

# Dynamics of Entanglement between Linearly Polarized Photons and $\pi$ -Electrons in Graphene

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**ABSTRACT**— In the present article we report the dynamical behavior of entanglement between  $\pi$ -electrons and photons in Graphene. It is shown that the degree of such entanglements depend on the orientation of  $\pi$ -electron momenta relative to the photonic polarization. Moreover, we show that as the detuning between the  $\pi$ -electron transition frequencies and that of the photons is increased, the degree of entanglement decreases.

**KEYWORDS:** Graphene,  $\pi$ -electron, Photon, Entanglement.

## I. INTRODUCTION

Graphene, a two-dimensional sheet of graphite, because of its extraordinary properties, has recently attracted much interest [1-7]. Such an interest originates from the fact that in graphene the so-called  $\pi$ -electrons are almost free with long-mean-free path, phase-coherence length, etc. [2-4]. These properties have made it feasible to use graphene, or layers of it, to develop spintronic devices [5-7]. Furthermore, nanoscaled structures (wires, dots, etc.) made of graphene have been proposed for use in the development of quantum computers, quantum communication channels and so forth [8-10]. A major role in these applications is played by quantum entangled states [11-13]. Quantum entangled states have been well established to form the corner stone of quantum information processing, including quantum communication, quantum teleportation, cryptography, etc. [14-17]. In fact, for such purposes it is required that physical information contained in a composite system

be either local or distributed amongst the subsystems, giving rise to entangled states [18, 19]. The research on the entanglement then focuses on two subjects: How the entanglement is implemented and how it may be quantified [20, 21]. It is therefore the main aim of the present work to report the properties of entanglement between  $\pi$ -electrons and photons in graphene [22, 23]. Neglecting the spin states, the state of  $\pi$ -electrons in graphene, near the Dirac points, can be described by the eigenstates of single-particle Hamiltonian,  $H = v_f(\boldsymbol{\sigma}\cdot\mathbf{p})$  where  $v_f$ ,  $\boldsymbol{\sigma}$  and  $\mathbf{p}$  denote the Fermi speed, *pseudo-spin* and the momentum, respectively [1, 24]. Here the *pseudo-spin* operators act on the graphene "lattice sites" [1]. When the  $\pi$ -electron interacts with an external electromagnetic field, the lattice sites and the field are coupled via  $\boldsymbol{\sigma}$  and  $\mathbf{p} \rightarrow \mathbf{p} + \frac{e}{c}\mathbf{A}$ , where  $\mathbf{A}$  is the vector potential [25]. This fact, as we shall see, leads to the entanglement between  $\pi$ -electrons and photons in graphene.

In this article, therefore, we consider a  $\pi$ -electron in graphene, interacting with a one-mode quantized linearly polarized electromagnetic field and report the dynamical behavior of entanglement between *pseudo-spin* (conduction and valance bands) [26, 27] and photonic states. To this end, we add the interaction Hamiltonian to that of  $\pi$ -electron's and proceed to calculate the time-evolution operator (TEO). In what follows, however, dissipative effects (such as losses in the graphene itself, lossy medium where the

interaction occurs, etc.) have been ignored. In this regard we emphasize that the inclusion of any type of dissipation into the system leads to mixed states (ensembles) for which one has to appeal to, for instance, a master equation to determine the density operator [28].

Using the so calculated TEO, we then determine the von-Neumann entropy, a measure of entanglement [29, 30], as a function of time. We thus show that the degree of entanglement between  $\pi$ -electron's states and photons in graphene strongly depends upon the photonic frequency and its polarization. From an analysis of the von-Neumann entropy, we demonstrate that by adjusting the photonic frequency and its polarization, one can control the degree of entanglement. In fact, when the photonic frequency is in resonance with inter-band transitions, the entanglement is drastically enhanced. Moreover, when the electronic momentum is along the photonic polarization the entanglement vanishes, while in the perpendicular case it is maximal. We further show that in graphene the entanglement is stronger than its ordinary counterparts, such as the entanglement of two-level atoms and photons [31].

