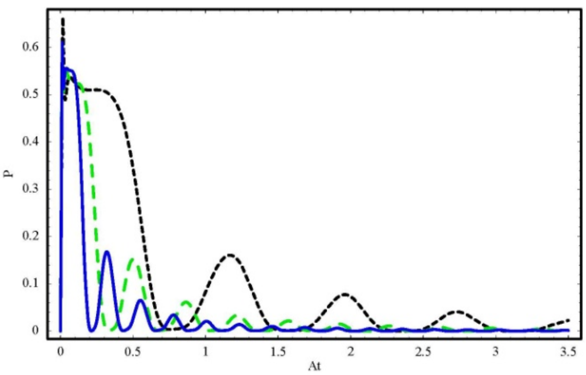

Fig. 8. Purity in the presence of detuning and exciton-exciton interaction versus A_t. For all plots $n=5$ and $g/A=20$. For dashed line, long dashed line and solid line we have, respectively, $\Delta/A=10$, $\Delta/A=20$ and $\Delta/A=30$.

Fig. 7 shows dynamical evolution of the Mandel parameter. As shown, the photon-field shows sub-Poissonian statistics, while the oscillatory behavior of this parameter is suppressed and eventually the Mandel parameter is stabilized at an asymptotically value. Similar to the squeezing case, the detuning affects the periods of oscillation. It is apparent that in this case the cavity field statistics leads to the number state statistics. In Fig. 8 we have plotted the purity. This figure shows that the amplitude of oscillations is relatively small and the exciton-exciton interaction prevents the oscillation between pure and mixed states. After some time this parameter becomes stable. In the case of specific initial conditions, the exciton-exciton interaction prevents oscillations in photon statistics and it tries to make photon statistics stable.

To achieve a more clear insight to the effects of the exciton-exciton interaction and initial state on the photon statistics, we consider another initial state for the system. We assume at $t=0$ the quantum well be in the ground state and the cavity field be in a coherent state $|\alpha\rangle$. Then the initial state of the system is given by $|\psi(0)\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}} \left| \frac{n}{2}, -\frac{n}{2} \right\rangle$. By the same procedure as before we can derive the quantum state of system at time *t*. The quadrature squeezing is plotted in Fig. 9 for a fixed value of the detuning parameter and coupling constant and definite values of the intensity of initial photon coherent state, $|\alpha|^2$.

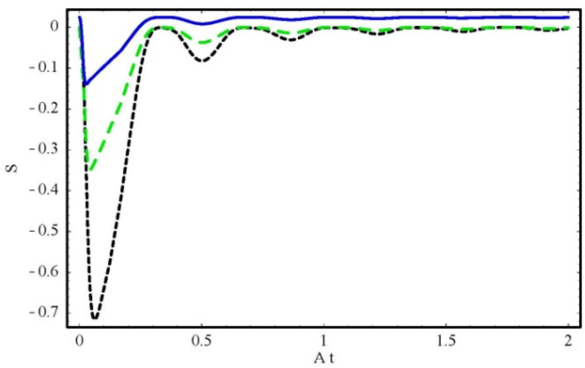

Fig. 9. Squeezing in the presence of detuning and exciton-exciton interaction versus A_t for initial coherent state. In all plots, $g/A=20$ and $\Delta/A=20$. Dashed line, long dashed line and solid line, respectively, corresponding to intensity of initial field as $|\alpha|^2=2.5$, $|\alpha|^2=4$ and $|\alpha|^2=9$.

As shown, when the initial intensity of the field is small enough, the state exhibits squeezing. In contrast to the previous cases, in this case we can generate squeezed light. As mentioned before, theoretical and experimental schemes for generation of squeezed state are based on nonlinear interactions. In the presence of the exciton-exciton interaction, this interaction insures the nonlinear nature of the interaction and hence, the cavity-field exhibits quadrature squeezing. On the other hand, when we choose the initial state of the cavity-field as a number state, the cavity-field did not show quadrature squeezing. This is related to the statistical properties of the number states which they do not show quadrature squeezing. Then we consider the effects of detuning while the intensity of initial coherent state, $|\alpha|^2$, is constant. In Fig. 10 the plots of squeezing parameter for different values of ratio Δ/A are depicted. As shown, with increasing the value of Δ/A the oscillation of squeezing becomes more pronounced and it becomes stable more rapidly. We see that by decreasing the value of detuning the quadrature squeezing occurs more frequently.

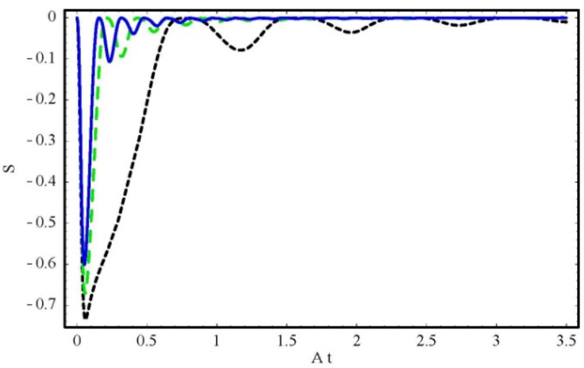

Fig. 10. Squeezing in the presence of detuning and exciton-exciton interaction versus A_t for initial coherent state (detuning effects). In all plots, $g/A = 20$ and $|\alpha|^2 = 2.5$. Dashed line, long dashed line and solid line, respectively, correspond to $\Delta/A = 10$, $\Delta/A = 20$ and $\Delta/A = 30$.

V. Conclusion

In this paper, we considered the interaction of a single-mode cavity-field with a quantum well in an ideal microcavity in the absence of damping effects. We have seen, in the case of resonant interaction and absence of the exciton-exciton interaction, the state of the total system can be considered as a $su(2)$ coherent state. Then we take into account the influence of the detuning and the exciton-exciton interaction on the photon statistics. We have found that when the initial state of the cavity-field is prepared in a Fock state, there is no quadrature squeezing, while with choosing the initial state as a coherent state with small intensity, the quadrature squeezing occurs in the course of the time evolution. On the other hand, we showed that by changing the detuning parameter, we can maintain squeezing in more time intervals. Also, we considered the exciton-exciton interaction and its effects on the photon statistics. We showed that this interaction makes the photon statistics stable and suppresses its oscillatory behavior.

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Malek Bagheri Harouni was born in Isfahan in 1981. He was graduated from Bu-Ali-Sina University in Hamedan in 2001. Moreover, he finished his master thesis in theoretical physics at Shahid Beheshti University of Tehran in 2003. He got his Ph.D. degree in theoretical physics from the University of Isfahan in 2008.

Malek Bagheri Harouni is a theoretical physicist in the field of quantum optics. His Ph.D. thesis is about quantum optical properties of nano-structures, specially semiconductor nano-structures. At the present, he works at the University of Isfahan as an assistant professor. His researches are focused on quantum optics of nano-structures and quantum dynamics in complex systems.

Rasoul Roknizadeh is professor of theoretical physics in Department of Physics, University of Isfahan. Quantum Optics, generalized coherent states and theoretical and mathematical foundation of quantum mechanics are some of his research areas.

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