The present article is organized in the following manner. After the introduction, we devote section *II* to a discussion of the minimal coupling Hamiltonian which governs the dynamical behavior of the system of photons and  $\pi$ -electrons. We then proceed to the procedure with which the dynamics of entanglement is determined in section *III*. Section *IV* is devoted to the numerical results of our calculations and concluding remarks are made in section *V*.

## II. THE HAMILTONIAN MODEL

The states of  $\pi$ -electrons in a graphene sheet, lying in the x-y plane, is governed by the Hamiltonian [1]:

$$H = v_f |\mathbf{p}| (\boldsymbol{\sigma} \cdot \mathbf{p}) \quad (1)$$

where  $v_f$  is the electronic Fermi speed,  $|\mathbf{p}|$  and  $\hat{p}$  represent magnitude and direction of momentum, respectively.  $\boldsymbol{\sigma}$  represents the Pauli *pseudo-spin* matrices, acting on graphene sublattices [24]. In writing Eq. (1) it is assumed that the momentum lies around a Dirac point and tight-binding approximation has been employed [32]. The eigenvalues and eigenstates of the Hamiltonian in Eq. (1) are easily obtained as,

$$|\psi_j\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \beta e^{i\varphi} \end{pmatrix} \quad (2)$$

with  $E_j = \beta v_f |\mathbf{p}|$  and  $\beta = \pm 1$  for  $j = 1, 2$ , respectively. It is noted that the eigenstates given in Eq. (2) are, in fact, particular combinations of sublattice sites and the angle  $\varphi$  is defined as  $\tan^{-1}(p_y/p_x)$ . When the graphene sheet is influenced by a single mode quantized electromagnetic field, the Hamiltonian of Eq. (1), in the minimal coupling regime,  $\mathbf{p} \rightarrow \mathbf{p} + \frac{e}{c} \mathbf{A}$ , becomes

$$H = \hbar\omega \left( a^\dagger a + \frac{1}{2} \right) + v_f (\boldsymbol{\sigma} \cdot \mathbf{p}) + \hbar g (\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\varepsilon}}) (a^\dagger + a) \quad (3)$$

where the electromagnetic field is assumed to propagate in a direction normal to graphene sheet with its polarization coplanar with it. In Eq. (3)  $a^\dagger(a)$  and  $\hat{\boldsymbol{\varepsilon}}$  represent the photonic creation (annihilation) operator and polarization unit vector, respectively. It is clear from the last term of Eq. (3) that the  $\pi$ -electrons and photons are coupled through  $g (= \frac{e}{\hbar c} v_f \sqrt{2\pi\hbar c^2/\omega V}$  in standard notations).

It is noted that this coupling is, under the same conditions, about ten times stronger than that of photon-two-level atoms, consequently, a larger photon- $\pi$ -electron entanglement is expected. This point will be further addressed in section IV.

To clarify the effect of photon and  $\pi$ -electron interaction, the last term of Eq. (3), we cast the Hamiltonian into a different form. Noting that,

$$(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\varepsilon}})|\psi_1\rangle = -i \text{Sin}\gamma |\psi_2\rangle + \text{Cos}\gamma |\psi_1\rangle \quad (4)$$

and

$$(\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\varepsilon}})|\psi_2\rangle = -\text{Cos}\gamma |\psi_2\rangle + i \text{Sin}\gamma |\psi_1\rangle \quad (5)$$

where  $\gamma$  is the angle between photonic polarization and electronic momentum, one finds,

$$\begin{aligned} H = & \hbar\omega \left( a^\dagger a + \frac{1}{2} \right) + \frac{1}{2} \hbar\Omega \tau_z \\ & + \hbar g \text{Cos}\gamma (a^\dagger + a) \tau_z \\ & + i \hbar g \text{Sin}\gamma (a^\dagger + a) (\tau_+ - \tau_-) \end{aligned} \quad (6)$$

In obtaining Eq. (6) new Pauli matrices,  $\boldsymbol{\tau}$ , with bases  $|\psi_j\rangle$ , such that

$$\tau_z = |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2| \quad (7)$$

and

$$\tau_+ = \tau_x + i\tau_y = |\psi_1\rangle\langle\psi_2| = \tau_-^\dagger \quad (8)$$

have been introduced. The transition frequency between  $\pi$ -electron states,  $2v_f |\mathbf{p}|/\hbar$ , is denoted by  $\Omega$ . From Eq. (6) it is clear that the bases are formed by  $|n, \psi_j\rangle = |n\rangle \otimes |\psi_j\rangle$ ;  $j=1,2$ , where  $|n\rangle$  denotes a photonic state. We further note that the last two terms of Eq. (6) mix the photonic and electronic states, giving rise to photon and  $\pi$ -electron entanglement in graphene.

As a matter of comparison with (two-level) atom-photon Jaynes–Cummings (J-C) Hamiltonian, in the rotating wave approximation [31], Eq. (6) contains two additional terms of the forms  $(a^\dagger + a)\tau_z$ ,  $a\tau_-$ , etc. This fact, in addition to a stronger photon and  $\pi$ -electron coupling, is then expected to enhance the corresponding entanglements. The strength of these two

terms involves the angle  $\gamma$  which in turn maybe used to control the entanglement. This fact indicates that, generally, photons and  $\pi$ -electrons in graphene are more correlated (entangled) than atoms and photons in standard J-C models. In the next section we use the Hamiltonian of Eq. (6) to construct the time evolution operator and, consequently, determine the dynamics of photon and  $\pi$ -electron entanglement.

### III. DYNAMICS OF THE VON NEUMANN ENTROPY

Although there are a number of measures to quantify entanglement [29], we choose to use the von Neumann entropy which maybe directly related to information theories [30]. For a bipartite system containing parts A and B, the von Neumann measure of entanglement is defined as,

$$\begin{aligned} S_{AB} = & -\text{Tr}[\rho_A \log_2 \rho_A] \\ = & -\text{Tr}[\rho_B \log_2 \rho_B] = S_{BA}, \end{aligned} \quad (9)$$

where the reduced density matrices are

$$\rho_{A(B)} = -\text{Tr}_{B(A)}[\rho_{AB}] \quad (10)$$

In Eq. (10), the total density operator (matrix) is defined as  $\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|$ , with  $|\psi_{AB}\rangle$  being the state of the combined system. If the state of the composite is not a stationary one, from the definition of  $\rho_{AB}$ , it is clear that the degree of entanglement evolves with time. The evolution of entanglement is then determine by the action of time-evolution operator,  $U = \exp(-iHt/\hbar)$ , on the initial state. In what follows we describe the procedure of computing the evolution of entanglement.

When the Hamiltonian of Eq. (6) acts upon the identity in the Hilbert space of the whole system  $\sum_{n,j=1,2} |n, \psi_j\rangle\langle n, \psi_j|$  one finds that it has non-vanishing matrix elements with respect to the states,

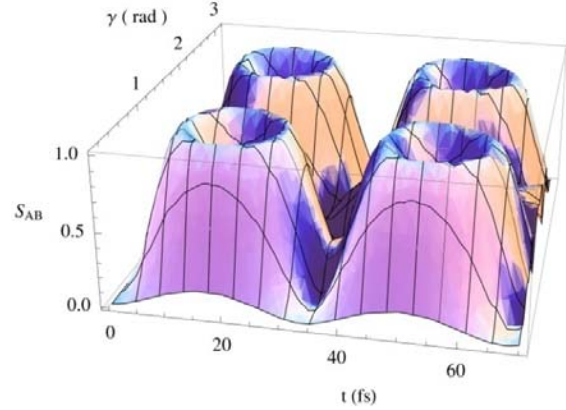
$$\begin{aligned}
|1\rangle &= |n, \psi_1\rangle & |2\rangle &= |n-1, \psi_1\rangle, \\
|3\rangle &= |n+1, \psi_1\rangle & |4\rangle &= |n, \psi_2\rangle, \\
|5\rangle &= |n-1, \psi_2\rangle & |6\rangle &= |n+1, \psi_2\rangle,
\end{aligned} \tag{11}$$

These states are marked "original" in what follows. Parenthetically, we mention that the energy nonconserving states, like  $|2\rangle, |3\rangle$ , etc. in Eq. (11) originate from operators,  $a\tau_z, a^\dagger\tau_z$ , etc., in the Hamiltonian, which, in turn, derives energy nonconserving transitions. The  $6 \times 6$  matrix representation of  $H$  is then diagonalized giving 6 "dressed" states. In the dressed state representation the time-evolution operator is also diagonal with elements  $U_{ii} = \exp(-iE_i t/\hbar)$ , where  $E_i, i=1, \dots, 6$  are the energy eigenvalues. Transformation of the diagonal representation,  $U_{ii}$ , to the original bases is then achieved by the action of the transformation matrix, formed by inner products of the "original" and "dressed" states. In the next section the aforementioned procedure is employed to investigate the entanglement between photonic and  $\pi$ -electron states in graphene.

#### IV. NUMERICAL RESULTS

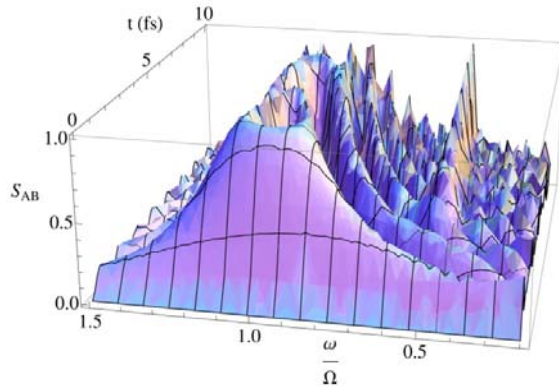
As was mentioned in the previous section, the time evolution of entanglement, as given in Eq. (10), is determined by the eigenvalues of the Hamiltonian in Eq. (6). To calculate these eigenvalues one encounters a sixth-order algebraic equation, for which no analytical solutions exist. In this section, therefore, we numerically present the time variation of von Neumann entropy, Eq. (10), as the degree of photon and  $\pi$ -electron entanglement. To this end, we assume that initially the system is prepared in a separable (unentangled) state  $|\Psi_0\rangle = |5\rangle \times |\psi_1\rangle$ . This defines a state in which the  $\pi$ -electron is described by  $|\psi_1\rangle$  of Eq. (2) and five photons are present. The time evolution of the photon and  $\pi$ -electron entanglement is then computed *via* the procedure described at the end of the previous section. This procedure and a glance at the Hamiltonian of Eq. (6) indicate that the dynamics of entanglement between photons and  $\pi$ -electrons strongly depends upon the

angle  $\gamma$  and the detuning,  $\Delta = \Omega|1 - \omega/\Omega|$ , measured in units of  $\omega/\Omega$ . We note that in this unit the detuning is symmetric about the resonance,  $\omega/\Omega = 1$ , and increases away from this value. Therefore, in Fig. 1 we illustrate the behavior of entanglement versus  $\gamma$  and time, and in Fig. 2 the same is depicted for different detuning, respectively.



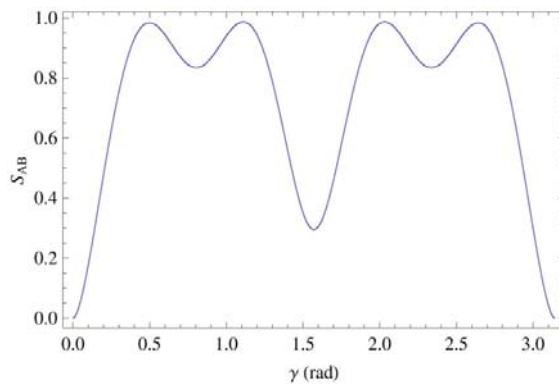
**Fig. 1.** Degree of entanglement,  $S_{AB}$ , versus time and the angle between photonic polarization and  $\pi$ -electronic momentum.

For clarity, the behavior of entanglement, at a particular instant of time (projections of Fig. 1, Fig. 2 onto the time axis) is depicted in Fig. 3, Fig. 4, as functions of  $\gamma$  and  $\omega/\Omega$ , respectively. It is observed from these that, as expected for bipartite systems, the entanglement behaves oscillatory whose characteristics, period, extrema, *etc.* depend upon  $\gamma$  and  $\omega/\Omega$ . Consideration of Fig. 1, Fig. 3 indicates that when photonic polarization and  $\pi$ -electronic momentum are parallel,  $\gamma = 0, \pi$ , no entanglement occurs. This is a reasonable result since the last term of Eq. (6), that mixes the  $\pi$ -electron and photonic states, vanishes at these angles. An increase in this angle, generally, enhances the entanglement up to its maximum value of one.



**Fig. 2.** Degree of entanglement,  $S_{AB}$ , versus time and detuning of photon and  $\pi$ -electron frequencies, as measured in units of  $\omega/\Omega$ , at  $\gamma = \pi/3$ .

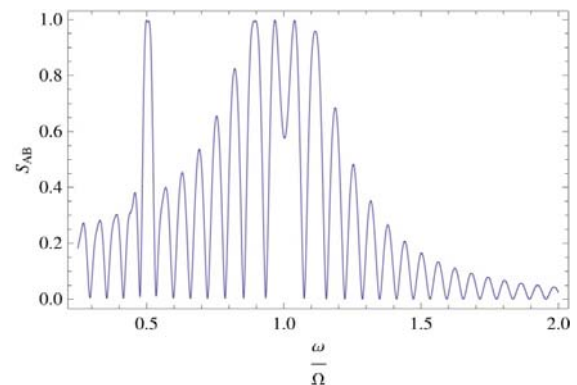
Moreover, it is observed from Fig. 2, Fig. 4 that as the detuning is decreased the photon and  $\pi$ -electron states become more entangled, again reaching its maximum value of one at the resonances,  $\omega/\Omega = 1$ . This result is also reasonable since as the detuning increases then transition between states become unlikely, giving smaller entanglements.



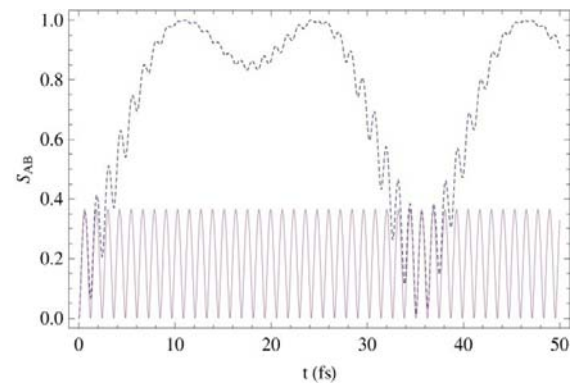
**Fig. 3.** Degree of entanglement,  $S_{AB}$ , versus the angle between photonic polarization and  $\pi$ -electronic momentum at a fixed time,  $t=10$  fs, and detuning,  $\omega/\Omega = 0.5$ .

As a matter of comparison, the behavior of entanglement between two-level atoms (atomic qubits) and photons, in the celebrated Jaynes–Cummings model along with that of  $\pi$ -electron (electronic qubits) in graphene is also illustrated in Fig. 5. From this figure it is concluded that the  $\pi$ -electrons and photons become more correlated in graphene than the system of atom-photon. The reason for such enhancement in the entanglement, as

compared to that of two level atoms and photons is the fact that  $\pi$ -electron is coupled to the photons more strongly along with additional terms in the interaction (consequently more state are involved in the entanglement). Moreover, the entanglement oscillations occur at a smaller frequencies (longer periods) for  $\pi$ -electrons compare to that of two-level atoms. This is again, as a simple calculation starting from partial tracing of the density matrix and using completeness of the eigenstates indicates, is due to the stronger coupling for the former.



**Fig. 4.** Degree of entanglement,  $S_{AB}$ , versus detuning, as measured in units of  $\omega/\Omega$  at a fixed time,  $t=10$  fs and  $\gamma = \pi/3$ .



**Fig. 5** Degree of entanglement,  $S_{AB}$ , for the system of atom-photon (solid curve) and  $\pi$ -electron-photon (dashed curve). The curves are drawn for equal coupling strengths and  $\gamma = \pi/3$  and  $\omega/\Omega = 0.5$  (dashed curve).

## V. CONCLUSION

In the present work we report the dynamics of photon and  $\pi$ -electron entanglement in graphene. Although the results of our

investigation are thoroughly discussed in section *IV*, we outline the more important aspects of this report in this section.

- The degree of entanglement between the  $\pi$ -electrons and photons strongly depends upon the orientation of electronic momentum and photonic polarization, when they are parallel no entanglement occurs.
- The degree of such an entanglement also depends on the photon and  $\pi$ -electron energy detuning; away from resonance it decreases, while at resonance the entanglement reaches its maximum.
- The system of  $\pi$ -electrons (qubits) and photons in graphene is more correlated (entangled) than that of two-level atoms (atomic qubits) and photons in the Jaynes–Cummings models.

In brief, the material presented in this article provides novel means of creation and control of entanglement

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#### REFERENCES

- [1] A.K. Geim and K.S. Novoselov, “The rise of graphene,” *Nat. Mater.*, Vol. 6, pp. 183-191, 2007.
- [2] F. Miao, S. Wijeratne, Y. Zhang, U.C. Coskun, W. Bao, and C.N. Lau, “Phase-coherent transport in graphene quantum billiards,” *science*, Vol. 317, pp. 1530-1533, 2007.
- [3] D. Huertas-Hernando, F. Guinea, and A. Brataas, “Spin-orbit-mediated spin relaxation in graphene,” *Phys. Rev. Lett.* Vol. 103, pp. 146801 (1-4), 2009.
- [4] W. H. Wang, K. Pi, Y. Li, Y.F. Chiang, P. Wei, J. Shi, and R.K. Kawakami, “Magnetotransport properties of mesoscopic graphite spin valves,” *Phys. Rev. B*, Vol. 77, pp. 020402 (1-4), 2008.
- [5] J. Maassen, W. Ji, and H. Guo, “Graphene spintronics: the role of ferromagnetic electrodes,” *Nano Lett.*, Vol. 11, pp. 151-155, 2011.
- [6] J. Csert, and G. David, “Unified description of Zitterbewegung for spintronic, graphene, and superconducting systems,” *Phys. Rev. B*, Vol. 74, 172305 [4 pages], 2006.
- [7] P. San-Jose, E. Prada, E. McCann, and H. Schomerus, “Pseudospin valve in bilayer graphene: towards graphene-based pseudospintronics,” *Phys. Rev. Lett.* Vol. 102, pp. 247204 (1-4), 2009.
- [8] S.C. Benjamin, A. Ardavan, G.A.D. Briggs, D.A. Britz, D. Gunlycke, J. Jefferson, M.A. G. Jones, D.F. Leigh, B.W. Lovett, A.N. Khlobystov, S.A. Lyon, J.J.L. Morton, K. Porfyakis, M.R. Sambrook, and A.M. Tyryshkin, “Towards a fullerene-based quantum computer,” *J. Phys.: Condens. Matter*, Vol. 18, pp. 867-878, 2006.
- [9] T.D. Ladd, F. Jelezko, R. Laflamme, Y. Nakamura, C. Monroe, and J.L. O’Brien, “Quantum computers,” *Nature*, Vol. 464, pp. 45-53, 2010.
- [10] W. Harneit, “Fullerene-based electron-spin quantum computer,” *Phys. Rev. A*, Vol. 65, pp. 032322 (1-6), 2002.
- [11] D.S. Naik, C.G. Peterson, A.G. White, A.J. Berglund, and P.G. Kwiat, “Entangled state quantum cryptography: eavesdropping on the Ekert protocol,” *Phys. Rev. Lett.* Vol. 84, pp. 4733-4736, 2000.
- [12] W.K. Wootters, “Entanglement of formation of an arbitrary state of two qubits,” *Phys. Rev. Lett.* Vol. 80, pp. 2245-2248, 1998.
- [13] Z. Ficek, “Quantum Entanglement processing with atoms,” *Appl. Math. Inf. Sci.*, Vol. 3, pp. 375-393, 2009.
- [14] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wootters, “Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels,” *Phys. Rev. Lett.* Vol. 70, pp. 1895-1899, 1993.
- [15] A.K. Ekert, “Quantum cryptography based on Bells’s theorem,” *Phys. Rev. Lett.* Vol. 67, pp. 661-663, 1991.

- [16] R. Safaiee, M.M. Golshan, N. Foroozani, "Temporal behavior of entanglement between electronic spin and subband states in a Rashba nanoloop," *J. Stat. Mech.* Vol. 2009, P11014, Nov. 2009.
- [17] L.C. Venuti, S.M. Giampaolo, F. Illuminati, and P. Zanardi, "Long-distance entanglement and quantum teleportation in XX spin chains," *Phys. Rev. A*, Vol. 76, pp. 052328 (1-9), 2007.
- [18] H.J. Briegel, W. Dür, J.I. Cirac, and P. Zoller, "Quantum repeaters: the role of imperfect local operations in quantum communication," *Phys. Rev. Lett.* Vol. 81, pp. 5932-5935, 1998.
- [19] S. Bandyopadhyay, "Teleportation and secret sharing with pure entangled states," *Phys. Rev. A*, Vol. 62, pp. 012308 (1-7), 2000.
- [20] D. Bouwmeester, A. Ekert, and A. Zeilinger, *The physics of quantum information*, Springer, Berlin, Chap. 8, 2000.
- [21] L.H. Sun, G.X. Li, and Z. Ficek, "Continuous variables approach to entanglement creation and processing," *Appl. Math. Inf. Sci.* Vol. 4, pp. 315-339, 2010.
- [22] Z. Papić, R. Thomale, and D.A. Abanin, "Tunable electron interactions and fractional quantum Hall states in graphene," *Phys. Rev. Lett.* Vol. 107, pp. 176602 (1-5), 2011.
- [23] M. Tokman, X. Yao, and A. Belyanin, "Generation of entangled photons in graphene in a strong magnetic field," *Phys. Rev. Lett.*, Vol. 110, pp. 077404 (1-5), 2013.
- [24] N. Shibata, and K. Nomura, "Coupled charge and valley excitations in graphene quantum Hall ferromagnets," *Phys. Rev. B*, Vol. 77, pp. 235426 (1-5), 2008.
- [25] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, "The electronic properties of graphene," *Rev. Mod. Phys.* Vol. 81, pp. 109-162, 2009.
- [26] M.Y. Han, B. Özyilmaz, Y. Zhang, and P. Kim, "Energy band-gap engineering of graphene nanoribbons," *Phys. Rev. Lett.* Vol. 98, pp. 206805 (1-4), 2007.
- [27] E. McCann, "Asymmetry gap in the electronic band structure of bilayer graphene," *Phys. Rev. B*, Vol. 74, pp. 161403 (1-4), 2006.
- [28] S.K. Goyal, and S. Ghosh, "Quantum-to-classical transition and entanglement sudden death in Gaussian states under local-heat-bath dynamics," *Phys. Rev. A*, Vol. 82, pp. 042337 (1-4), 2010.
- [29] V. Vedral, M.B. Plenio, M.A. Rippin, and P.L. Knight, "Quantifying entanglement," *Phys. Rev. Lett.* Vol. 78, pp. 2275-2279, 1997.
- [30] T.C. Wei, K. Nemoto, P.M. Goldbart, P.G. Kwiat, W.J. Munro, and F. Verstraete, "Maximal entanglement versus entropy for mixed quantum states," *Phys. Rev. A*, Vol. 67, pp. 022110 (1-12), 2003.
- [31] S.B. Zheng and G.C. Guo, "Efficient scheme for two-atom entanglement and quantum information processing in cavity QED," *Phys. Rev. Lett.* Vol. 85, pp. 2392-2395, 2000.
- [32] S. Reich, J. Maultzsch, and C. Thomsen, "Tight-binding description of graphene," *Phys. Rev. B*, Vol. 66, pp. 035412 (1-5), 2002.



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